BPE BULLETIN OF POLITICAL ECONOMY
11:2 (2017): 75-104

Mathematical Principles of Monetary Econophysics with Application to Problem of Financial Stabilization

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This paper presents a mathematical solution of the problem for financial stabilization. The exact statement of the problem is carried out in terms of the four conventional market values, involved in the famous Fisher equation of monetary circulation. The latter is subjected to sush named dynamic extension. Then, the conditions for occurrence of economic destabilization and cyclicality are deduced analytically. At the end, the final conclusion is made, that to escape the occurrence of economic crisis cycles, it is necessary to sustain sufficiently high progressive taxation and respectively enough mass consumption.

INTRODUCTION

The methodological approache of analytical empiricism and its possible impact towards constructing monetary econophysics

This paper belongs to neither theoretical physics, nor such named *mainstream economics* (Colander, 2003; Krugman, 2009). It tries to solve the problem for emergence of cyclicality in the monetary circulation by applying the *method of principles* in physics (but not the very physical principles). This task has not analogue in the literature of the mentioned sciences. In view of that, the paper might be called an initial work in the specific context of *mathematical principles of monetary econophysics*, where the interdisciplinary approaches of both theoretical physics and monetary economics are acceptable. The possibly useful purpose of the work is to argue in a mathematically exact way the validity of the next sentence:

> *In order to escape the occurrence of Marx's realistic prediction for inevitability of market destabilization and cyclicality, it is necessary to sustain sufficiently high progressive taxation and respectively enough mass consumption.*

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To achieve an exact argumentation of this sentence in mathematical terms, we are motivated to accept a generally enough approach both in philosophical and scientific sense. The most valuable formulation of such a motivation, for specifying appropriate approach in the present work, is contained in the following characteristic paragraph of Russell (1959) for the modern analytical empiricism:

> *"Modern analytical empiricism [...] differs from that of Locke, Berkeley, and Hume by its incorporation of mathematics and its development of a powerful logical technique. It is thus able, in regard to certain problems, to achieve definite answers, which have the quality of science rather than of philosophy. It has the advantage, in comparison with the philosophies of the system-builders, of being able to tackle its problems one at a time, instead of having to invent at one stroke a block theory of the whole universe. Its methods, in this respect, resemble those of science. I have no doubt that, in so far as philosophical knowledge is possible, it is by such methods that it must be sought; I have also no doubt that, by these methods, many ancient problems are completely soluble."*

It is known that during the last 100 years, the dominant theories in macroeconomics were those of Marx (Marx, 1864), Keynes (Keynes, 1936) and the so-called quantity theory of money, having several centuries of history, but being associated with the names especially of Fisher (Fisher, 1933) and Friedman (Friedman, 1987). The comprehensions on the main problem for the macroeconomic destabilization and cyclicality in these theories have such a long history of mutual differences, contradictions and rejections, that the question arises: It is may be necessary to apply the *method of principles* of the exact sciences, to a theoretical construction in macroeconomics for determining mathematically precise these differences? Actually, by treating the macroeconomics as *philosophical science*, the systematic application of this method to the monetary circulation would be a substantive application of the analytical empiricism in terms of Russell. Responding positively to the above question, the present study makes use of the method of principles to provide a more accurate (mostly qualitative, ie geometric) language for mathematical formulation and exact solution of the following

Problem of market equilibrium destabilization and cyclicality emergence of an autonomous macroeconomic system: *To define the conditions for stabilization and destabilization of market equilibrium in an autonomous macroeconomic system, respectively - the disappearance and appearance of cyclicality in it. To compare the obtained mathematical results with the similar verbal formulations in the dominant macroeconomic theories.*

Terms stabilization, destabilization, equilibrium, autonomous system and cyclicality, can be interpreted both in specific mathematical sense and general economic context. This work aims to make a meaningful connection between the two possible extensions (mathematical and economic) of the above-formulated problem. Furthermore, this study is dedicated to not just imitate cyclicality (periodic and even chaotic) in appropriate terms. Similar modeling is already achieved (Ercolani, 2007), (Reitz and Westerhoff, 2003). The efforts, in the present work, are focused both on the modeling and the more difficult task of defining the conditions of appearance and disappearance of the macroeconomic cycles, described in terms of market variables, involved in the famous Fisher equation. These terms, in their quasi-static sense, are common in the theories of Marx, Fisher - Friedman and Keynes. To solve the above evidently non-static problem, we need to make a dynamic extension of the very Fisher equation. This would make it possible to compare the dynamic aspects of the mentioned quasi-static theories in the context of the mathematical theory of dynamical systems. Similar purpose differs essentially from the known publications on business cyclicality (Prokhorov, 2001).

The use of the term for autonomous macroeconomic system gives some economic certainty to the above-formulated problem in the following sense: In view of the term *macroeconomic*, the object of study is not directed towards the activities of the various actors (individuals and legal entities, companies, institutions or states) in the market for goods and services, but towards the behavior on time of the main economic indicators of all set of the market agents (actors). So, macroeconomic *autonomous* nature of the system requires its definition, at least as a national one.

Later in this article, specific *qualitative* issues follow, whose meaning requires preliminary historical and logical explanations:

The terms *qualitative* and *geometrical* are used in special scientific literature as synonyms. Precise qualitative identification of new truths has been so productive in the geometry of Antiquity, that during the Renaissance, it is seen as the main method in the so-called rational philosophy (Descartes, 1988; Spinoza, 2002; Wolff, 1985). In natural philosophy (the name of physics then) axiomatic approach is used in rather modified form, significantly supplemented by the quantitative experimental method. Both methods (axiomatic and experimental), in interaction with each other, have formed the method of principles which has proven to be extremely fruitful to explain and predict a wide variety of events and facts in the world around us (Poincarre, 1952). In this combined form, the method of principles exists in physics to present days.

It is known that political economy was initially developed in collaboration with philosophy, which, as already noted, is axiomatically built by the rationalists of 17th century. Unfortunately, they do not include the political economy as part of their axiomatic constructions, probably due to the underdevelopment of this scientific domain at their time. Later, the classicists of political economy Smith (2005), Ricardo (1817) and Marx (1864) do not attempt any axiomatic constructions in this area of knowledge. They preferred the ideological approach based on empirical data and observations, and free of any even minimal axiomatics. So far, in macroeconomics (and political economy) it has been no systematic attempts to build it on the method of principles.

Throughout its history, the political economy (named economy in the new age) remains away from the triumph of axiomatic method, that of principles and the experimental one. Additional and sufficient reason for this is in the circumstance that in most cases the phenomena studied by the economy are characterized by pluralism of the participating factors, ambiguity of the identified trends, lack of repeatability of the quantitative relationships and other features, which necessitated adherence to qualitative language instead to quantitative one of the exact sciences. This was due to the fact that the studied systems in the economy were *complex*, which was not inherent to the systems in the natural science then.

In the first half of the 20th century, however, in physics, theoretical chemistry, and even in mathematical biology, it occured a kind of explosion of research into systems characterized by a complex behavior, which is connected with the establishment of qualitative features, such as: existence and stability of the equilibrium (steady state) of the system, reversible or irreversible loss of this equilibrium, regular or random behavior of the system, (which has lost equilibrium), and other ones, making the terminology in these sciences similar to those of political economy, i.e. - *qualitative*. The methods of so-called *qualitative* theory of dynamical systems have emerged and recently been applied to the study of economic systems (Prokhorov, 2001).

The novelty in the present work is the obtained qualitatively exact comparison between the basics of quasi-static theories of Marx (1864), Fisher (1933), Friedman (1987) and Keynes (1936) for monetary circulation, in terms of the dynamic system, synthesized here.

Necessary Considerations Concerning the Notion Evidence (Clarity)

As below, it will come to obviously true statements (axioms), it is necessary to introduce some concepts of refinement of the notion *evidence (clarity)*. The latter is primarily a *relative* term. What is apparent to a subject, it may not be so to another. For example, the axioms of Euclidean geometry are obvious to most people, but there are some for which they are not. Children under a certain age did not perceive geometric axioms as self-evident. Therefore, the study of this branch of mathematics begins in the upper class of the school. Until then, in one form or another, the efforts are to develop key concepts of the students for point, line, plane, curve, shape, geometric dependence, etc. For this purpose they give them examples, problem solutions, etc. that develop geometric intuition of children, their observation and spatial imagination.

It applies the same to the axioms in the other mathematical disciplines and the underlying principles (laws) of mechanics and physics at all. There are students unwilling to science that did not reach the perception for the evidently true first principle of mechanics (inertia) for example. Not accidentally Galilei wrote a treatise on the inertia principle, that based on numerous examples (thought experiments), observation and reasoning to evolve the ability of his contemporaries (the characters - Salviati, Sagredo and Simplicio) for perceiving this principle as an obvious truth (Galilei, 1638).

It was simlar the story of the second and third principle of mechanics. Based on observations and reflections on suitable real and imaginary (idealized) situations, it was developed the ability of followers of the Newton's theory to adopt its principles as obviously true, not experimentally verifiable. Tens and perhaps hundreds of collections of scholar problems in mechanics, published in the last three centuries, confirm this trend of the education. In this case, experimental verification of the mechanical principles is principally impossible, because one can not put a *real* experiment under *ideal* conditions, for direct confirmation or refutation of any of the three principles (Poincarre, 1952). It can be only made observations and experiments, to be used as leading (indirect) considerations of the truth of the principles, but not as their direct empirical evidence.

There is no doubt that the ability of a well-trained physicist to perceive as obvious the basic principles of mechanics is more developed than that of a humanitarian person, ignorant in this science. For the uninitiated people in physics, it is needed some preparation (such as that which Galilei applies to his contemporaries) to develop their intuition, imagination, knowledge and the ability to perceive the relevant principles as obvious ones. The same is valid for the readers, uninitiated in the theoretical economy, which have not the necessary observations and reflections on the obvious abstract relationships in the market. This might be a problem, when they try to perceive the evidence of formulated here definitions and postulates. The latters would possibly seem non-obvious to some readers at first sight, which would mean that they should make further considerations for the reasons discussed above in this paragraph.

Deterministic and Stochastic Components of Market Variables. Fast and Slow Variations. Dynamics and Quasistatics

This article offers a regular (ie non-stochastic and non-chaotic) theory of monetary circulation. It is known, however, that the time series of market variables contain random (stochastic) component superimposed over an alleged deterministic (including regular, ie non-chaotic) one. This fact, in the considered case requires the separation of the random component from the regular, which is accepted mathematically in the following manner:

Let the function $\tilde{\varphi}$ presents an arbitrary market vriable, depending on the time t. According to the above, the function is presented in the form

$$\tilde{\varphi} = \varphi + \varphi' , \qquad (1.1)$$

where φ is a regular component, and φ' is a random variable of the considered market variable. We suppose the time interval of random change of every market variable is a small value with respect to the supposed period τ of cyclicality of its regular component. The last is obtained as a result of time averaging of the function $\tilde{\varphi}$ by the formula

$$\varphi(t) = \frac{1}{\theta} \int_{\xi = t - \frac{\theta}{2}}^{\xi = t + \frac{\theta}{2}} \tilde{\varphi}(\xi)\, d\xi , \qquad (1.2)$$

Where ξ is an integration variable, θ is a time interval, which is large with respect to the time interval of random change, and small comparing to the period τ. Moreover we suppose that, the average value of the random component in the interval θ is equal to zero.

We assume that, the regular component $\varphi(t)$ of the arbitrary market variable is a continous and smooth (differentiable) enough to apply the methods for analyzing dynamical systems we consider further. So, the regular market variables, we deal, are averaged by the formula (1.2.). Moreover, these averaged functions have the necessary properties for applying the mathematical procedures in the next part of this work. It should be noted yet, that the market variables satisfy (1.1), because their regular components are *stable* solutions (oscillatory and equilibrium ones in our case) to the corresponding dynamic system. Small deviations from these solutions in the form of random components φ', fade with time and does not change the average regular components. This is valid for cases when the regular components are constants and constitute corresponding equilibrium solutions of the system, but only at the condition that the equilibrium is stable. Otherwise, the relation (1.1) expires and the system leaves the equilibrium state to go on periodic solution. The latter is precisely the case of emerging periodic cycles which is the subject of this paper.

Further we introduce the well known values used in the famous quantity theory of money (Fisher, 1933), (Friedman, 1987). We define *circulatory money* as *all money participating in the real transactions of stocks and services*, and denote it by M with dimension [*money*]. Moreover we suppose M is *slowvaring* value. Specifically to quantity theory of money, we do not include the well known *bank reserve* in the *circulatory money* M. The whole *money supply* is a sum of *circulatory money* and *bank reserve*. Following quantity theory, we accept that, the basic market variables of the monetary circulation are: the velocity of monetary circulation X with dimension [$time^{-1}$] presenting number of turns per unit time, the amount of commodity transactions Y, contracted per unit time, with dimension [$transaction \times time^{-1}$] and the average price Z of a single commodity with a dimension [$money \times transaction^{-1}$], which are regular values in the sense of (1.1) and are *fast-varying*. But we assume yet that, they contain *slowvaring* parts V (Velocity of money turns), T (Transactions of stoscks), P (Price averaged), ie the equalities $X = V + x$, $Y = T + y$ and $Z = P + z$ are valid. The variables V, T and P, as well as x, y and z have the necessary dimensions. The measurability of M, V, T and P is considered on page 9. Basic units of measure are *unit of money*, *transaction*, *turn* and *year*. We imagine yet, that slowvaring values satisfy the following dynamical system, written in a general form

$$\frac{dM}{dt} = \mu . f(M,V,T,P), \qquad (1.3$$

$$\frac{dV}{dt} = \varepsilon . g(M,V,T,P), \qquad (1.4)$$

$$\frac{dT}{dt} = \delta . h\left(M, V, T, P\right), \tag{1.5}$$

$$\frac{dP}{dt} = \nu . s\left(M, V, T, P\right), \tag{1.6}$$

For the fastvaring values we accept they satisfy the next system:

$$\frac{dX}{dt} = p\left(X, Y, Z, M, V, T, P, p_1, p_2, \ldots\right), \tag{1.7}$$

$$\frac{dY}{dt} = q\left(X, Y, Z, M, V, T, P, p_1, p_2, \ldots\right), \tag{1.8}$$

$$\frac{dZ}{dt} = r\left(X, Y, Z, M, V, T, P, p_1, p_2, \ldots\right). \tag{1.9}$$

The behavior (set of solutions) of the system (1.3-6) for the four slowvaring variables is called quasistatic behavior and the system itself - quasistatic (degenerate) system. In it, the parameters μ, ε, δ и *v* are small and this fact predetermines the slowvaring character of the changes of values *M*, *V*, *T*, *P*. We assume that the equations (1.3-6) describe changes of those values which are close to equilibrium behavior, ie to invariability of variables on time, so that the left sides of equations are exactly or almost equal to zero. In view of that the slowvaring values *M*, *V*, *T*, *P* are called yet quasistatic. The behavior of system (1.7-9) for the three fastvaring values *X*, *Y* and *Z*, is called *dynamical* one, and the very system is named also *dynamical (attached)*. The combination of the two (quasi-static (1.3-6) and dynamic (1.7-9)) systems form the so-called complete system (1.3-9), which equilibrium solution coincides with that of (1.3-6). That is, at zero left parts of (1.7-9), the fastvaring values become equal to the slowvaring. The parameters p_1, p_2,... depend on the other slowvaring variables. It is valid the following

Theorem of Tichonov (Tichonov (1952: 575-586)):

At sufficiently small values of the parameters μ, ε, δ u v, the solution of the full system (1.3-9) tends to the solution of the degenerate (quasistatic) system (1.3-6), if the following conditions are valid:

(a) The equilibrium (steady state) solution V, T, P for the attached (dynamic) system (1.7-9) is isolated and stable.

(b) The initial conditions for the full system (1.3-9) lie in the attraction of the equilibrium of dynamic (attached) system (1.7-9).

(c) The right sides of the complete system (1.3-9) are continuous functions and the solution of that system is singlevalued.

The importance of this theorem for the problem, formulated in the introduction, is that the quasi-static dependencies analyzed in the theories of Marx (1864), Keynes (1936) and Friedman (1987) are valid provided that there is a stable equilibrium of the market. The latter may lose or acquire stability, and this may be related to the appearance or disappearance of a cyclicality, which has been studied by the above mentioned authors, on the basis of empirical observations and ideological considerations. With the appearence of cycling, however, according to Tikhonov theorem, it disappears the reason in the above mentioned authors treatment, to apply the quasi-static dependencies for analyzing the macroeconomic system. This is a common deficiency in the mentioned quasistatic theories of economic cyclicallity, which can be overhelmed only by their dynamical extension, i.e. by including them in a more general dynamical sheme.

In the present work cyclicality is analyzed in terms of the theory of dynamical systems. For this purpose, we apply the wellknown method of principles for dynamic extension of quasi-static Fisher equation for monetary circulation. So we follow the general approach for research synthesis

valuable definitions ⇒ evidently true postulates ⇒ well grounded principles

The resultant of this scientific synthesis will be treated as *attached* (within the meaning of the above theorem of Tikhonov) system to mathematically implicit, *degenerate* systems. Quasi static behavior of each generate system is seen as verbally formulated and logically justified (based on empirical observations and ideological reasons) in the theories of Marx (1864), Friedman (1987) and Keynes (1936).

In conclusion of this section, it is appropriate to note the following:

The whole mathematics, without any *restrictions*, is built on assertions of the type of above mentioned *evidently true postulates* (axioms). Moreover, the whole theoretical physics is based on the method of principles (postulates being not always evident even, but taken as results from the mathematics). The method of principles is created just by analogy with the axiomatic method. Recently, many other branches in the exact science are also in the process of matematization, i.e. in using results from these *evidently true postulates*. This means the idea of *evidently true postulates* is exclusively fruitful and useful for the whole human knowledge. It would be rather strange for the economics to make exception from this general trend. The latter, certainly, should be neither exaggerated nor neglected.

BASIC DEFINITIONS AND POSTULATES

Definitions of Market Equilibrium, Money Supply and Consumer Demand

Firstly we accept the following

Definition 1: *Equilibrium is called a correspondence (equality) between the demand and supply of total goods and services on the market.*

As it will be specified later in relation to the well known Fisher equation (look at Definition 2 and Definition 3 below), the values *D* and *S* have one and the same dimension [$money \times time^{-1}$]. It means market equilibrium can be tought also as equilibrium of *money demand and supply* with alternative change of notations – *S* for money demand and *D* for supply. Definition 1 presents notion of *global equilibrium*, formulated by analogy with the similar understanding of Marshall (Marshall, 1879) and gives a global (for all goods and services in general) sense of supply and demand. The lasts, in this sense, can not be considered as distributed per separate products with their different prices, as in Marshall (well-known curves of supply and demand with their intersection point as a graphic image of the market equilibrium for individual commodities). The difference between the behavior of particular (local) and global values is essential. For example, the changes of *local* demand and stock price are always opposite (Marshall, 1879), but it seems realistic that, sometimes (not always), *global* demand and price vary unidirectionally (Gokal, 2004).

Further, we introduce dynamic understanding of the above defined global equilibrium that can be stable (and therefore - practically realizable) and unstable (therefore – not practically realizable). In this sense, the lack of market equilibrium, ie practical inequality between supply and demand, does not mean non-existence of equilibrium. In all cases, the equilibrium exists, but for various reasons it may be unstable, so - not practically realizable.

It is worth noting that, in accordance with the method of principles we apply, it is not necessary to outline a procedure by which every introduced variable can be measured (or observed). The values *D* and *S* are *auxiliary* variables, which we need neither to measure nor to observe even. In this respect there are many examples in the exact science (for example - quantum mechanics) for introducing auxiliary variables, which are immeasurable and unobservable.

Consider a second definition for money supply (Fisher, 1933, and Marx, 1864) in the following form

Definition 2: *The money supply is equal to the* ***multiplication*** *of the purchased product and the average price for a single transaction,* ***divided*** *by the number of turnsof the money supply.*

That is, we accept as definitively valid the equality

$$0 = YZ - MX \tag{2.1}$$

Purchased goods (and servises) Y are also called *consumption*. In the quantity theory of money, (2.1) is called *Fisher equation* and by the usual symbols it is recorded in the form

$$0 = TP - MV, \tag{*}$$

As in section 1.3., here the well known values used in the famous quantity theory of money (Fisher, 1933), (Friedman, 1987) are introduced. We denote *circulatory money* by M with dimension [*money*] and suppose it is *slowvaring* value. Specifically to quantity theory of money, we do not include the well known *bank reserve* in the *circulatory money* M. The whole money supply is a sum of circulatory money and bank reserve. Following quantity theory, we accept that, the basic units of measure are *unit of money* (dollar, euro, etc.), *transaction* (having different price depending on the quantity stocks and services), *turn* (unit for measure of money circulation) and *year* (usual unit of mesure for time in the economy). The basic market variables of the monetary circulation are: the velocity of monetary circulation X with dimension [$time^{-1}$] presenting number of turns per unit time, the amount of commodity transactions Y, contracted per unit time, with dimension [$money \times transaction^{-1}$] and the average price of a single commodity Z with a dimension [$money \times transaction^{-1}$], which are regular values in the sense of (1.1) and are *fast-varying*. But we assume yet that, they contain *slowvaring* parts V (*V*elocity of money turns), T (*T*ransactions of stoscks), P (*P*rice averaged), ie the equalities $X = V + x$, $Y = T + y$ and $Z = P + z$ are valid. The variables V, T and P, as well as x, y and z have the necessary dimensions. Thus the formula (2.1) is true only when $x = 0$, $y = 0$ and $z = 0$, ie for the *static* values of $X = V$, $Y = T$ and $Z = P$. Now, the term *static* has a concrete sense of satisfying (2.1) or (*).

As for the essential question of *measurability* of the parameter M and variables V, T and P, there are huge amount of information on this subject in the book of (Boumans, M. ed., 2007). Most suitable for the present consideration could be the following:

The circulatory money *M* is difficult to be measured, because the macroeconomic circulation is not closed system. There are many inflows and outflows of money, which are not controlled. Nevertheles, an essential part of *M* can be determined as a difference between the *total money supply* and the *bank reserve*. If this essential part increases or decreses, we can conclude that the whole *M* does the same. For the purposes of *exact qualitative* analysis, such measurement is enough.

The circulation velocity *V* is measured as a *number* of passes (exchanges) per unit time (usually 1 year) of fixed coin (or banknote) through separate market participant (agent). The number is different for various fixed coins and separate participants. That's why, the circulation velocity is taken as an *average* value of many numbers determined in this way.

In accordance with the traditional sense of Fisher equation (*), the average price *P* is trivially determined as an average value of all registered prices of *transactions*, being scalar measures (possibly deviated by the market) of some abstract *labour values* embedded in the corresponding *stocks* and *services*. Certainly, the price *P* can be also considered as a vector (Froehlich, 2011).

The global transaction *T* is difficult to be directly determined, but it s possible to do this indirectly in the following way: The multiplication *TP* is directly measurable value (this the national income, usually defined per year). If the price *P* is determined, then the transaction *T* can be calculated too.

Therefore, at least qualitatively, the values *M*, *V*, *T* and can be approximately measured or calculated, which is enough for the quality mathematical modeling, we apply.

Equation (2.1), is called yet the *law of monetary circulation*. It claims that the money measure of commodity turnover *YZ* is equal to the cash turnover *MX*. It is assumed to be true in the quantity theory of money, in the Keynesian theory, and also in Marxian one. The latter, however, consider this law systematically distorted in the private propriatory market.

Further, we accept yet the following

Definition 3: *The commodity turnover is a quantitative measure of the global consumer demand.*

We notice that from the three definitions 1, 2 and 3, it follows logically that the equality (2.1) makes sense of *market equilibrium* between demand *D* and supply *S* as well as between *commodity turnover YZ* and *cash turnover MX*. In (2.1), the first member is defined as a quantitative measure of *demand (ie commodity turnover)*, and for the second term it is obtained (logically), that it is a quantitative measure of *cash turnover*. In this extension of the economic sense of the members in equation (2.1), there is an element of scientific convention, which follows from the sense of definition 3. The last, as every definition, can not be true or false (Poincarre, 1952), but consistent or inappropriate with the meaning of the concept of demand in this case. In accordance with Poincarre conseption for a scientific convention, it should be noted that the definition 3 is *not arbitrary*, but it is economically *valuable* as well as mathematically *convenient*. It will be shown later, in the present paper, it is, besides, definitively *fruitful*. Moreover – together with the definitions 1 and 2, as it is seen here, the definition 3 gives a quantitative meaning to the notion of *market supply S*.

Through equality (2.1), we determine the equilibrium value of each of the variables X, Y and Z, when the other two are fixed. The equilibrium also means that the above-defined market variables X, Y and Z, are constant, independent of time and therefore are denoted as V, T and P respectively. Geometric image of the latter is presented in the space of market variables X, Y and Z as an intersection point of the three mutually perpendicular planes $X = V$, $Y = \frac{MV}{P}$ and $Z = P$. This point being defined in an evidently different way, than the usual graphic image of equilibrium of a particular product, ie than *local* equilibrium in Marshall (1879) terms.

Postulates for the Non-equilibrium Market Behavior Distorting Fisher Equation

It is a general principle in the present paper, that the *law of monetary circulation* (i.e. the market equilibrium, expressed by Fisher equation (2.1)) is actually distorted in the market practice (Marx, 1864). From a general scientific point of view, there is not something unusual in such distortion: The equality (2.1) remains valid definition for the money circulatory money, but just ceases to be accomplished as a condition for market equilibrium. The case is similar to that in the non-equilibrium thermodynamics where the thermal equilibrium is used as definition of the temperature, but the very equilibrium is distorted when describing the heat transfer by using such defined notion of temperature (Lebon et al. 2008). Many other examples from the exact sciences can be presented.

Further we accept this principle of equilibrium distortion in the form of three postulates for the market variables, which deviate from their steady state values and distort in this way Fisher equation (2.1). The posulates are formulated in the following form

Postulate 1: *The demand activates the supply.*

Postulate 2: *The supply selfinhibits itself (ie without demand, the supply vanishes).*

Postulate 3: *Unidirectional changes of the market variables,andare sometimes realizable simultaneously.*

The first two postulates conserve their evident validity, if the terms *demand D* and *supply S* are replaced by the equivalent notions - *commodity turnover YZ* and *cash turnover MX*, respectively. The three postulates follow from the practical experience in the real market. All of them distort Fisher equation and the definition of money supply, it presents, as well as - the market equilibrium it expresses. Moreover, they appear as obviously true. Especially, the first two seem almost inherent to the commercial instinct of market participants. The third postulate only approve implementation of the widely observed possibility that the velocity of cash turnover *X*, purchased goods (transactions) *Y* and the average price *Z* of the product, may increase or decrease simultaneously (and unidirectonally). This simultaneity of their one-way variations is observed in practice in the form of *economic growth and decline*, without excluding occurence of the alternative behavior (divergence of changes in opposite directions). The aim here is: By starting from the conceptual basis of the definitions 1, 2 and 3 and the practical validity of the postulates 1, 2 and 3, to derive appropriate dynamic principles of monetary circulation.

DYNAMICAL PRINCIPLES OF THE NON-EQUILIBRIUM MONETARY CIRCULATION

From the Postulates 1, 2 and the economical sense of *YZ* and *MX* (demand and supply, respectively), it follows logically the next differential equation for the non-equilibrium monetary circulation

$$M\tau\frac{dX}{dt} = YZ - MX, \tag{3.1}$$

Indeed, in accordance with Postulate 1, the first member *YZ* in the right side presents the demand (commodity turnover), which activates the supply (money multiplication) in the left one (the derivative of *MX*). Because of the negative sign in front of the second member in the right side, this member inhibits itself as Postulate 2 claims. The variables *X*, *Y* and *Z* are functions of time *t*. The variable *t* is attached to the end of a moving time interval, at which market variables are defined in every moment. The length of interval is fixed and can be a month or year, for example. The characteritic period τ appears necessarily in the left side of (3.1) as a coefficient of proportionality, which, in view of the reasons for one and the same dimension (money) of individual members in the two sides of (3.1), has dimension of time. The parameter τ is appropriate to be *time unity*, which is numerically equal to 1. Further, we keep writing τ in the equations and formulas, to underline the dimensional validity of (3.1). We accept the equation (3.1) as an extension of the Fisher equation (2.1), to be the first *dynamical* principle of monetary circulation.

Further, we derive equations for the speeds of changes in purchased goods Y and stock price X. For this purpose, from postulate 3 we derive the following consequences:

Statement 1. *It is realizable, that an increase (decrease) of the purchased goods Y is simultaneous with an increase (decrease) of number of turns X.*

Statement 2. *It is realizable, that an increase (decrease) of the stock price Z is simultaneous with an increase (decrease) of number of turns X.*

The derivations of the other two equations (principles) are the following:

With increasing (decreasing) the goods purchased or consumption Y, in view of Statement 1, the number of money turns X also increases (decreases). This means that the derivative of X in the left side of equation (3.1) is positive (negative) and the right side also satisfies the positive (negative) inequality with respect to zero. From this inequality it follows the conclusion, that *the value Z will be greater (less) than the equilibrium value P, determined at a sign of equality instead of inequality*. The simplest equation satisfying this conclusion at sufficiently small deviations of Z from P (the infinitezimal requirement is connected with the neglection of nonlinear members of the Taylor's series of growing function) $Y = g(Z - P)$, in this case is

$$\frac{dY}{dt} = \beta(Z - P). \tag{3.2}$$

Indeed, the linear approximations of Taylor's series of Y, on t and Z, lead to the relation $\frac{dY}{dt}(t - t_0) = \frac{dY}{dZ}(Z - Z_0)$, which is equivalent to (3.2) for $\beta = \frac{1}{t - t_0}\frac{dY}{dZ}$ and $Z_0 = P$. The coefficient β have a dimension of "stock quantity per unit of time and per unit of money" (ie β is *consumption power*) and on this basis we can assume it to be constant (or slowvaring parameter) in view of requirement of simplicity. The latter means Y is linear function of Z with coefficient of proportionality $\beta(t - t_0)$, depending also linearly on time. An additional reason to accept (3.2) is the fact, that it describes the well known growth of consumption Y at relatively high values of the average price $Z > P$, and its decrease at low ones $Z < P$. These are such named economic growth and decline (Gokal, 2004). Of course, there are other equations, compatible with the above conclusion. It deserves attention the circumstance that from equations (3.1) and (3.2), it does not follow Statement 1. That is, the equations (3.1-2) could be in accordance with a behavior of market variables X, Y and Z, which may be opposite to that in Statement 1.

We make analogical considerations to derive the third equation (principle):

With an increase (decrease) of stock price Z, according to Statement 2, cash turnover also increases (decreases). This means that the derivative ofof the left side of equation (3.1) is positive (negative) and the right side also satisfies the positive (negative) inequality with respect to zero. From this inequality it follows that the value of X will be less (bigger) than the equilibrium value V, determined at an equality sign instead of inequality. The simplest equation satisfying this conclusion at sufficiently small deviations of X from V (here we again neglect nonlinear members of the Taylor's series of decreasing function) $Z = d(V - X)$, in this case

$$\frac{dZ}{dt} = \gamma(V - X). \qquad (3.3)$$

Here γ has a dimension of *price depreciation* of the global commodity, and on this basis, it is assumed to be approximately constant (similarly to that of β), what is connected to the requirement for simplest possible form of (3.3). An additional reason to accept (3.3) is the fact, that it describes the well known rise of price Z at relatively low values of the circulation velocity $X < V$, and its decline at high ones $X > V$. This circumstance has essential application in the Marxian and Keynesian points of view about the role of monetary circulation in the economics (Marx, 1864, Keynes, 1936). Similarly to the equation (3.2), we pay attention to the fact that from the equations (3.1) and (3.3) it does not follow Statement 2. That is, the equations (3.1), (3.2), beside Statement 2, may be in accordance with other behavior of market variables X, and Z.

In so made derivations of the equations (3-2) and (3-3), the latter are in accordance with the postulate 3, but their content is more specific than this postulate. These equations are valid only for sufficiently small deviations of the dynamic variables X and Y from their equilibrium values. Further, we accept that this requirement is satisfied in practice, so that dynamic values of the market variables can be only positive (since the equilibrium values V, T and P are large positive quantities with respect to their deviations x, y and z). Thus, the postulate 3 is a leading consideration in finding the equations (3.2-3), which together with the equation (3.1) form a closed system of three ordinary differential equations, with determined first derivatives of X, Y and Z. Such a system is called *dynamic* in the qualitative theory of ordinary differential equations, which is a reason to call the equations (3.1-3) - *dynamic principles*.

QUALITATIVE ANALYSIS OF DYNAMICAL SYSTEMS FOR WELL-KNOWN QUASISTATIC MODELS

As it is already noted above, the quasi-static parameters M, V, T and P, change slowly in comparison with the dynamic variables X, Y and Z. Slow change is called *trend* in the economy or *quasi-static* (or degenerate in Tikhonov's terms) in exact sciences. As it was accepted above, the dynamic parts (deviations) x, y and z of the corresponding variables X, Y and Z are superposed on their quasi-static ones, so the following formulas hold: $X = V + x$, $Y = T + y$ and $Z = P + z$.

Therefore, we examine the impact of changes in the quasi-static variations of M, V, T and P, on the dynamic changes of X, Y и Z. For this purpose, let us write our system of differential equations (3.1-3) in the canonical form

$$\frac{dX}{dt} = \frac{1}{M\tau} YZ - \frac{1}{\tau} X, \tag{4.1}$$

$$\frac{dY}{dt} = \beta(Z - P), \tag{4.2}$$

$$\frac{dZ}{dt} = \gamma(V - X). \tag{4.3}$$

We consider the differential equations (4.1-3) as a concrete form of the system (1.7-9), attached to the degenerate system (1.3-6). The latter, further, being only taken in view, as general form of the abovementioned dominant theories of Fisher-Friedman, Keynes and Marx. These authors claim some verbal assertions about the relationships between quasistatic variables M, V, T and P. Here, these relationships are taken as qualitative extensions of some concrete solutions of degenerate system (1.3-6). To define whether or not the solutions of (4.1-3) are compatible with the mentioned quasistatic relationships, we apply the theorem of Tichonov. The first step in such an application is to analyze the stability of the equilibrium of (4.1-3).

By applying the standard analysis to steady-state values $X = V$, $Y = \frac{MV}{P}$ and $Z = P$, we get the following conditions for equilibrium stability using the coefficients of Routh-Hurwitz, p, q, r и $R = pq - r$.

$$p = \frac{1}{\tau} > 0, \tag{4.4}$$

$$q = \gamma \frac{T}{M\tau} > 0, \tag{4.5}$$

$$r = \beta\gamma \frac{P}{M\tau} > 0, \tag{4.6}$$

$$P < \frac{T}{\beta\tau} \tag{4.7}$$

The first three relations (4.4) (4.5) and (4.6) are obviously always satisfied. The fourth condition (4.7) of stability is valid for not very large values of the average stock price P, and for high enough consumption T. If the consumption is low enough, and the average stock price is high enough, the condition (4.7) would not be satisfied and the equilibrium represented by equality (2.1) would not be stable. If the average price is high enough, it will be satisfied even the opposite of (4.7) inequality, which means instability of equilibrium $X = V$, $Y = \frac{MV}{P}$ and $Z = P$. According to the theory of bifurcations, at transition through the critical point $P = \frac{T}{\beta\tau}$, the sign of inequality between the two sides of the equation changes and as a result, the character of the equilibrium state (2.1) also changes. Around the changed steady state, a limit cycle occurs, corresponding to self-excitation of periodic oscillations of the dynamic variables of the system under consideration. This type of behavior is just a mathematical analogue of the economic cyclicality.

Further, it follows a mathematical application of the dynamical system (4.1-3) to the well known quasi-static theories of the monetary circulation.

Dynamical Extension of the Quasi-static theory of Fisher-Friedman

To describe correctly the transition from the equilibrium

$$X = V, Y = \frac{MV}{P} \text{ and } Z = P, \tag{4.8}$$

to another steady state, corresponding to money supply $M + \Delta M$, we apply certain quantity theory of money (Friedman, 1987). According to this theory (model), we can assume that the change ΔM in the money supply is associated with a proportional change of average market price ΔP and the factor of proportionality is equal to the ratio of the equilibrium consumption T to the steady speed of circulation V, at condition that T and V can be considered as constant. That is, it is valid the formula

$$\Delta M = \Delta P \frac{T}{V}, \tag{4.9}$$

The last is a fundamental relationship (accepted as unconditionally true in all cases) in the quantity theory of money. Given the ratio (4.9), after entering in the system changed by more money supply, the equilibrium acquires the form

$$X = V,\ Y = \frac{MV}{P}\ ,\ Z = P + \Delta M \frac{V}{T}. \tag{4.10}$$

In this way in the changed system, the velocity of cash turnover and consumption are the same as in the initial system, but the stock price has increased in proportion to the increased money supply. Changed (increased with money supply) system will have the form

$$\frac{dX}{dt} = \frac{1}{(M + \Delta M)\tau} YZ - \frac{1}{\tau} X, \tag{4.11}$$

$$\frac{dY}{dt} = \beta\left(Z - P - \Delta M \frac{V}{T}\right), \tag{4.12}$$

$$\frac{dZ}{dt} = \gamma\,(V - X). \tag{4.13}$$

For the initial values of the dynamic variables in this system we choose the equilibrium values (4.8) of the output system (4.1). Depending on the size of the money supply growth ΔM, for the solutions of the system (4.11 -13) we have the following three possible cases

(a) it is valid the condition

$$\frac{\beta\tau(P + \Delta P)}{T} < 1 \tag{4.14}$$

The latter corresponds to inequality (4.7), ie - to the case of stable equilibrium state (4.10), which means the solution of (4.11-13) presents *damped oscillations* at steady-state values , numerically illustrated below in Figure 1.

(b) it is valid the condition

$$\frac{\beta\tau(P + \Delta P)}{T} = 1 \tag{4.15}$$

It corresponds to the case of indifferent equilibrium (4.10), which means a solution of (4.11-13) in the form of undamped oscillations.

(c) it is valid the condition

$$\frac{\beta\tau(P + \Delta P)}{T} > 1 \tag{4.16}$$

This corresponds to the inequality, opposite to (4.7), i.e. – to the case of unstable equilibrium (4.10), which means the solution of (4.11-13) is a self-excited stable oscillations (selfoscillations) around the equilibrium condition, numerically illustrated below in Figure 2.

Clearly, if ΔM is not large enough, then (4.14) holds. If it is large enough – it will be valid (4.16). And since the bifurcation at the point (4.15) is reversible (ie soft loss of stability occurs), that means it is possible in principle to restore the lost stability by inverse transition, that is - by largely reducing the money supply. Thus, according to the quantitative theory of money (Fisher-Friedman), positive and negative monetary values appear as a sure tool to control money supply and through it - the stable and unstable behavior of market variables, including damped and periodic changes in the form of an alternating sequence of growth and decline.

Below, they are shown two numerical illustrations (Figures 1 and 2) obtained at MatLab for the dimensionless form of the dynamic system (3.1). The procedure of removing dimensions is accomplished in such a way that all variables and parameters in the dimensionless system have one and the same order of magnitude equal to 1. Below, two numerical solutions are shown: In fig. 1, under satisfied condition for stability of equilibrium (4.7), $\left(\frac{\beta\tau\ P}{T}=0{,}5\right)$, it is seen a solution in the form of damped oscillations of market variables towards their equilibrium values, in this case being of order of 1. In figure 2,[1] at not satisfied condition (4.7) $\left(\frac{\beta\tau\ P}{T}=1{,}1\right)$ they are shown selfoscillations (selfinduced undamped oscillations) of the system, when its equilibrium state is unstable. The bifurcation point in the parameter space is $\frac{\beta\tau\ P}{T}=1$. From the numerical illustrations below, it can be concluded that this is part of the so-called supercritical bifurcation of the Andronov-Hopf, which is *reversible* (Guckenheimer and Holme, 1983). This reversibility is in favour of the basic claim of quantitative theory of money for the decisive role of supply emissions in controlling the monetary circulation.

The emergence of the business cycle is not inevitable, according to the quasi-static model of Fisher-Friedman and this is transferred as inference in this dynamic extension. The latter is not even limited by the fact that the *consumption power* β increases. Once the milestone formula (4.9) is valid, then this unfavorable grouth of β can be compensated by negative value of ΔM (respectively of Δ*P*).

Certainly, in some sense the Kondratieff"s curve is more informative, in view of the historical facts contained in it and disposed on the real time (years). However, in the figure 2, the behaviors of three market variables are presented. It is seen that, on one hand they are slightly displaced each the other, but on the other, some time intervals are seen, where the the different variables change simultaneously in one and the same direction (increasing or decreasing) as it is claimed in Postulate 3.

Dynamical Extension of the Quasistatic Theory of Keynes

The Keynesian theory does not unconditionally accept the validity of the formula (4.9) of the quantity theory of money of Fisher and

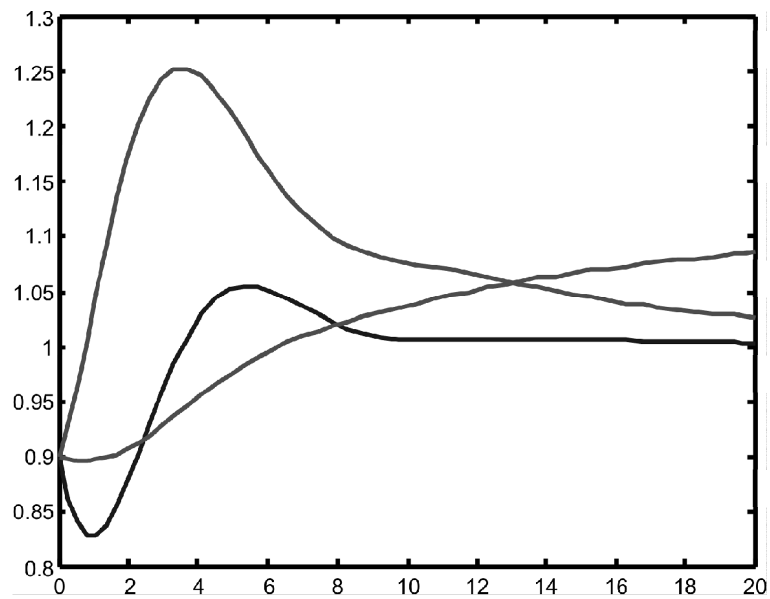

Figure 1: Damped oscillations of the three market variables of Fischer equation

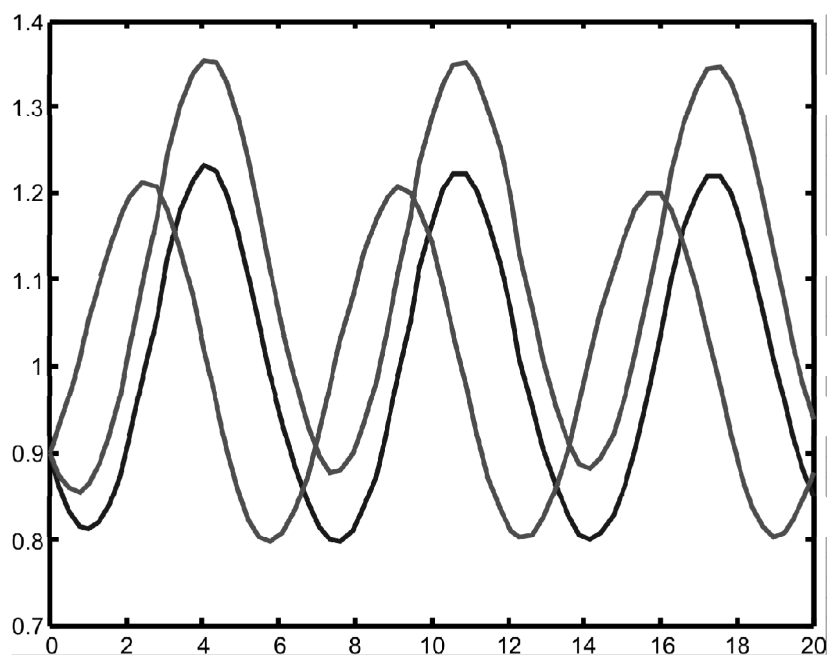

Figure 2: Selfoscillations of the three market variables of Fischer equation

Friedman, but it is assumed that there is some lag in the effect of monetary emission on the average price. Moreover, firstly monetary circulation delays with a magnitude, proportional to the increase in the money supply. Then the delay is likely to affect for increasing the average price. So, a mediated (indirect) impact of monetary emission on the behavior of average price is not seen as surely defined in all cases and therefore the ability to control the monetary stock market through changes in the money supply is under question.

The quasi-static model of Keynes *deals with (4.9) and (4.12)*: After entering an additional money supply ΔM into economic system, the increased money supply becomes $M + \Delta M$, and the equilibrium state takes the form, accounting the *reducing of aggregate national income* Y

$$X = V - \Delta V,\ Y = \frac{M(V - \Delta V)}{P},\ Z = P + \Delta M\frac{(V - \Delta V)}{T}. \tag{4.17}$$

The changed (with the increased money supply) system will have a form

$$\frac{dX}{dt} = \frac{1}{(M + \Delta M)\tau} YZ - \frac{1}{\tau} X, \tag{4.18}$$

$$\frac{dY}{dt} = \beta\left(Z - P - \Delta M\frac{V - \Delta V}{T} \right), \tag{4.19}$$

$$\frac{dZ}{dt} = \gamma (V - \Delta V - X). \tag{4.20}$$

For the initial values of the dynamic variables of the system, we choose the equilibrium values (4.8) of the initial system (4.1). Again, depending on the magnitude of the increase in the money supply, for the type of solution of the system (4.11 -13), the three cases (4.14-16) are possible.

The difference here, in comparison with the theory of the Fischer - Friedman, lies in the fact that the presence of ΔV in the formula for the price (4.17) has an opposite sign in comparison with that of ΔM. This imports the uncertainty (the size of ΔV is principally unknown) in the dependence of price on the money supply growth. Hence, the conditions of stability and their loss (4.14-16) have no fixed extension when the money supply increases, which is in accordance with the general view of Keynes on the subject. He does not also consider money separation, but tends rather to the view of Marx (as we will see below) rather than to Fisher and Friedman conception. However, Keynes believes that the increase in consumption may counteract rising prices and thus stabilize the market equilibrium, avoiding the emergence of macroeconomic cyclicality. In a dynamic extension of mathematical terms, this means keeping the validity of (4.14) and not enetering into force of (4.15-16). As will be shown below, this recommendation of Keynes to avoid the economic cycle, does not contradict the substance of Marx's teaching about money and can also be justified in terms of the mathematical theory proposed here.

Comparitive Analysis of Quasistatic Theories of Marx, Keynes and Fisher-Friedman

In this section, the main findings of the present paper are compared with the most famous macroeconomic theories of Fisher - Friedman, Marx and Keynes in respect to their attitudes for the financial destabilization and emergence of economic cyclicality.

It is known that Marx also doubts in the ability to manage the money supply in the capitalistic state, long before Keynes. Indeed, he acknowledged the impact of changes in money supply on the price level, but insists that this happens not always and not in all situations. So, the dynamical system (4.18-20) can be *attached* (in terms of Tichonov's theorem) to the Marx's *quasistatic* model too. The specificity, however, is that Marx considers the increase in the average stock price P as the *most essential reason* for increasing the money supply, rather than vice versa, as Fisher-Friedman theory claims later.

In othis section we apply this important specificity of the price P to the basic result of the stability analysis in this paper: On page 11, it was defined that β is *consumption power*. So, the consumption T is proportional to β, and we can replace the approximative formula $T = k\beta$ in (4.7). As a result we obtain

$$P < \frac{k}{\tau}, \tag{4.21}$$

where k is a constatnt coefficient of proportionality. Namely, condition (4.21) allows us to compare exactly the dynamical consequences from Marx, Keynes and Fisher-Friedman theories.

In accordance with the political economy of Marx (1864), the stabilization is possible only at some initial stage of decreasing M, which if continuing, can cause destabilization. The same is asserted in the neoclassical economy of Keynes [Keynes, 1936]. The quasistatic understanding of Marx for the emergence of economic cyclicality can be presented by the following sheme

$$\textit{cash turnover decrease} \Rightarrow \textit{profit decrease} \Rightarrow \textit{price increase} \Rightarrow \textit{cycles emergence} \qquad (4.22)$$

The first two implications present the behavior of a quasistatic system, describing stationary monetary circulation in the context of Tichonov's theorem. The last one, shows the issue from the qualitative analysis of the dynamic (attached) system (4.1-3), having as a result the stability condition (4.21). In this sense the implications (4.22) present transition from quasistatic system to its dynamic distortion. They are not ideological, but are both realistically confirmed by Marx data description, as well as mathematically wellgrounded here. Another thing is that they can be accepted or rejected for ideological usage.

The application of formula (4.21) to the quantity theory of money (Fisher, 1933, Friedman, 1987) is the following: In accordance with the basic formula of the theory (4.9), the increasing of the circulatory money M is always accompagned with a proportinl increase of the price P, at condition that T and V are constant. In this case, in accordance with (4.21) destabilization occurs, because the left side increses, and the right one is constant. Vice versa, at decreasing of M, in view of (4.9), P will decrease too, and the inequality (4.21) will enforce itself, which means stabilization of the stationary cash turnover. These conclusions are in full accordance with the understanding of the mentioned quantity theory of money.

In this way, Fisher and Friedman reach in their monetary theory relatively optimistic conclusions for possible conserving stable equilibrium and thus avoiding the economic cyclicality provoked by monetary emissions. In their quasistatic scheme, the second implication of Marx in (4.22) is replaced by opposite one and as a result the final implication is opposite too: In their theory the circulatory money decrease does not lead to economic cyclicality, but on the contrary – it stabilizes the monetary circulation. *However, the economic practice confirms the predictions of Marx for ciclicality, but not that of the monetary theory for this phenomenon.* In terms of the proposed here mathematical interpretation of Marx theory, the market *equilibrium destabilization* and economic *cyclicality* inevitably occur, when we follow Friedman recommendation for restriction of taxation and state expenditures for consumption.

Keynes's optimism is more realistic (in comparison with the monetary theory) because of his basic idea for the economic stabilizing role of high consumption. He proposes progressive taxation for social purposes. In such a way, his approach consists in effectively countering the first datum (*circulatory money decrease*) in the abovepresented (by us) scheme of mechanism, which Marx claims for the emergence of market destabilization and cyclicality.

The conclusion from the above-quoted analysis here is that the dynamic system (4.1-3) can be used in each of these theories, as they deals only with parametric relationships between steady-state values satisfying Fisher equation.

The discussed economic doctrines do not consider dynamic deviations from the steady circulation, as required by the theorem of Tikhonov. Therefore, the comprehensive stability analysis of the process of monetary circulation remains outside their field of vision . Only Marx insists that the law of monetary circulation is systematically distorted. That's why he considers it as quasistatic process. The latter term was used at that time in thermodynamics, but not in the economy. So, Marx did not use the term too. In modern physical terms, this means that due to the changable nature of market variables, stationarity is distorted in one direction or another and so the law remains valid in an average sense. The opposite violations on the stationarity pass continously each into other and determine the non-stationary behavior of monetary circulation. In terms of modern dynamic theory [Tikhonov, 1952], this behavior is modeled as it is explained in section (1.3): Each market variable is represented as a sum of quasistationary (degenerate) component and dynamic (attached) components. For example, in the previous section 5, the components M, V, T and P, are also defined as quasi stationary, and X, Y and Z as dynamical ones.

The conseption of Marx for systematic distortion of the law of monetary circulation is extraneous to the later authors (monetarists and neo-classicists). For them, the Fisher equation and related stationarity of cash turnover is considered to be indisputable truth that is impossible to be distorted both practically and theoretically.

From the viewpoint of dynamical theory, the distortion of stationary circulation is a central problem of economics. As already stated, Marx tried to solve this problem by applying quasi-static (in terms of modern science) approach, since at his time the dynamic theory of stability was not existed yet. (For example, the mathematical definition of stability was given only after his death).

Neoclassicism and modern monetarism do not deny that the economy is a dynamic science. They recognize even the mere monetary circulation is also dynamic. The latter, of course, is difficult to be denied, in view of the presence of ubiquitous measured time series of varing market values. But when it comes to statistical average values, they systematically accept (without arguing) that these values (although variable, ie dynamic ones) satisfy the equation of Fisher. That means they consider only undistorted stationary circulation. The latter, of course, gives a reason to deny the existence of surplus value, to not intodicing the investors total profit, and hence to not considering the growing income disparity in the market society.

According to contained herein analysis of the dynamic system (4.1-3), the existence alternation of economic cycles during the last centuries (in a form of growth rates and downs of the market) is an undeniable evidence for the claim that the so-called stationary circulation is unstable (though theoretically existing), so that it is practically not realizable. Therefore, the corresponding equation of Fisher, (which as a mathematical definition of the money in circulation describes exactly this stationarity), is also practically invalid. This equation, as Marx himself argued, is continuously distorted in terms of so-called free private proprietary market.

Therefore, the understanding of modern neoclassical economic monetarism for "validity of the Fisher equation" is unacceptable from both an economic point of view (it actually ignores the most essential mechanism of property stratification) and in view of the modern dynamic theory (it assumes to be "practically realizable" an unstable stationary circulation of money). The evidence shows that *monetary circulation is a non-stationary oscillatory process* and this is a circumstance unconditionally supporting the views of Marx on the issue.

The basis of the examination in this article is that there are two types of distortions of the stationary (equilibrium) money circulation: (a) Dynamic variations with time of fast variables X, Y and Z, from their stationary values M, V and T, satisfying the equation of Fischer; (b) Quasi-static distortion of Fisher equation, due to the slow variation with time of the circulatory money, treated in the theories of Fisher - Friedman, Keynes , Marx and throughout macroeconomics at all. The most common link between dynamic and quasistationary distortion is given by the famous theorem of Tichonov. For a description of dynamic disturbances is derived dynamic system (4.1-3) of three ordinary differential equations, solved with respect to the first order derivatives of the dynamic variables. For this purpose, it is started from three evidently true postulates for qualitative relationship between market values. In the particular case of steady state (stationary) circulation, the mentioned autonomous, non-stationary system is reduced to the

Fisher equation. The exact qualitative analysis of the system (4.1-3) shows that the stationary circulation could be destabilized because of the inevitable decrease of *circulatory money*. Indeed, the *reserve money* in the bank system constitutes the whole *money supply*. The slow accumulation of *total profit* (actually in *unavailable money* of bank deposits) directly affects the structure of the *available money supply*, by increasing the amount of *bank reserve* at the expense of reduction of *circulatory money M*. This reduction (money deficit in the circulation) is naturally associated with a corresponding decrease of total consumption. In view of the stability condition (4.7), this leads to approaching stationarity destabilization and to increasing prices (from the side of business to compensate the lower placement). In result a new consumer downturn occurs and so on, the cycle repeats. On this quasi-static cycle, the dynamical (bifurcation) analysis of the system (4.1-3) shows that the price increase inevitably occurs and leads to loss of stability of the stationary circulation, around which periodic oscillations (economic cycles) arise. The latter represent a series of consecutive periods of economic gains and declines. All these events are evidently accelerated when we follow Friedman recommendation for restriction of taxation and state expenditures for consumption, because in this case the quantity of *circulatory money*decreases faster.

CONCLUSIONS

From the considerations of the present work, it can be drawn three main conclusions:

1. The theory of Marx for the inevitably emerging economic cyclicality is compatible with the results, obtained here from the qualitative analysis of the dynamical system (4.1-3).
2. In terms of the proposed here theory, the market equilibrium destabilization and economic cyclicality inevitably occur, when we follow Friedman recommendation for restriction of taxation and state expenditures for consumption.
3. The economic cyclicality can be avoided by sticking to Keynesian measures (progressive taxation and high social expenditures) to invalidate the implication from price increase to cash decrease in the mechanism leading to destabilization of market equilibrium and selfoscillation emergence.

Actually, the conclusions 1, 2 and 3, are more in detail presentation of the single sentence formulated in the introduction of this paper as its purpose.

Achowledgements

The author expresses his gratitude to the referee for the valuable comments and useful recommendations, contributing to the improvement of the paper.

Notes

1. The curves in the last figure 2 are similar to that of the famous Kondratieff"s one, published on the following web-address:

   http://www.google.bg/imgres?imgurl=http://www.earthsharing.org.au/wp-content/uploads/kondratieff.gif&imgrefurl=http://www.earthsharing.org.au/facts-and-figures/collapsing-economies-and-national-resource-rents/&h=348&w=537&sz=8&tbnid=xBd3zsL8FrxmmM:&tbnh=89&tbnw=137&zoom=1&usg=__H8X_OQzyd4xUIU3vWJf5XmLcfBg=&docid=2PD0EuaefRS2aM&sa =X&ei=nuoyUpzdMZKHswaVyYCgBQ&ved=0CGgQ9QEwDw&dur=2021

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