αGR-CLOSED SETS AND αGR-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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ABSTRACT: *The aim of this paper is to introduce the new classes of sets, (τ_1, τ_2)-αgr-losed set, (τ_1, τ_2)-αgr-open set, (τ_1, τ_2)-αgr-$T_{1/2}$ space, (τ_1, τ_2)-αg-closed set, (τ_1, τ_2)-αg-closed set, (τ_1, τ_2)-αgpr-closed set, (τ_1, τ_2)-αgr continuous function and to study some of their basic properties among themselves and with other known concepts.*

Keywords and Phrases: (τ_1, τ_2)-αgrc-*set,* (τ_1, τ_2)-$grT_{1/2}$ *spaces,* (τ_1, τ_2)-αgc *sets,* (τ_1, τ_2)-*gprc sets,* (τ_1, τ_2)-αgrc-*continuous functions and* (τ_1, τ_2)-$\alpha gprc$-*continuous functions*

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1. INTRODUCTION AND PRELIMINARIES

The concept of bitopological space was first introduced by Kelly [5] in 1963. Levine [6], Njastad [11] and Mashhour *et al.* [10] introduced g-closed sets, α-open sets and preopen sets, respectively. Palaniappan and Rao [12] introduced regular generalized closed sets. Maki *et al.* [9] introduced and studied genealised α-closed sets. Recently Veera Kumar [15] introduced α-generalized closed sets, α-generalized regular $T_{1/2}$-spaces and characterized them. In this paper we try to extend them in bitopological spaces and study some of their basic properties and their relationship with other known axioms. The space (X, τ_1, α_2) means a bitopological space in which no separation axioms are assumed unless otherwise explicitly stated.

Definition 1: A subset A of a topological space (X, τ) is said to be:

(i) α-open [11] if $A \subset Int(Cl(Int(A)))$ and α-closed if $Cl(Int(Cl(A))) \subset$ A,

(ii) preopen [10] if $A \subset Int(Cl(A))$ and preclosed if $Cl(Int(A)) \subset$ A,

(iii) semi-preopen [1] if $A \subset Cl(Int(Cl(A)))$ and semi-preclosed if $Int(Cl(Int(A))) \subset A$,

(iv) regular open if $A = Int(Cl(A))$ and regular closed if $A = Cl(Int(A))$.

The α-closure (resp. preclosure, semi-pre-closure, regular closure) of A in (X, τ) is the intersection of all α-closed (resp. preclosed, semi-preclosed, regular closed) sets of (X, τ) that contains A and is denoted by $\alpha Cl(A)$ (resp. $pCl(A)$, $spCl(A)$, $rCl(A)$).

The union of all α-open sets of (X, τ) contained in A is called the α-interior of A and is denoted by $\alpha Int(A)$.

2. (τ_1, τ_2)-αGR-CLOSED SETS

Definition 2: A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-α-generalized regular closed set (briefly, (τ_1, τ_2)-τgr-closed set) if τ_2-$Cl(A) \subset U$ whenever $A \subset U$ and U is τ_1-regular open.

Definition 3: [3]. A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-generalized closed set (briefly, (τ_1, τ_2)-g-closed) if τ_2-$Cl(A) \subset U$ whenever $A \subset U$ and U is τ_1-open.

Definition 4: A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-α-generalized closed set (briefly, (τ_1, τ_2)-αg-closed) if τ_2-$Cl(A) \subset U$ whenever $A \subset U$ and U is τ_1-open.

Definition 5: A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-generalized α-closed set (briefly, (τ_1, τ_2)-αg-closed) if τ_2-$Cl(A) \subset U$ whenever $A \subset U$ and U is τ_1-α-open.

Definition 6: [13] A subset A of a bitopological space (X, τ_1, τ_2) is called a

(i) (τ_1, τ_2)-regular-generalized closed set (briefly, (τ_1, τ_2)-$r\hat{g}$-closed) if τ_2-$Cl(A) \subset U$ whenever $A \subset U$ and U is τ_1-regular open.

(ii) (τ_1, τ_2)-regular-generalized star closed set (briefly, (τ_1, τ_2)-rg^*-closed) if τ_2-$rCl(A)$ $\subset$ U whenever $A \subset U$ and U is τ_1-regular open.

(iii) (τ_1, τ_2)-regular-generalized star regular closed set (briefly, (τ_1, τ_2)-g^*r-closed) if τ_2-$rCl(A) \subset U$ whenever $A \subset U$ and U is τ_1-open.

(iv) (τ_1, τ_2)-regular-generalized pre-regular closed set (briefly, (τ_1, τ_2)-gpr-closed) if τ_2-$pCl(A) \subset U$ whenever $A \subset U$ and U is τ_1-regular open.

(v) (τ_1, τ_2)-generalized preclosed (briefly, (τ_1, τ_2)-gp-closed) if τ_2-$pCl(A) \subset$ U whenever $A \subset U$ and U is τ_1-open.

Remark 2.1: The collection of all (τ_1, τ_2)-αgr-closed (resp. (τ_1, τ_2)-$g\alpha$-closed, $((\tau_1, \tau_2)$-g-closed, (τ_1, τ_2)-$r\hat{g}$-closed, (τ_1, τ_2)-rg^*-closed, (τ_1, τ_2)-g^*r-closed, (τ_1, τ_2)-gpr-closed, (τ_1, τ_2)-gp-closed) sets of (X, τ_1, τ_2) will be denoted by $\alpha GRC(\tau_1, \tau_2)$ (resp. $GC(\tau_1, \tau_2)$, $GC(\tau_1, \tau_2)$, $R\hat{G}C(\tau_1, \tau_2)$, $RG^*C(\tau_1, \tau_2)$, $G^*RC(\tau_1, \tau_2)$, $GPRC(\tau_1, \tau_2)$, $GPC(\tau_1, \tau_2)$).

Proposition 2.2: If A is a τ_2-closed subset of a bitopological space, then A is (τ_1, τ_2)-g-closed and (τ_1, τ_2)-$g\alpha$-closed.

Proposition 2.3: Let A be a subset of a bitopological space (X, τ_1, τ_2). If A is (τ_1, τ_2)-αg-closed, then A is (τ_1, τ_2)-αgr-closed.

Proof: Let U be a τ_1-regular open set such that $A \subset U$. Since every regular open set is open, by given hypothesis τ_2-$Cl(A) \subset U$. Hence A is (τ_1, τ_2)-αgr-closed.

The following example shows that the converse of Proposition 2.3 is need not be true.

Example 2.4: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. Then $\alpha GRC(\tau_1, \tau_2) = P(X) - \{c\}$, $\alpha GC(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\} = GC(\tau_1, \tau_2)$, $GPRC(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$, $GC(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$. Consider a set $A = \{a, b\}$. Then A is a (τ_1, τ_2)-αgr-closed set but not a (τ_1, τ_2)-αg-closed set.

Proposition 2.5: Let A be a subset of a bitopological space (X, τ_1, τ_2).

(i) If $A \in GC(\tau_1, \tau_2)$, then $A \in GC(\tau_1, \tau_2)$.

(ii) If $A \in GC(\tau_1, \tau_2)$, then $A \in GC(\tau_1, \tau_2)$.

Proof:

(i) Suppose that A is a (τ_1, τ_2)-g-closed set. Let U be a τ_1-open set such that $A \subset U$. By given hypothesis, τ_2-$Cl(A) \subset U$. From τ_2-$\alpha Cl(A) \subset \tau_2$-$Cl(A)$, it follows that τ_2-$\alpha Cl(A)) \subset U$ and A is (τ_1, τ_2)-αg-closed.

(ii) Let U be a τ_1-open set such that $A \subset U$. Since every open set is α-open and A is (τ_1, τ_2)-$g\alpha$-closed, τ_2-$\alpha Cl(A) \subset U$. Then A is (τ_1, τ_2)-αg-closed.

The converse statements of Proposition 2.5 need not be true as can be seen from the following example.

Example 2.6: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then $\alpha GRC(\tau_1, \tau_2) = P(X) - \{b\}$, $\alpha GC(\tau_1, \tau_2) = P(X) - \{\{b\}, \{a, b\}\}$ and $G\alpha C(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. Consider a set $A = \{a, b\}$. Then A is a (τ_1, τ_2)-αgr-closed set but it is neither (τ_1, τ_2)-gr-closed nor (τ_1, τ_2)-g-closed set.

Proposition 2.7: Let A be a subset of a bitopological space (X, τ_1, τ_2).

(i) If $A \in \alpha GRC(\tau_1, \tau_2)$, then $A \in GPRC(\tau_1, \tau_2)$.

(ii) If $A \in \alpha GC(\tau_1, \tau_2)$, then $A \in GPC(\tau_1, \tau_2)$.

(iii) If A $\in GPC(\tau_1, \tau_2)$, then $A \in GPRC(\tau_1, \tau_2)$.

Proof: (i) Suppose that A is a (τ_1, τ_2)-αgr-closed set in (X, τ_1, τ_2). Let U be a τ_1-regular open set such that $A \subset U$, then τ_2-$Cl(A) \subset U$. Since τ_2-$pCl(A)$ τ_2-$Cl(A)$, it follows that τ_2-$pCl(A) \subset U$. Then A is (τ_1, τ_2)-gpr-closed.

(ii) and (iii) are similar and hence omitted.

The following example shows that the converses of the above proposition is not true in general.

Example 2.8: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Then $\alpha GRC(\tau_1, \tau_2) = \alpha GC(\tau_1, \tau_2) = G\alpha C(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $GPRC(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. Consider the set $A = \{c\}$. Then $A \in GPRC(\tau_1, \tau_2)$, but it is not in any one the collections $\alpha GRC(\tau_1, \tau_2)$, $\alpha GC(\tau_1, \tau_2)$ and $G\alpha C(\tau_1, \tau_2)$.

Proposition 2.9: Let A be a subset of a bitopological space (X, τ_1, τ_2).

(i) If $A \in R\hat{G}C(\tau_1, \tau_2)$, then $A \in \alpha GRC(\tau_1, \tau_2)$.

(ii) If $A \in RG^*C(\tau_1, \tau_2)$, then $A \in R\hat{G}C(\tau_1, \tau_2)$.

(iii) If $A \in G^*RC(\tau_1, \tau_2)$, then $A \in RG^*C(\tau_1, \tau_2)$.

(iv) If $A \in RG^*C(\tau_1, \tau_2)$, then $A \in GC(\tau_1, \tau_2)$.

Proof: Let U be a τ_1-regular open set such that $A \subset U$. Since A is (τ_1, τ_2)-$r\hat{g}$-closed, τ_2-$Cl(A) \subset U$. Since τ_2-$\alpha Cl(A)$ τ_2-$Cl(A)$, we have τ_2-$Cl(A) \subset U$. Therefore, $A \in \alpha GRC(\tau_1, \tau_2)$.

(ii) (iii) and (iv) are easy and hence omitted.

The converse statements of Proposition 2.9 are not true as can be seen from the following example.

Example 2.10: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Then $R\hat{G}C(\tau_1, \tau_2) = \alpha GRC(\tau_1, \tau_2) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $RG^*C(\tau_1, \tau_2) = G^*RC(\tau_1, \tau_2) = \{\emptyset, \{a, b\}, \{a, c\}, X\}$. Consider the set $A = \{a\}$. Then A is a (τ_1, τ_2)-αgr-closed set, but it is neither a (τ_1, τ_2)-rg^*-closed set nor a (τ_1, τ_2)-g^*r-closed set.

From propositions 2.2, 2.3, 2.5, 2.7, and 2.9 we have the following diagram:

$$\begin{array}{ccccccc}
 & & \tau_2\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}g\alpha\text{-}closed & & \\
 & & \downarrow & & \downarrow & & \\
(\tau_1,\tau_2)\text{-}g^*r\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}g\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}\alpha g\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}gp\text{-}closed \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
(\tau_1,\tau_2)\text{-}rg^*\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}r\breve{g}\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}\alpha gr\text{-}closed & \rightarrow & (\tau_1,\tau_2)\text{-}gpr\text{-}closed
\end{array}$$

Proposition 2.11: Let A be a subset of a bitopological space (X, τ_1, τ_2).

(i) If $A \in \alpha GRC(\tau_1, \tau_2)$, then τ_2-$Cl(A)$–A does not contain any non-empty τ_1-regular closed set.

(ii) If $A \in GC(\tau_1, \tau_2)$, then τ_2-$\alpha Cl(A)$–A does not contain any non-empty τ_1-closed set.

(iii) If $A \in G\alpha C(\tau_1, \tau_2)$, then τ_2-$Cl(A) - A$ does not contain any non-empty τ_1-α-closed set.

Proof: (i) Suppose that A is a (τ_1, τ_2)-αgr-closed set in (X, τ_1, τ_2). Let F be a τ_1-regular closed set such that $F \subset \tau_2$-$\alpha Cl(A) - A$. Since A is (τ_1, τ_2)-αgr-closed, $A \subset F^c$ and we have τ_2-$\alpha Cl(A) \subset F^c$. Consequently $F \subset \alpha\tau_2$-$\alpha Cl(A) \cap (\tau_2$-$\alpha Cl(A))^c = Ø$.

(ii) and (iii) are similar and hence omitted.

The following Example shows that the converses of the above proposition is not true in general.

Example 2.12: Consider the bitopological space described in the Example 2.4.

Here $\alpha GRC(\tau_1, \tau_2) = P(X) - \{c\}$, $\alpha GC(\tau_1, \tau_2) = \{Ø, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $G\alpha C(\tau_1, \tau_2) = \{Ø, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$,

Consider the set $A = \{c\}$ which does not it belong to any one the collections $\alpha GRC(\tau_1, \tau_2)$, $\alpha GC(\tau_1, \tau_2)$ and $G\alpha C(\tau_1, \tau_2)$. Where as τ_2-$\alpha Cl(A)$ – A does not contain any non-empty τ_1-regular closed set.

τ_2-$Cl(A)$ – A does not contain any non-empty τ_1-open set.

τ_2-$Cl(A)$ – A does not contain any non-empty τ_1-α-open set.

Proposition 2.13: Let A and B be any two subsets of a bitopological space (X, τ_1, τ_2). If $A, B \in \alpha GRC(\tau_1, \tau_2)$ then $A \cup B \in \alpha GRC(\tau_1, \tau_2)$.

Proof: Let U be a τ_1-regular open set such that $A \cup B \subset U$. Consequently $A \subset G$ and $B \subset G$. Since $A, B \in \alpha GRC(\tau_1, \tau_2)$, we have τ_2-$Cl(A) \subset U$ and τ_2-$Cl(B) \subset U$. Therefore τ_2-$\alpha Cl(A) \cup \tau_2$-$\alpha Cl(B) \subset U$, which implies that τ_2-$\alpha Cl(ASB) \subset U$. Then $A \cup B \in GRC(\tau_1, \tau_2)$.

Remark 2.14: In the Example 2.12 we see that the intersection of two (τ_1, τ_2)-αgr-closed sets need not be a (τ_1, τ_2)-αgr-closed set in general.

Theorem 2.15: Let A be a (τ_1, τ_2)-αgr-closed set in a bitopological space (X, τ_1, τ_2). If $B \subset X$ such that $A \subset B \subset \tau_2$-$\alpha Cl(A)$, then B is also (τ_1, τ_2)-αgr-closed in (X, τ_1, τ_2).

Proof: Let U be a τ_1-regular open set such that $B \subset U$. Since A is (τ_1, τ_2)-αgr-closed and $A \subset U$, we have τ_2-$\alpha Cl(A) \subset U$. From $B \subset \tau_2$-$\alpha Cl(A)$, we have τ_2-$\alpha Cl(B) \subset \tau_2$-$\alpha Cl(A)$ and hence τ_2-$\alpha Cl(B) \subset U$. Therefore B is (τ_1, τ_2)-αgr-closed.

3. (τ_1, τ_2)-α*GR*-OPEN SETS

Definition 7: A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-α-generalized regular open (briefly, (τ_1, τ_2)-αgr-open) set if its complement $X - A$ is a (τ_1, τ_2)-αgr-closed set in (X, τ_1, τ_2).

Theorem 3.1: A subset A of (X, τ_1, τ_2) is (τ_1, τ_2)-αgr-open if and only if $F \subset \tau_2$-$\alpha Int(A)$ whenever F is τ_1-regular closed and $F \subset A$.

Proof: Suppose that $F \subset \tau_2$-$\alpha Int(A)$ whenever F is τ_1-regular closed such that $F \subset A$. Let G be a τ_1-regular open set in X such that $(X - A) \subset G$. Then $(X - G) \subset A$ and $X - G$ is τ_1-regular closed. Therefore by hypothesis $(X - G)$ τ_2-$Int(A)$, then $X - (\tau_2$-$\alpha Int(A))$ $\subset G$. But $X - (\tau_2$-$Int(A)) = \tau_2$-$Cl(X - A)$. Thus τ_2-$Cl(X - A) \subset G$ whenever $(X - A) \subset G$ and G is τ_1-regular open. Then $X - A$ is (τ_1, τ_2)-αgr-closed and hence A is (τ_1, τ_2)-αgr-open. Conversely Suppose that A is (τ_1, τ_2)-αgr-open. Let F be a τ_1-regular closed set such that $F \subset A$. By the Definition 4 and since $(X - A)$ is (τ_1, τ_2)-αgr-closed, $(X - A)$ $\subset (X - F)$ and $(X - F)$ is τ_1-regular open, we have (τ_1, τ_2)-$\alpha Cl(X - A) \subset (X - F)$. But $X - (\tau_2$-$\alpha Int(A)) = \tau_2$-$\alpha Cl(X - A) \subset (X - F)$. Therefore $F \subset \tau_2$-$\alpha Int(A)$.

Theorem 3.2: Let A and B be any two subsets of a bitopological space (X, τ_1, τ_2) such that is τ_2-$\alpha Int(A) \subset B \subset A$ and A is (τ_1, τ_2)-αgr-open, then B is (τ_1, τ_2)-αgr-open.

Proof: From τ_2-$\alpha Int(A) \subset B \subset A$, we have $(X - A) \subset (X - B)$ $X - (\tau_2$-$\alpha Int(A))$, that is $(X - A) \subset (X - B)$ τ_2-$\alpha Cl(X - A)$. Since $(X - A)$ is (τ_1, τ_2)-αgr-closed, by Theorem 2.15, if follows that $(X - B)$ is (τ_1, τ_2)-αgr-closed and hence B is (τ_1, τ_2)-αgr-open.

Theorem 3.3: Let A be a subset of a bitopological space (X, τ_1, τ_2). If A is (τ_1, τ_2)-αgr-closed then $(\tau_2$-$\alpha Cl(A) - A)$ is (τ_1, τ_2)-αgr-open.

Proof: Suppose that A is a (τ_1, τ_2)-αgr-closed set. Let F be a τ_1-regular closed set such that F τ_2-$Cl(A) - A$. Since A is (τ_1, τ_2)-αgr-closed, by Theorem 2.11, τ_2-$\alpha Cl(A) - A$ does not contain any non-empty τ_1-regular closed set. Thus $F = \emptyset$. Then $F \subset \tau_2$- $\alpha Int(\tau_2$-$\alpha Cl(A) - A)$. Therefore by Theorem 3.1, $(\tau_2$-$\alpha Cl(A) - A)$ is (τ_1, τ_2)-αgr-open.

4. (τ_1, τ_2)-*GR*-$T_{1/2}$ SPACES

Definition 8: A bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-αgr-$T_{1/2}$ space if every (τ_1, τ_2)-αgr-closed set is a τ_2-α-closed set.

Theorem 4.1: Let (X, τ_1, τ_2) be a bitopological space. Then the following are equivalent:

(i) (X, τ_1, τ_2) is a (τ_1, τ_2)-αgr-$T_{1/2}$ space

(ii) For each $x \in X$, $\{x\}$ is either τ_1-regular closed or τ_2-α-open.

Proof:

(i) $\rightarrow$ (ii): Let $x \in X$. Suppose $\{x\}$ is not τ_1-regular closed. Then $X - \{x\}$ is not a τ_1-regular open set and a τ_1-regular open set containing $X - \{x\}$ is X only. From τ_2-$\alpha Cl(X - \{x\}) \subset X$, it follows that $X - \{x\}$ is (τ_1, τ_2)-αgr-closed. Since (X, τ_1, τ_2) is a (τ_1, τ_2)-αgr-$T_{1/2}$ space, $X - \{x\}$ is τ_2-α-closed and hence $\{x\}$ is τ_2-α-open.

(ii) $\rightarrow$ (i): Let F be (τ_1, τ_2)-αgr-closed. Suppose $\{x\}$ is τ_2-α-open or τ_1-regular closed for any $x \in \tau_2$-$\alpha Cl(F)$.

Case (1): Suppose $\{x\}$ is τ_2-α-open. Since $x \in \tau_2$-$\alpha Cl(F)$, $\{x\} \cap \tau_2$-$\alpha Cl(F) \neq \emptyset$. Hence $x \in F$.

Case(2): Suppose $\{x\}$ is τ_1-regular closed. If $\{x\} \neq F$, then $\{x\} \in \tau_2$-$\alpha Cl(F) - F$, that is τ_2-$\alpha Cl(F) - F$ contains a non-empty τ_1-regular closed set $\{x\}$ which is a contraction due to Proposition 2.11 and F is (τ_1, τ_2)-αgr-closed. Therefore our assumption is wrong and hence $x \in F$. From the above two cases, we conclude that $x \in F$ for any $x \in \tau_2$-$\alpha Cl(F)$ and hence F is τ_2-α-closed.

Definition 9: [7] A bitopological space (X, τ_1, τ_2) is pairwise semi-T_0 if for each pair of distinct points of X there is a set which is either τ_1-semi-open or τ_2-semi-open containing one of the points but not the other.

Theorem 4.2: If a bitopological space (X, τ_1, τ_2) is (τ_1, τ_2)-αgr-$T_{1/2}$, then it is pairwise semi-T_0.

Proof: This is obvious by the definitions.

5. αGR-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

Definition 10: [15] A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called an gr-continuous if $f^{-1}(V)$ is αgr-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 11: A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be:

(1) (τ_1, τ_2)-σ_k-αgr-continuous if the inverse image of each σ_k-closed set is (τ_1, τ_2)-αgr-closed,

(2) (τ_1, τ_2)-σ_k-αg-continuous if the inverse image of each τ_k-closed set is (τ_1, τ_2)-αg-closed,

(3) (τ_1, τ_2)-α_k-g-continuous if the inverse image of each α_k-closed set is (τ_1, τ_2)-g-closed,

(4) (τ_1, τ_2)-α_k-gpr-continuous if the inverse image of each α_k-closed set is (τ_1, τ_2)-gpr-closed.

Theorem 5.1: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (τ_1, τ_2)-σ_k-αg-continuous, then f is (τ_1, τ_2)-σ_k-αgr-continuous.

Proof: Let F be a α_k-closed set in (Y, σ_1, σ_2). Since f is (τ_1, τ_2)-σ_k-αg-continuous, $f^{-1}(F)$ is (τ_1, τ_2)-αg-closed in (X, τ_1, τ_2). By Proposition 2.3, $f^{-1}(F)$ is (τ_1, τ_2)-αgr-closed in (X, τ_1, τ_2). Hence f is (τ_1, τ_2)-σ_k-αgr-continuous.

The converse of Theorem 5.1 is not true as can be seen from the following example.

Example 5.2: Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$, $\sigma_1 = \{\emptyset, \{a\}, \{b, c\}, Y\}$, $\sigma_2 = \{\emptyset, \{a\}, \{a, c\}, Y\}$. Here $\alpha GRC(\tau_1, \tau_2) = P(X) - \{b\}$, $\alpha GC(\tau_1, \tau_2) = P(X) - \{\{b\}, \{a, b\}\}$. Define a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is (τ_1, τ_2)-σ_k-αgr-continuous. But f is not (τ_1, τ_2)-σ_k-αg-continuous.

Theorem 5.3: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (τ_1, τ_2)-σ_k-αgr-continuous, then f is (τ_1, τ_2)-σ_k-gpr-continuous.

Proof: Let V be a σ_k-closed set in (Y, σ_1, σ_2). Since f is (τ_1, τ_2)-σ_k-αgr-continuous, $f^{-1}(V)$ is (τ_1, τ_2)-σ_k-αgr-closed in (X, τ_1, τ_2). By Proposition 2.7(i), $f^{-1}(V)$ is (τ_1, τ_2)-σ_k-gpr-closed set in (X, τ_1, τ_2). Hence f is (τ_1, τ_2)-σ_k-gpr-continuous.

From the following example, it is clear that the converse of Theorem 5.3 is not true.

Example 5.4: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b, c\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$, $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{r\}, Y\}$, and $\sigma_2 = \{\emptyset, \{p\}, \{p, q\}, Y\}$. Define a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = q$, $f(b) = p$ and $f(c) = r$. Then f is (τ_1, τ_2)-σ_k-gpr-continuous. But f is not (τ_1, τ_2)-σ_k-αgr-continuous.

Corollary 5.5: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be (τ_1, τ_2)-σ_k-αg-continuous, then f is (τ_1, τ_2)-σ_k-gpr-continuous.

Proof: This follows from Theorems 5.1 and 5.3.

Theorem 5.6: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (τ_1, τ_2)-σ_k-g-continuous, then f is (τ_1, τ_2)-σ_k-αg-continuous.

Proof: Let V be a σ_k-closed set in (Y, σ_1, σ_2). Since f is (τ_1, τ_2)-σ_k-g-continuous, $f^{-1}(V)$ is (τ_1, τ_2)-g-closed in (X, τ_1, τ_2). By Proposition 2.5(i), $f^{-1}(V)$ is a (τ_1, τ_2)-αg-closed set in (X, τ_1, τ_2). Then f is (τ_1, τ_2)-σ_k-αg-continuous.

The following example shows that the converse of Theorem 5.6 need not be true.

Example 5.7: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}, X\}$. Here $\alpha GC(\tau_1, \tau_2) = P(X) - \{b\}$ and $GPRC(\tau_1, \tau_2) = P(X)$. Let $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{q, r\}, Y\}$, and $\sigma_2 = \{\emptyset, \{q\}, \{p, r\}, Y\}$. Define a function $f : (X, \tau_1, \tau_2) \to$

(Y, σ_1, σ_2) by $f(a) = q, f(b) = p$ and $f(c) = r$. Then f is (τ_1, τ_2)-σ_k-gpr-continuous. But f is not (τ_1, τ_2)-σ_k-g-continuous.

Corollary 5.8: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an (τ_1, τ_2)-σ_k-g-continuous, then f is an (τ_1, τ_2)-σ_k-gpr-continuous.

Proof: This follows from Theorems 5.3 and 5.6.

Definition 12: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be τ_2-σ_k-continuous if the inverse image of every σ_k-closed set is a τ_2-closed set.

Proposition 5.9: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is τ_2-σ_k-continuous, then f is (τ_1, τ_2)-σ_k-g-continuous.

Proof: Let V be a σ_k-closed set. Since f is τ_2-σ_k-continuous, $f^{-1}(V)$ is τ_2-closed. By Proposition 2.2, $f^{-1}(V)$ is a (τ_1, τ_2)-σ_k-g-closed set in (X, τ_1, τ_2). Then f is (τ_1, τ_2)-σ_k-g-continuous.

The converse of Proposition 5.9 need not be true as be seen from the following example.

Example 5.10: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{b, c\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$, $Y = \{p, q, r\}$ and $\sigma_2 = \{\emptyset, \{q, r\}, Y\}$. Here $CL(X, \tau_2) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Define a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = q, f(b) = r$ and $f(c) = p$. Then f is (τ_1, τ_2)-σ_k-αgr-continuous. But f is not τ_2-σ_k-continuous.

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