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# Forward and Inverse Kinematic analysis of a spatial tensegrity mechanism with translational degrees of freedom

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**ABSTRACT**

Tensegrity mechanisms are novel type of mechanisms with amazing characteristics which make them an alternative to conventional mechanism for certain type of applications, such as pick and place application. In this paper, a spatial tensegrity mechanism is considered and its kinematics is studied in details. Due to the presence of compliant limbs, kinematic analysis is a challenging task. In this research, static equations are derived by minimizing the potential energy. In a special case, where gravitational and external forces aren't considered, an analytical solution for forward and inverse kinematics is obtained and some numerical examples are reported.

***Keywords:*** Forward and inverse kinematic, Tensegrity mechanism, Static analysis

## 1. INTRODUCTION

The word "tensegrity" which is a conjunction of the two words tension and integrity was introduced by R. B. Fuller [1]. With a general definition, tensegrity structures consist of some ties and struts which are in purely axial loads while ties are in tension and struts are in compression [2]. A detail review of tensegrity structures is given by Hernandez and Mirats-Tur [3].

In recent years, some researchers use the idea of tensegrity structures and proposed some new type of robots which are called tensegrity mechanism. This object can be achieved by adding some appropriate actuators in a system [4]. A number of advantages can be mentioned for tensegrity mechanism such as low mass, rapid movement and high resistance to weight ratio [5]. As a result, tensegrity mechanism can be considered as an optimum alternative for conventional robots.

The triangular tensegrity prism [6], which is consist of three struts and nine cables , is considered as the simplest form of spatial tensegrity systems. Since now some types of tensegrity prism have been proposed from triangular tensegrity prism.

Tran [7] used T-3 tensegrity prism and proposed a parallel device. Three of the side ties consist of non-elastic cables in series with elastic members. The length of noncompliant struts can be varied to control the shape of tensegrity mechanism.

Marshal [8] developed the mechanism by replacing top and bottom ties with platforms and struts with prismatic actuators. Length of each compliant member can be adjusted. Arsenault and Gosselin [9] introduced a spatial three degrees of freedom tensegrity mechanism. It contains nine tensile and three compressive components. Three of tensile components are cables and the remaining six tensile components are springs. The compressive components are replaced with prismatic actuators that are used to change the shape of the mechanism. Furthermore, Arsenault and Gosselin [10-11] studied two planar tensegrity mechanisms, and analyzed them.

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Shekarforoush [12] developed a 6-DOF tensegrity mechanism. It has six limbs connecting the moving platform to the fix base. Three of them are similar limbs consisting of a cylinder and a piston and the remaining are active compliant limbs. The mobility of the mechanism is achieved by three prismatic actuators and by adjusting the lengths of the active compliant components.

In recent years, some underactuated tensegrity mechanism are developed. Arsenault and Gosselin [13] proposed a new spatial mechanism. They have solved the direct and inverse problem of the mechanism and in a special case an analytical solution is proposed. Also, a realistic tensegrity based robot composed of 3-bar is presented by Mirats-Tur [14]. Furthermore, Ji etal [15] had studied the stiffness and dynamics of a planar class-2 tensegrity mechanism.

In this research, a particular mechanism from the family of triangular tensegrity prism is investigated. The three lateral ties in the conventional tensegrity prism T-3 are replaced by springs and the struts are replaced by hydraulic actuators. The length of three actuators can be modified to position the center of the platform in the cartesian space.

The main implication emerging from this research is to study the kinematic of an underactuated tensegrity mechanism. In contrast with conventional mechanisms, for the purpose of kinematic analysis, static equation must be incorporated which make the static analysis a complicated task. In this paper the kinematic of the considered mechanism is solved for the first time and in a special case, analytical solution is presented. An appropriate law for actuator forces is obtained in such a way that the mechanism follows a desired trajectory in its workspace.

## 2. GEOMETRY OF THE MECHANISM

A diagram of the mechanism discussed in this paper is shown in Fig. 1. The upper triangle is connected to the fixed base by six limbs. Three of the limbs, connecting nod pairs $b_i p_{i+1}$ are made up of a cylinder and a piston that are connected together by a prismatic joint (henceforth, $i = 1, 2, 3$ with $i + 1 = 1$ if $i = 3$). The movement of the moving platform is achieved by actuating these prismatic joints. The length of each hydraulic actuator is showed with $I_{pi}$.

The remanding limbs, joining node pairs $b_i p_i$ are similar springs whose lengths are $l_{si}$. Without lose of generality, it is assumed that springs have zero free length [16]. Also, as shown in the next sections, the strings are subjected to tension and the shape of triangle $p_1 p_2 p_3$ does not alter. Thus, it is feasible to replace it with a triangular plate.

**Figure 1: Schematic of the spatial mechanism**

The degree of freedom of a spatial mechanism can be predicted using Kutzbachs' equation. In this equation, the value of $J_i$ is the number of DOF joints and $L$ is number of links.

$$M = 6\,(L-1) - 5\,J_1 - 4\,J_2 - 3\,J_3 - 2\,J_4 - J_5 \tag{1}$$

The mechanism has 8 rigid bodies $L = 8$, three prismatic joint with 1 degree of freedom $J_1 = 1$, three universal joints with 2 degrees of freedom and three spherical joints with 3 degrees of freedom. Substituting these values in Eq. 3 it can be shown that the mechanism has six degrees of freedom. Nevertheless, three hydraulic actuators set the geometric center of triangle $p_1 p_2 p_3$ in a given position, in the workspace of the mechanism. For this reason, it is noted that the mechanism has three degrees of freedom. In this paper, the input vector, $l = [l_{p1}, l_{p2}, l_{p3}]$ actuate coordinates of the centroid of upper triangle $r = [x, y, z]^T$, which is controllable or output vector. Also, the orientation of moving platform can be expressed by three Euler angles φ, θ and ψ, which are considered as uncontrollable variables.

For the analysis of the mechanism, a reference frame $O(X_b, Y_b, Z_b)$ with origin $O$ is attached at the geometric center of $\Delta b_1 b_2 b_3$ where $X_b$-axis directed toward node $b_1$ and $Z_b$-axis perpendicular to plane defined by points $b_1$, $b_2$ and $b_3$, see Fig, 3. Moreover, a moving coordinate system $p(X_t, Y_t, Z_t)$ is located at the centroid of $\Delta p_1 p_2 p_3$. As shown in Fig. 3, $X_t$ and $Y_t$ axes lie in the plane containing the nodes $p_i$, also $Y_t$-axis pointing to the center point of $p_1 p_2$.

Let $\mathbf{r}_{bi} = \left[D\cos(\gamma_i), D\sin(\gamma_i), 0\right]^T$ be the position vector of nods $b_i$ in the reference coordinate system. Also, the vector $\mathbf{U}_{pi} = \left[d\cos(\gamma_i + \pi/6), d\sin(\gamma_i + \pi/6), 0\right]^T$ is the position of nods $p_i$ with respect to the geometric center of moving plate in the mobile reference frame. As shown in Fig. 2, the base and moving plate are equilateral triangles with $Ob_i = D$ and $pp_i = d$, respectively. Also, in these vectors $\gamma_i$, is considered as

$$\gamma_1 = 0, \gamma_2 = 2\pi/3, \gamma_3 = 4\pi/3 \tag{2}$$

In order to determine the orientation of hydraulic actuators, the unit vectors, $\mathbf{s}_{pi}$ are defined. This vector can be obtained as

$$\mathbf{s}_{pi} = (\mathbf{r}_{pi+1} - \mathbf{r}_{bi})/l_{pi} \tag{3}$$

Similarly, for three compliant limbs, $\mathbf{s}_{pi}$ is a unit vector pointing from $b_i$ to $p_i$.

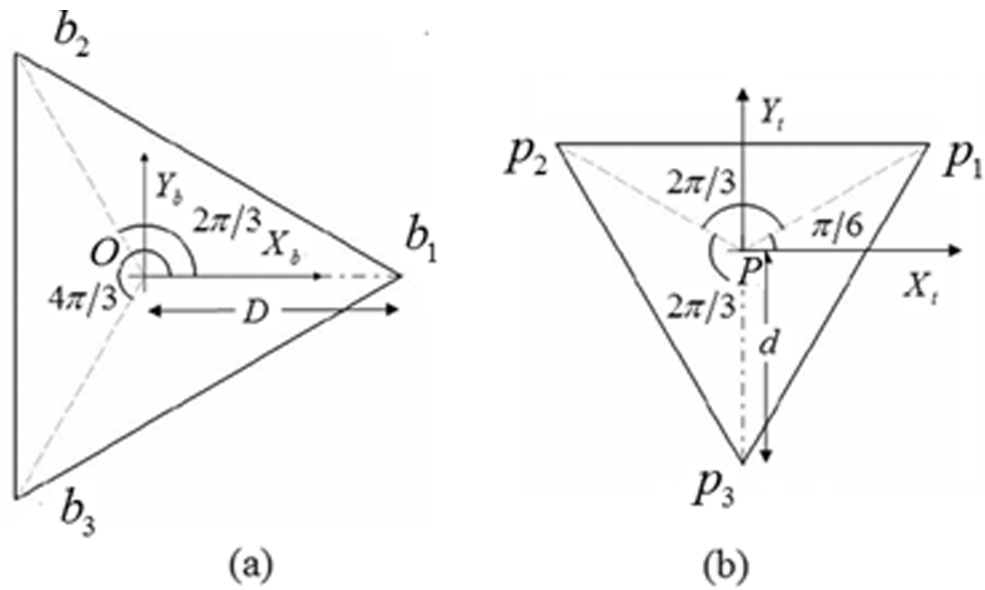


**Figure 2: Tensegrity mechanism: (a) base and (b) moving plate.**

$$\mathbf{s}_{si} = (\mathbf{r}_{pi} - \mathbf{r}_{bi}) / l_{si} \tag{4}$$

Also, the position vector of nodes $p_i$ in the reference frame can be written as

$$\mathbf{r}_{pi} = \mathbf{r} + \mathbf{A}\mathbf{U}_{pi} \tag{5}$$

In the above relation, for a 3-2-1 sequence of Euler angles, the transformation matrix from moving frame to the reference frame is

$$\mathbf{A} = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\phi + c_\psi s_\theta s_\phi & s_\psi s_\phi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{6}$$

## 3. STATIC ANALYSIS

### 3.1. Static analysis – general case

In this section, a relation between inputs and outputs variables is obtained. For this purpose, in addition to kinematic relations, the static equations must be incorporated because of the presence of spring in the structure of the mechanism.

In what follow, static equilibrium conditions by considering external forces are obtained by minimizing potential energy of the mechanism. For obtaining static equilibrium conditions, the length of spring limbs, are considered as the independent generalized coordinates. Using these variables, while gravitational forces are neglected, the potential energy of the mechanism is obtained as

$$V = \frac{1}{2}\sum_{i=1}^{3} k l_{si}^2 \tag{7}$$

Taking partial derivative of $V$, the equilibrium conditions for the tensegrity mechanism are obtained

$$\frac{\partial V}{\partial l_{si}} = k l_{si} = Q_i \tag{8}$$

In the above equation, the generalized forces due to the external forces, $Q_i$ can be obtained as

$$Q_i = \mathbf{F} \cdot \frac{1}{3}\sum_{i=1}^{3} \frac{\partial \mathbf{r}_{pi}}{\partial l_{si}} = \mathbf{F} \cdot \frac{\mathbf{s}_{si}}{3} \tag{9}$$

In the above equation, the external force, $\mathbf{F} = \left[F_1, F_2, F_3\right]^T$ is applied at the geometric center of moving plate. In sequence, by evaluating the value of $\mathbf{s}_{si}$ and $l_{si}$ in term of controllable and non-controllable variables, the static equations are rewritten

$$\begin{aligned} & x^2 + y^2 + z^2 + d^2 + D^2 - xD + d(x - D)(\mathrm{c}_\psi\, \mathrm{s}_\theta\, \mathrm{s}_\phi - \mathrm{s}_\psi\, \mathrm{c}_\theta) \\ & -\sqrt{3}d\, \mathrm{c}_\theta\, \mathrm{c}_\psi (x - D) + yd(\mathrm{s}_\psi\, \mathrm{s}_\theta\, \mathrm{s}_\phi + \mathrm{c}_\psi\, \mathrm{c}_\phi - \sqrt{3}\, \mathrm{s}_\psi\, \mathrm{s}_\theta) \\ & + zd(\mathrm{c}_\theta\, \mathrm{s}_\phi + \sqrt{3}\, \mathrm{s}_\theta) + \frac{\sqrt{3}}{2} d^2\, \mathrm{s}_\psi\, \mathrm{c}_\psi\, \mathrm{c}_\theta\, \mathrm{c}_\phi = Q_1 \end{aligned} \tag{10}$$

$$\begin{aligned} & x^2 + y^2 + z^2 + d^2 + D^2 + d(2x + D)(\mathrm{s}_\psi\, \mathrm{c}_\phi - \mathrm{c}_\psi\, \mathrm{s}_\theta\, \mathrm{s}_\phi) \\ & + xD - \sqrt{3}yd - 2yd(\mathrm{s}_\psi\, \mathrm{s}_\theta\, \mathrm{s}_\phi + \mathrm{c}_\psi\, \mathrm{c}_\phi) - 2zd\, \mathrm{c}_\theta\, \mathrm{s}_\phi = Q_2 \end{aligned} \tag{11}$$

$$x^2+y^2+z^2+d^2+D^2-xD+d(x-\frac{D}{2})(\mathrm{c}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+\sqrt{3}\,\mathrm{c}_\psi\,\mathrm{c}_\theta)$$

$$-d\,\mathrm{s}_\psi\,\mathrm{s}_\phi(x-\frac{D}{2})+yd(\mathrm{s}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+\mathrm{c}_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{s}_\psi\,\mathrm{c}_\theta)-\frac{\sqrt{3}}{2}yD$$

$$+\frac{zd}{2}(\mathrm{c}_\theta\,\mathrm{s}_\phi-\sqrt{3}\,\mathrm{s}_\theta)+\frac{\sqrt{3}}{4}d^2\,\mathrm{s}_\theta\,\mathrm{c}_\theta\,\mathrm{s}_\phi-\frac{Dd}{2}(3\,\mathrm{s}_\psi\,\mathrm{c}_\theta+\mathrm{c}_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{s}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)=Q_3 \tag{12}$$

where

$$Q_1=\frac{1}{3}F_1[x+\frac{D}{2}+\frac{\sqrt{3}d}{2}\mathrm{c}_\psi\,c_\theta+\frac{d}{2}(-s_\psi c_\phi+\mathrm{c}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)]+$$

$$\frac{1}{3}F_2[y+\frac{\sqrt{3}D}{2}+\frac{\sqrt{3}d}{2}s_\psi c_\theta+\frac{d}{2}(c_\psi c_\phi+s_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)]+$$

$$\frac{1}{3}F_3[z-\frac{\sqrt{3}d}{2}s_\theta+\frac{d}{2}c_\theta\,\mathrm{s}_\phi] \tag{13}$$

$$Q_2=\frac{1}{3}F_1[x+D-\frac{\sqrt{3}d}{2}\mathrm{c}_\psi\,c_\theta+\frac{d}{2}(-s_\psi c_\phi+\mathrm{c}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)]+$$

$$\frac{1}{3}F_2[y-\frac{\sqrt{3}d}{2}s_\psi c_\theta+\frac{d}{2}(c_\psi c_\phi+s_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)]+$$

$$\frac{1}{3}F_3[z+\frac{\sqrt{3}d}{2}s_\theta+\frac{d}{2}c_\theta\,\mathrm{s}_\phi] \tag{14}$$

$$Q_3=\frac{1}{3}F_1[x+\frac{D}{2}-d(-s_\psi c_\phi+c_\psi s_\theta s_\phi)]+$$

$$\frac{1}{3}F_2[y-\frac{\sqrt{3}D}{2}-d(c_\psi c_\phi+s_\psi s_\theta s_\phi)]+$$

$$\frac{1}{3}F_3[z-dc_\theta\,\mathrm{s}_\phi] \tag{15}$$

The above equations contain output and uncontrollable variables. For the inverse kinematic analysis, the output variables are considered known and the input variables and the length of springs are to be determined. The conditions expressed by Equations (10)-(12) are a system of three nonlinear equations in three unknowns. The uncontrollable variables can be obtained by numerical methods. In the next step, the input variables can be calculated as

$$l_{p_i}=\sqrt{(\mathbf{r}_{pi+1}-\mathbf{r}_{bi})\cdot(\mathbf{r}_{pi+1}-\mathbf{r}_{bi})}\quad i=1,\ 2,\ 3 \tag{16}$$

By using the above equation, the lengths of hydraulic actuators can be calculated as follow

$$x^2+y^2+z^2+d^2+D^2-2xD-xd(\mathrm{s}_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{c}_\psi\,\mathrm{c}_\theta)+$$

$$yd(\mathrm{s}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+\mathrm{c}_\psi\,\mathrm{c}_\phi-\sqrt{3}\,\mathrm{s}_\psi\,c_\theta)+zd(\mathrm{c}_\theta\,\mathrm{s}_\phi+\sqrt{3}\,\mathrm{s}_\theta)+$$

$$Dd(s_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{c}_\psi\,\mathrm{c}_\theta-\mathrm{c}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi)=l_{p1}^2 \tag{17}$$

$$x^2+y^2+z^2+d^2+D^2-xD+$$

$$\frac{d}{2}\mathrm{s}_\theta\,\mathrm{s}_\phi(x\,\mathrm{c}_\psi+y\,\mathrm{s}_\psi)+z\frac{\sqrt{3}}{2}d\,\mathrm{s}_\theta+z\frac{d}{2}c_\theta\,\mathrm{s}_\phi$$

$$-\frac{\sqrt{3}}{2}dc_\theta(xs_\psi+xc_\psi)+\frac{d}{2}c_\phi(y\,\mathrm{c}_\psi-x\,\mathrm{s}_\psi)=l_{p2}^2 \tag{18}$$

$$x^2+y^2+z^2+d^2+D^2+xd(c_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi-\mathrm{s}_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{c}_\psi\,\mathrm{c}_\theta)$$

$$xD+\sqrt{3}Dy+yd(\mathrm{s}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+\mathrm{c}_\psi\,\mathrm{c}_\phi+\sqrt{3}\,\mathrm{s}_\psi\,c_\theta)+$$

$$zd(\mathrm{c}_\theta\,\mathrm{s}_\phi-\sqrt{3}\,\mathrm{s}_\theta)+Dd(\frac{\sqrt{3}}{2}\mathrm{s}_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+\frac{1}{2}c_\psi\,\mathrm{s}_\theta\,\mathrm{s}_\phi+$$

$$+\frac{\sqrt{3}}{2}\mathrm{c}_\theta\,c_\psi+\frac{\sqrt{3}}{2}\mathrm{c}_\psi\,\mathrm{c}_\phi+\frac{3}{2}s_\psi c_\theta-\frac{1}{2}s_\psi c_\phi)=l_{p3}^2 \tag{19}$$

For the forward kinematic analysis, the input variables are given and the output variable and the lengths of springs are to be determined. The forward kinematic analysis of the introduced mechanism is more challenging in comparison with conventional mechanism. If the lengths of hydraulic actuators are given, the configuration of the mechanism cannot be specified. Therefore, geometric equations and static equations must be considered simultaneously. In this case, the static conditions expressed by Equations (10)-(12) are three equations in six unknowns.

The static equilibrium equations and the three geometric equations, Equations (17)-(19), yield a system of nonlinear equations which can be solved for the forward kinematic analysis.

### 3.2. Static analysis–special case

The obtained equilibrium equations, in the previous section, are highly nonlinear and must be solved by numerical methods. In this section, in a special case that gravitational and external forces are neglected, an analytical solution for the static analysis of the mechanism is presented.

By considering translatory motion for moving platform, the equilibrium equations for node $p_i$ are written. Figure (3) show the forces acting on node $p_i$. The summation of the forces in the ith joint is derived as follow

$$f_i\,\mathbf{s}_{pi}-k\,(l_{si+1}\mathbf{s}_{si+1})-T_i\mathbf{e}_i+T_{i+1}\mathbf{e}_{i+1}=\mathbf{0}\quad i=1,\ 2,\ 3 \tag{20}$$

In the above equation, the force in the ith hydraulic actuator is $f_i$. The force due to springs is $-k\,(l_{si}\mathbf{s}_{si})$. The sign for elastic forces are negative since, the force produced by spring is in the opposite direction of the unit vector $\mathbf{s}_{si}$. Also, $T_i$ is the tension force in cable jointing node pairs $p_i p_{i+1}$ and $\mathbf{e}_i$ is the unit vector pointing from $p_i$ to $p_{i+1}$. This unit vector can be obtained as

$$\mathbf{e}_i=(\mathbf{r}_{p_{i+1}}-\mathbf{r}_{pi})/2d\,\mathrm{s}_{\pi/3} \tag{21}$$

The above equation can be expanded in the X, Y and Z direction.

$$\frac{f_i}{l_{p_i}}(x+dc_{\gamma_{i+1}+\pi/6}-D\,\mathrm{c}_{\gamma_i})-K(x+d\,\mathrm{c}_{\gamma_{i+1}+\pi/6}-D\,\mathrm{c}_{\gamma_{i+1}})+$$

$$\left[T_{i+1}\,\mathrm{c}_{\gamma_{i+2}+\pi/6}-(T_{i+1}+T_i)\,\mathrm{c}_{\gamma_{i+1}+\pi/6}-T_i\,\mathrm{c}_{\gamma_i+\pi/6}\right]/\,2\,\mathrm{s}_{\pi/3}=0 \tag{22}$$

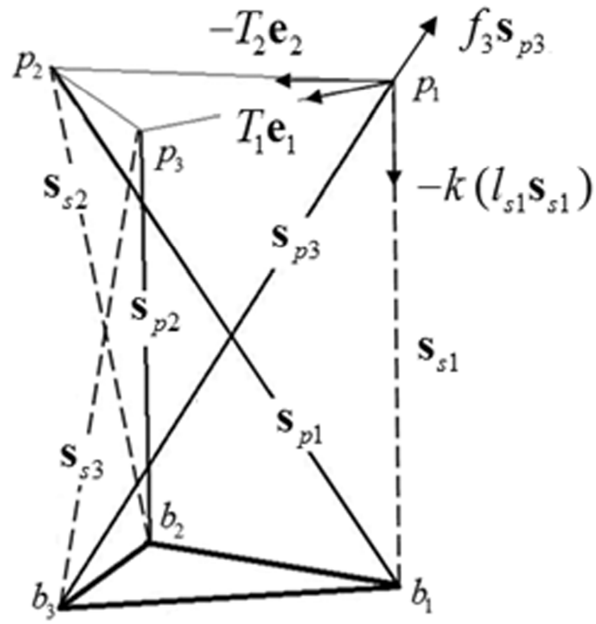


**Figure 3: Forces at nodes**

$$\frac{f_i}{l_{p_i}}(y+d\,\mathrm{s}_{\gamma_{i+1}+\pi/6}-D\,\mathrm{s}_{\gamma_i})-K(y+d\,\mathrm{s}_{\gamma_{i+1}+\pi/6}-D\,\mathrm{s}_{\gamma_{i+1}})+$$

$$\left[T_{i+1}\,\mathrm{s}_{\gamma_{i+2}+\pi/6}-(T_{i+1}+T_i)\,\mathrm{s}_{\gamma_{i+1}+\pi/6}-T_i\,\mathrm{s}_{\gamma_i+\pi/6}\right]/\,2\,\mathrm{s}_{\pi/3}=0 \tag{23}$$

$$f_i = kl_{pi} \tag{24}$$

The nonhomogeneous system of Equations, (22)-(24) can be written in the matrix form as

$$\mathbf{Ax}=\mathbf{b} \tag{25}$$

where

$$\mathbf{A}=\begin{bmatrix} -2\mathrm{d}\cos\pi/6 & 0 & -\mathrm{d}\cos\pi/6 \\ 0 & 0 & -\mathrm{d}(1+\sin\pi/6) \\ 2\mathrm{d}\cos\pi/6 & \mathrm{d}\cos\pi/6 & 0 \\ 0 & -\mathrm{d}(1+\sin\pi/6) & 0 \\ 0 & \mathrm{d}\cos\pi/6 & \mathrm{d}\cos\pi/6 \\ 0 & \mathrm{d}(1+\sin\pi/6) & \mathrm{d}(1+\sin\pi/6) \end{bmatrix} \tag{26}$$

and

$$\mathbf{b}=2\mathrm{KDd}\sin\frac{\pi}{3}\begin{bmatrix} -1-\cos\pi/3 \\ -\sin\pi/3 \\ 1+\cos\pi/3 \\ -\sin\pi/3 \\ 0 \\ 2\sin\pi/3 \end{bmatrix} \tag{27}$$

$$\mathbf{x} = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix}^T \tag{28}$$

The rank of matrix and the rank of augmented matrix (**A**, **b**) are identical and are three. So, the equations are consistent and the solution is unique. By performing finite sequences of elementary row operations, the above system of equations can be rewritten as

$$\begin{bmatrix} 0 & 0 & -1-\sin\frac{\pi}{6} \\ 2\cos\frac{\pi}{6} & \cos\frac{\pi}{6} & 0 \\ 0 & -1-\sin\frac{\pi}{6} & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 2KD\sin\frac{\pi}{3} \begin{bmatrix} -\sin\frac{\pi}{3} \\ 1+\cos\frac{\pi}{3} \\ -\sin\frac{\pi}{3} \end{bmatrix} \tag{29}$$

The above relation shows that in a special case that gravitational and external forces are not considered; the forces in hydraulic actuators can be calculated, for any set of output variables to pose the mechanism in equilibrium. In this case, an expression for the relations between input and output variables can be obtained as

$$\begin{aligned} l_{p_i}^2 &= x^2 + y^2 + z^2 + d^2 + D^2 + \sqrt{3}dD + \\ &2d(x\,\mathrm{c}_{\gamma_{i+1}+\pi/6} + y\,\mathrm{s}_{\gamma_{i+1}+\pi/6}) - 2D(x\,\mathrm{c}_{\gamma_i} + y\,\mathrm{s}_{\gamma_i}) \end{aligned} \tag{30}$$

## 4. SIMULATION AND RESULTS

In this section, some numerical examples are presented. The size of the upper and bottom triangle are $d = 0.2\ m$ and $D = 0.5\ m$, respectively. The spring constant which is identical for all limbs is determined by $k = 100\ N/m$. By using the static equations obtained in the previous section, movement of the mechanism in a quasi-static regime is studied. In the first simulation, the assumed trajectory for the center of mass of the mechanism is considered as

$$\mathbf{r} = \left[0.1\sin(2\pi t) + 0.2\cos(2\pi t), 0.1\cos(2\pi t) + 0.2\sin(2\pi t), 0.4\right]^T \tag{31}$$

The actuator forces are calculated from Eqs. (24) and (30) and they are plotted in Figure (4). In the next example, the trajectory is selected as

$$\mathbf{r} = \left[0, 0.1\cos(2\pi t), 0.7\right]^T \tag{32}$$

Similarly, the actuators forces are plotted in Figure (5). It is worth mentioning that one of the most challenging problems in designing tensegrity mechanisms is the assumption of tension in cables. In the proposed mechanism, the upper cables are in tension in the workspace of the mechanism. Also, tension forces are depends on the geometry of the mechanism.

## 5. CONCLUSIONS

Tensegrity concept has opened the doors to new technology for designing and construction novel mechanism. Tensegrity mechanisms are deformable and have a very high resistance to weight coefficient. Also, by using cables and springs in the structure of these robots the inertia of moving parts are reduced.

In parallel tensegrity mechanism, each actuator can affect all degrees of freedom. Thus, shape change and movement of the mechanism can be achieved with small number of actuators. In this regard, an underactuated tensegrity mechanism with three hydraulic actuators is studied. Kinematic and static analysis is done to find how to change the lengths of actuators to generate any arbitrary movement in the workspace of the mechanism.

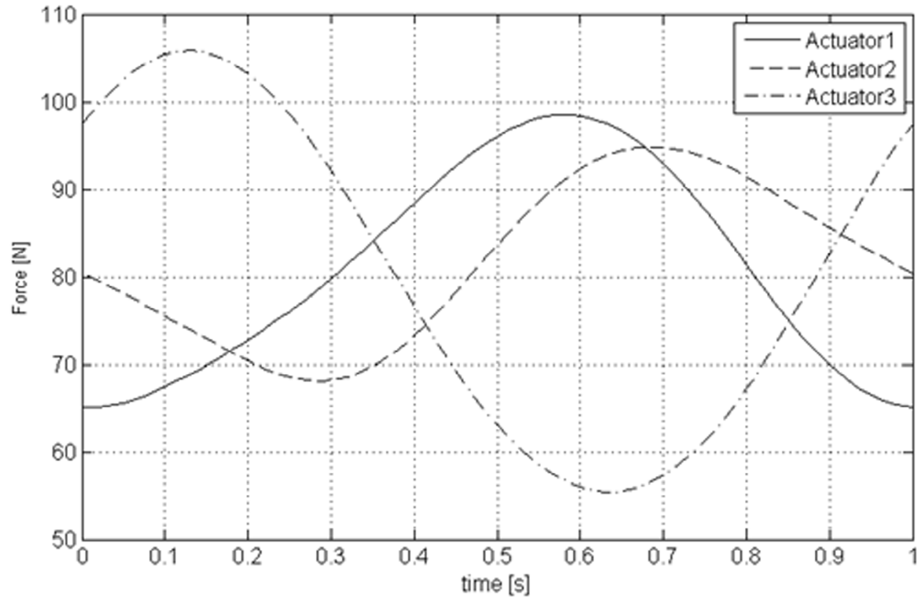


**Figure 4: Forces in the actuators (Example 1)**

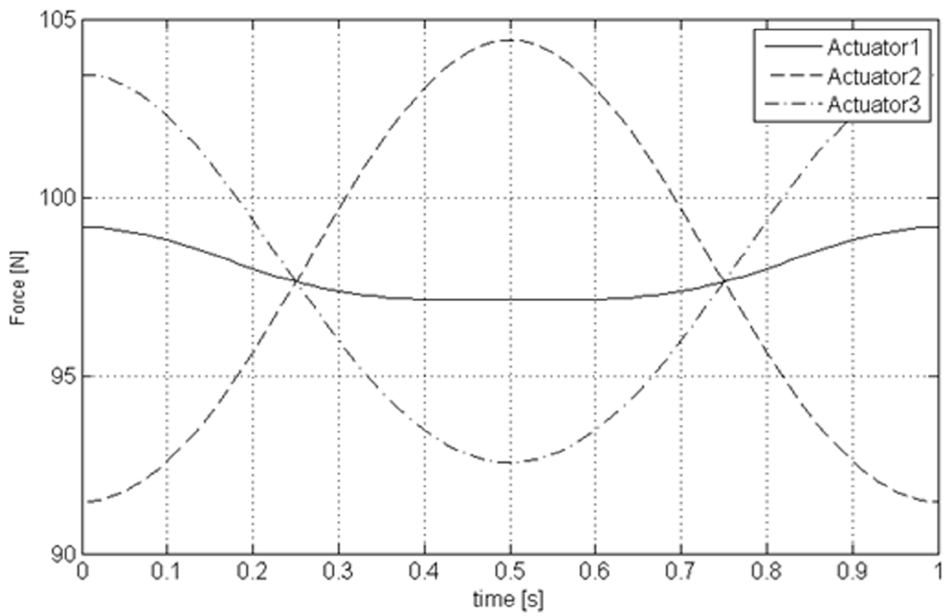


**Figure 5: Forces in the actuators (Example 2)**

For given length of actuators, external forces cause change in the structure of tensegrity mechanism. Thus, kinematic and static equations must be considered simultaneously. In this research, static equilibrium conditions are obtained by minimizing the potential energy in the presence of external forces and forward and inverse kinematic is studied. Then, it was illustrated that for translatory motion of moving plate the static equations can be satisfied while gravitational and external forces are not considered. In this special case, a simple relation between input and output variables is obtained.

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