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A New Measure of International Risk Sharing

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Abstract: Empirical anomalies are found with two conventional approaches to measuring extent of international risk sharing, a process in which country-specific income shocks being smoothed out via cross-border asset transactions. Regression approach, by running a regression of idiosyncratic consumption growth on idiosyncratic income growth, suggests that developing countries are better at sharing income risks than more-developed countries. Correlation approach, by comparing consumption growth correlation with income growth correlation, leads to the consumption correlation puzzle, and when being extended to less advanced countries shows that consumption correlation is even smaller than income correlation for advanced countries than for other less-developed counterparts. Our proposed single-country risk-sharing model shows that a country where productivity shocks driving income fluctuations would have a larger regression coefficient, i.e. a lower degree of risk sharing. Extending the model to a multi-country version shows that both global permanent and transitory income shocks would drive comovement of consumption growth across countries under perfect risk sharing condition. The fact that consumption only reacts to permanent shocks suggests that consumption correlation puzzle could be due to the relatively dominant role played by global transitory income shocks. Estimates of the new risk-sharing measure embedded in the single-country model show that advanced countries are on average able to share 86 percent of income risks while emerging and developing countries can eliminate 68 percent and 48 percent, respectively.

JEL Classification: F1, F4

Keyword: international risk sharing, consumption correlation puzzle, cointegrated trend-cycle decomposition, Gibbs sampling

1. INTRODUCTION

International risk sharing refers to the situation in which country-specific income shocks are smoothed out by buying and selling financial assets across borders. Knowledge of the extent of mitigated income shocks has important policy implications for the involved governments, the works of global institutions, e.g. the International Monetary Fund or the World Bank (see IMF (2013)), and basic economic research. There are a few approaches of how to measure the degree of International risk sharing in the literature. One well-known approach is to compare the correlation of consumption growth rate across countries with that of income growth rate proposed by Backus, Kehoe, and Kydland (1992). Another is to run a regression of consumption growth on income growth in the framework developed by Asdrubali, Sorensen, and Yosha (1996).

According to the standard theory, the consumption correlation should be larger than income correlation when risks are shared internationally (Backus *et al.*, 1992). Theoretical results also imply that consumption growth is much less dependent on country-specific income growth when risk sharing is complete (Artis & Hoffmann, 2008; Asdrubali *et al.*, 1996; Bai & Zhang, 2006; Baxter, 2011; Cochrane, 1991; Lewis, 1996; Mace, 1991; Obstfeld, 1994).

In fact, neither of the above properties holds in the observed data. The former is the well-known "consumption correlation puzzle" and the latter we call "over dependence puzzle". Table 1 and Table 2 show them. In addition, when we calculate the above correlations and regression in different income groups, we found additional empirical anomalies. First, the higher the income of a country, the larger will be the spread between income and consumption correlation (correlation reversal puzzle). Second, the higher the income of a country, the more dependence the consumption growth will be on the country-specific income growth (the dependence reversal puzzle).

The causes of the four puzzles are the issue addressed by several papers in the literature (Canova & Ravn, 1996; Chari, Kehoe, & McGrattan, 1997; Lewis, 1995, 1996, 1997; Obstfeld & Rogoff, 2000; Stockman & Tesar, 1995; Tesar, 1993). However, definite answers to the puzzles remain unfound.

The goals of this paper are to investigate the issue by focusing on the trend-cycle decomposition of income and consumption and provide a new measure of the extent of international risk sharing for individual countries. The idea behind this approach is that consumption and income are correlated (or consumption and income are cointegrated), and it must be the cyclical income shock to consumption that can be hedged by cross-border asset transactions. In contrast, permanent income shocks represent undiversifiable aggregate shocks. If consumption were a random walk *prior* to any risk sharing activity taken place, countries would not economically need international risk sharing activities because they would become useless. Consumption could be a random walk, as proposed by Hall (1978), since it is very likely that consumption data was observed *after* risk sharing activities occurred. Cochrane (1994) shows that consumption is nearly a random walk for US data, which would imply that risk sharing activities in the United States possibly have been done substantially. Results of test for random walk in consumption data of other countries indicate that consumption is not a random walk. The ultimate goal of international risk sharing activities is to allow countries to mitigate transitory consumption fluctuations, so that the countries can consume according to their long-term incomes. Consequently, post-risk-sharing consumption data would exhibit characteristics of a random walk if risk sharing was carried out perfectly, and measures of degrees of international risk sharing should indicate how well transitory shocks are reduced. Conventional approaches are likely unable to separate permanent and transitory shocks completely, therefore, incorrectly show the extent of mitigated transitory shocks for these countries.

The rest of paper is organized as follows. The next section presents the consumption correlation puzzle and other empirical anomalies using regression approach. Section 3 will present two semi-structural models to examine dependence puzzle and consumption correlation puzzle. Section 4 presents estimates of degrees of risk sharing for individual countries. Section 5 concludes.

2. EMPIRICAL ANOMALIES

In this section, we show the four puzzles regarding to international risk sharing and discuss them in details.

2.1. Correlation puzzle

The result from a benchmark world economy with complete markets shows that there would be very high consumption correlation compared with income correlation. Specifically, Backus *et al.* (1992) found that consumption correlation is 0.88, compared with -0.21 for income correlation from their model. Accounting for transport costs increases consumption correlation to 0.89 while decreasing income correlation to -0.5. The very high consumption correlation is interpreted as reflecting agent's ability to share risk internationally (Backus *et al.*, 1992). This approach leads to the consumption correlation puzzle that found that income correlation is much higher than that of consumption in observed data (Backus *et al.*, 1992; Obstfeld & Rogoff, 2000).

We demonstrate the puzzle by calculating all possible pairwise correlations for consumption growth and income growth for countries in our sample, then taking average of consumption and income correlations by country groups. Table 1 reports the results.

As can be seen from the first two columns for each income group from the table, consumption correlation is always smaller than income correlation across all country groups. The averages of consumption and income correlations for advanced countries are 0.07 and 0.13, respectively. In contrast, the two measures for emerging countries are 0.04 and 0.07, and for developing countries 0.06 and 0.08. Using median does not change the feature seen from using mean.

There are also two noticeable feature from the table. The correlations calculated from observed data are nowhere close to the ones generated from economic models. While the literature leaves open the question how much difference consumption and income correlation would be to be considered existence of risk sharing, it is counter-intuitive to see the difference between income and consumption correlations becomes larger when countries get richer.

2.2. Dependence puzzle

Regression approach is based on the observation that marginal utility growth of country i would be independent from country-specific risks if risk sharing were complete (Artis & Hoffmann, 2008; Asdrubali *et al.*, 1996; Bai & Zhang, 2006; Baxter, 2011; Cochrane, 1991; Lewis, 1996; Mace, 1991; Obstfeld, 1994). The coefficient $\hat{b}$ from a panel regression of idiosyncratic consumption growth on idiosyncratic income growth indicates the percentage of consumption growth remained dependent on domestic income fluctuations (Asdrubali *et al.*, 1996). Its complement, $1-\hat{b}$, therefore indicates the percentage of consumption growth being independent from country-specific income shocks. Perfect risk sharing would be achieved when $\hat{b} = 0$ or $1-\hat{b}=1$.

An economic issue with the regression approach using panel data is that countries are assumed to have equal degree of risk sharing. Grouping countries according to development levels, however, also suggests that the top-ranked country within a group can share income risks as well as the lowest-ranked country, e.g. the United States and Greece in OECD group. When running country-specific consumption growth on idiosyncratic income growth and taking average of the coefficient by country groups, it is intuitive to expect that the result would be larger for less developed countries. Nevertheless, it is seen in Table 2 that a reverse independence between country-specific income fluctuation and consumption growth when countries become more developed. The mean value of the regression coefficient for advanced economies are 0.88. In contrast, the values for emerging and developing economies are 0.81 and 0.69, respectively. Similar patterns can be seen when using median.

3. SEMI-STRUCTURAL APPROACH USING UNOBSERVED COMPONENT MODEL

3.1. Examining the dependence puzzle

The single-country model in this section and its extension presented in the next section draw their motivations and intuition from studies on international risk sharing and cointegration. We will use this model to investigate factors that influence the coefficient in the regression approach, and therefore are able to provide an explanation to the counter-intuition empirical results shown in Table 2.

The observed stability of the ratio of consumption over income (measured by GDP or GNP) over a long period is consistent with economic theories, e.g. permanent income hypothesis or balanced growth models. Statistically, the ratio indicates that consumption and income are cointegrated. Studies have shown cointegrated time series can be represented by a common stochastic trend and a transitory component (King, Plosser, Stock, & Watson, 1991; Stock & Watson, 1988), which could also be viewed as undiversifiable and diversifiable shocks, respectively. The transitory component (or diversifiable shocks) indicates short-run disequilibrium from the long-run trend. The common trend, on the other hand, indicates permanent undiversifiable shocks that changes steady state of the economy, hence represents the long-term prospect of the economy.

Notice that there would be no scope for international risk sharing activities if consumption fluctuations were entirely driven by undiversifiable shocks. Similar arguments apply when consumption fluctuations are only due to undiversifiable preference shocks, which are found to be undiversifiable (Stockman & Tesar, 1995). If consumption were not exactly a random walk, and its fluctuations were due to a mixture of preference shocks and transitory income shocks, there would be some room for international risk sharing activities. Cross-border asset transactions would reduce impacts of the transitory income shocks on consumption.

Let assume a country to be represented by an agent who attempts to maintain consumption and income in equilibrium in the long-run and to reduce transitory consumption fluctuations by engaging in cross-border financial transactions. While the agent is unable to diversify permanent productivity shocks, the shocks would alter the steady state of the economy. This would signal market participants the long-

Table 1
Consumption and income correlation for country group

Variable	*Obs*	*Mean*	*Median*	*Std. Dev.*	*Min*	*Max*
Advanced Economies						
Consumption correlation	23	0.07	0.07	0.04	0.00	0.15
Income correlation	23	0.13	0.13	0.04	0.04	0.18
Emerging Economies						
Consumption correlation	15	0.05	0.06	0.06	-0.06	0.14
Income correlation	15	0.08	0.08	0.05	-0.03	0.16
Developing Economies						
Consumption correlation	31	0.05	0.05	0.03	-0.03	0.12
Income correlation	31	0.06	0.07	0.04	-0.04	0.12

Table 2
Regression coefficient

Variable	*Obs*	*Mean*	*Median*	*Std. Dev.*	*Min*	*Max*
Advanced Economies	23	0.88	0.91	0.10	0.60	1.00
Emerging Economies	15	0.81	0.83	0.15	0.48	1.04
Developing Economies	31	0.69	0.72	0.24	0.00	1.13

term prospect of the country. Economically, countries are willing to engage in cross-border financial transactions to share income shocks if their counterparts have promising long-term macroeconomic prospects. In such situations, the latter would be able to reduce impacts of idiosyncratic income shocks by borrowing and lending with others on capital and credit markets. The degrees of shock reduction would be determined by institutional arrangements and the extent of their financial market developments and integration to the world's.

A parametric trend-cycle decomposition model capturing the above descriptions is presented in the following statistical form:

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1 & 0 \\ -\beta & 1 \end{bmatrix} \begin{bmatrix} \xi_t^y \\ \xi_t^c \end{bmatrix} \tag{1}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t \tag{2}$$

$$\begin{bmatrix} \phi_y(L)\xi_t^y \\ \phi_c(L)\xi_t^c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix} \tag{3}$$

where $$\begin{bmatrix} \eta_t \\ \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix} \sim \text{iid}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_c^2 \end{bmatrix} \right)$$

where y_t and c_t indicate logarithm values of annual GDP and consumption of the country (or agent) at time t, τ_t is the common stochastic trend or "permanent income", ξ_t^y is cyclical income shock, and ξ_t^c is cyclical consumption shock.

The model could be viewed as a decision-making scheme for the representative agent. We see that at each time t there are three uncorrelated shocks affecting the dynamics of stochastic trend, business cycle income and consumption fluctuations. The three shocks are transitory income, ε_t^y, consumption shocks, ε_t^c, and a permanent productivity shock, η_t. The shock ε_t^y could also be thought of as a structural transitory income shock influencing cyclical income fluctuations while ε_t^c could be viewed as a preference shock. The transitory income shock is a diversifiable shock that has no long-run effects on either output or consumption. In contrast, the productivity shock has long-term impacts on output and is undiversifiable by international risk sharing activities. The preference shock ε_t^c is undiversifiable (Stockman & Tesar, 1995). Consequently, international risk sharing activities are only able to eliminate impacts of transitory income shocks on consumption fluctuations.

The parameter β plays an economically important role in the decision scheme. It could be seen as the agent's hedging mechanisms against her/his income fluctuations by engaging in international portfolio diversification and other consumption smoothing activities, thanks to his economic prospect indicated by the long-term trend. In this perspective, the parameter would capture the agent's shock mitigation capability and willingness shown in economic policies and institutional arrangements. However, the development and global integration of financial markets of the country would impose some constraints on the risk-sharing willingness and capacity. Consequently, the agent may not be able to eliminate transitory income shocks fully, i.e. β would be less than its upper bound.

Dynamics of the common productivity shock is modeled to follow an unobservable random walk with deterministic drift that captures positive long-run growth rate in the economy. This form also reflects permanent effects of productivity shocks on the economy. Regarding transitory shocks, it is intuitive to realize that a consumption shock could affect business cycle dynamics of income. For instance, preference changes may cause positive and negative growth in some sectors in the economy simultaneously. Similarly, transitory income shocks could affect the cyclical dynamics of consumption. Looking at the economy as a whole, we however assume that only the latter occurrence, which could be thought of as the net impact of the two transitory shocks. In addition, thanks to the country's international risk sharing activities, the impacts of transitory income shock would be partially or fully mitigated if international risk sharing activities were performed well. Statistically speaking, the condition allows us to identify the impacts of transitory income shock in consumption dynamics and the parameter β's upper bound.

To make the model analytically tractable, we assume that the dynamics of the cyclical income and consumption follow finite-order autoregressive (AR) processes. In addition, our conclusions do not change under assumptions that transitory components follow $AR(1)$ processes and autoregressive coefficients are the same for both processes, i.e. $\phi_y(L) = \phi_c(L) = 1 - \phi L$.

We start our analysis of the model by noting that one of the special cases in this model is when $\beta = 0$. In that case, it is seen that $\text{var}(\Delta y_t) \leq \text{var}(\Delta c_t)$[1]. Another case is when $\beta = 0$ and there is a partial impact of transitory income shock on dynamics of consumption shocks. In this case, it is possible to have $\text{var}(\Delta y_t) \geq \text{var}(\Delta c_t)$ under condition that the variance of transitory income shock must be larger than variance of transitory consumption shock[2]. However, it is only economically sensible for this case to happen when the country manages to have partial transitory income shock on consumption dynamics initially.

When $|\phi| < 1$ so that $\lim_{k\to\infty} \phi^k \left(\Delta \xi_{t-k}^c - \beta \Delta \xi_{t-k}^y\right) = 0$, we can rewrite the equation of consumption growth as follows:

$$\Delta c_t = (\mu + \eta_t) + (1-\beta)\Delta \xi_t^y + \sum_{i=0}^{\infty} \phi^i \Delta \varepsilon_{t-i}^c \tag{4}$$

We can then see from the equation (4) the economically meaningful range for the shock mitigation parameter β is $0 \leq \beta \leq 1$. Full impact of contemporaneous income shock is seen when $\beta = 0$. In contrast, the income shock will be fully or partially canceled when $\beta = 1$ or $\beta \in (0, 1)$, respectively. It is also seen that consumption fluctuations would be driven by permanent income shocks and lags of preference shocks when full shock mitigation occurs, i.e. $\beta = 1$. In other words, consumption would be a random walk, as proposed in Hall (1978). Lastly, it is intuitive and economically sensible that the agent would not choose $\beta < 0$ as it would amplify income shocks.

We claim the following fact (see proofs in appendix). Under assumption $\phi_y = \phi_c = \phi$

Lemma: We have

$$\text{var}\begin{bmatrix}\Delta y_t \\ \Delta c_t\end{bmatrix} = \begin{bmatrix} \sigma_\eta^2 + \frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 & \sigma_\eta^2 + (1-\beta)\frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 \\ \bullet & \sigma_\eta^2 + \frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 + \frac{2}{1+\phi}\left(\sigma_{\varepsilon^y}^2 + \sigma_{\varepsilon^c}^2\right) - \left[1+\beta-\beta^2\right]\frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 \end{bmatrix}$$

Proposition 1: As directly implied by the lemma, we have

When $\beta = 0$, $$b = \frac{\text{cov}(\Delta y_t, \Delta c_t)}{\text{var}(\Delta y_t)} = \frac{\sigma_\eta^2 / \sigma_{\varepsilon^y}^2 + \frac{2}{1+\phi}}{\sigma_\eta^2 / \sigma_{\varepsilon^y}^2 + \frac{2}{1+\phi}} = 1$$

When $\beta = 1$, $$1 > b = \frac{\text{cov}(\Delta y_t, \Delta c_t)}{\text{var}(\Delta y_t)} = \frac{\sigma_\eta^2 / \sigma_{\varepsilon^y}^2}{\sigma_\eta^2 / \sigma_{\varepsilon^y}^2 + \frac{2}{1+\phi}} > 0$$

Proposition 2: The larger the variance ratio $\sigma_\eta^2 / \sigma_{\varepsilon^y}^2$, the more likely b is closer to 1.

As shown in proposition 1, the risk-sharing measure β and *b* are equivalent when countries are unable to share income risks. In contrast, the measure *b* will be strictly positive under perfect risk sharing. The size of measure *b* will be determined based on the relative sizes of $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}$ and $\frac{2}{1+\phi}$.

The proposition 2 can be partially seen from the formula of b. However, we will use simulations to complete and visualize the proof of the proposition. We show in **Figure 1** and **Figure 2** simulated patterns of b for different values of $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}$, and ϕ while assuming $\beta = 1$. Some important features can be seen from these figures as follows.

First, the variance ratio $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}$ determines the magnitude and distribution of b for various values of ϕ. Indeed, as shown in **Figure 1**, the magnitude of b becomes closer to one (1) as the ratio $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}$ becomes relatively larger and vice versa. The distribution of b becomes tighter or looser depending on the relative magnitude of $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}$ (see **Figure 2**). For instance, when $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}=5$, values of b runs from 0.73 to 0.82 (when ϕ varies from 0.1 to 0.9). The spread varies from 0.5 to 0.35 when the ratio $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}=1$. Values of b tightly varies from 0.05 to 0.09 when $\sigma_{\eta}^{2}/\sigma_{\varepsilon^y}^{2}=0.1$.

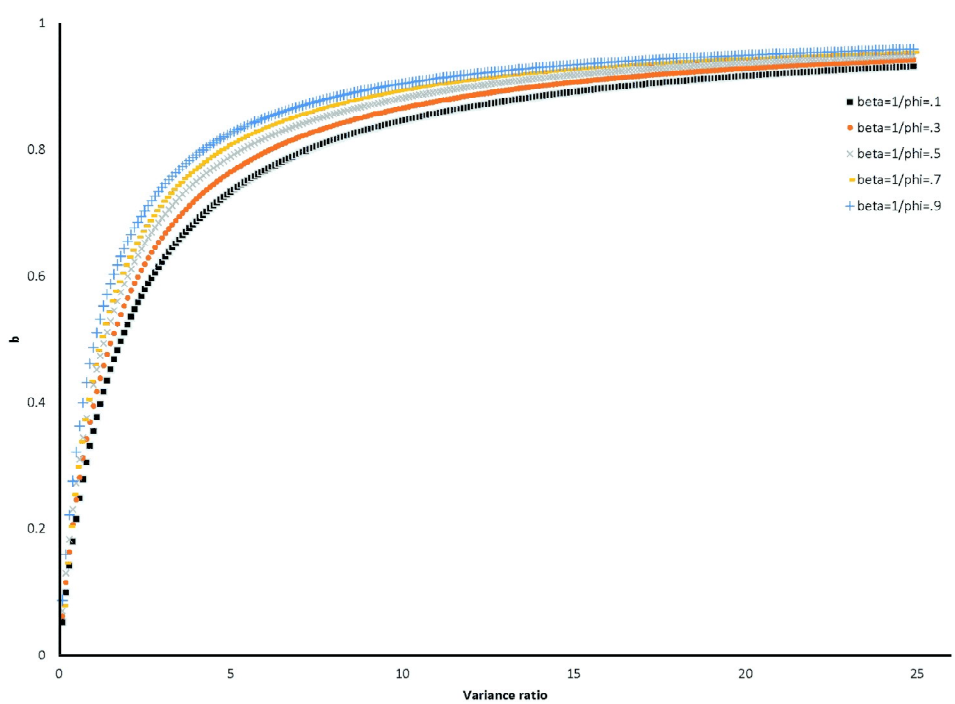

Figure 1: Patterns and distribution of simulated values of b when β = 1

Note: Simulated values of b^* for 5 different values of ϕ when holding β = 1.

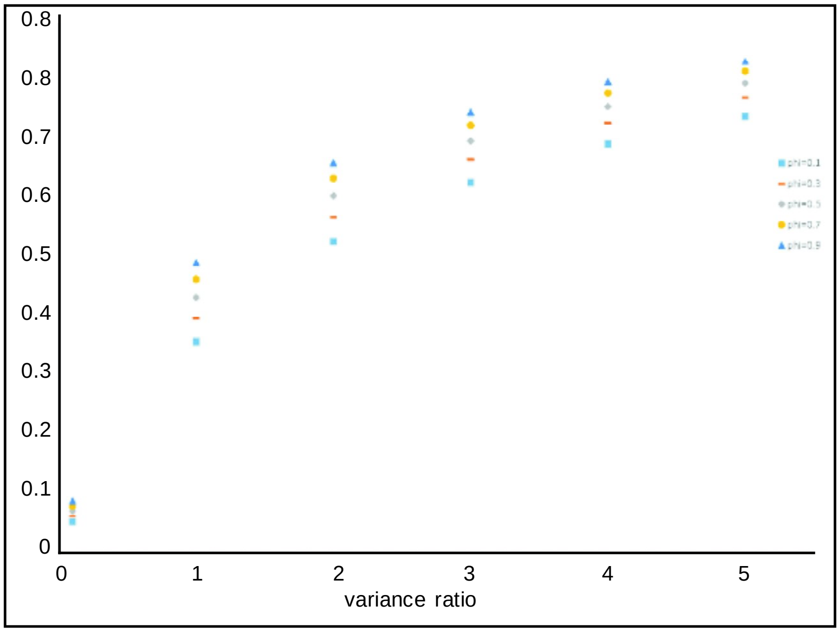

Figure 2: Distribution of b when β = 1

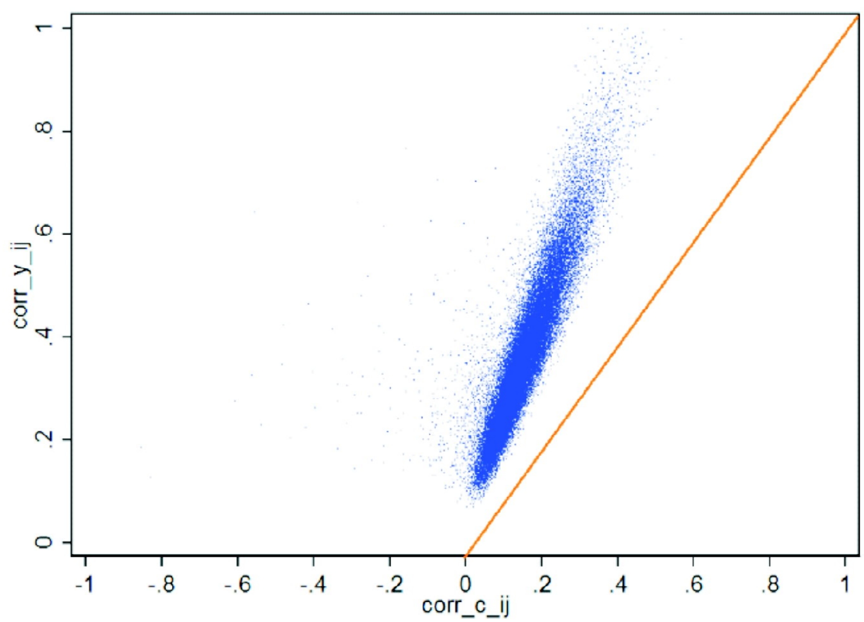

Figure 3: Simulated income and consumption correlation

Another subtle feature from **Figure 1** is that distribution of b is tighter below or beyond a threshold of $\sigma_{\eta}^{2}/\sigma_{\varepsilon^{y}}^{2}$. The more important feature from the figure is that, regardless of the values of ϕ, closer-to-zero b would be unlikely to obtain when the ratio $\sigma_{\eta}^{2}/\sigma_{\varepsilon^{y}}^{2}$ becomes relatively large and the shock mitigation parameter β to be 1. Lastly, larger ϕ leads to larger *b*.

These results suggest that a certain degree of shock mitigation measured by β could be mapped to many possible values of *b*, depending on the ratio $\sigma_{\eta}^{2}/\sigma_{\varepsilon^{y}}^{2}$ and ϕ. In other words, two countries having the same degree of risk sharing may have two very different values of *b* due to different values of ϕ, and $\sigma_{\eta}^{2}/\sigma_{\varepsilon^{y}}^{2}$.

The economic implications of the above propositions are as follows: a country experiencing more volatile business cycles would be likely to have smaller *b*, implying a higher degree of risk sharing for the country. This result suggests that *b* is not a reliable measure of international risk sharing. The proposition is also the analytical statement to the argument in Artis and Hoffmann (2008) in which they claims that the regression approach is unable to show that international risk sharing has been improved due to the fact that income became less volatile during the great moderation period in advanced countries. However, the argument is incomplete without taking into account the shock persistence.

The propositions also offer an insight into the empirical facts shown in Table 3 and possibly about the conclusion of lower degrees of risk sharing among advanced countries found in the literature. High fluctuations of income in developing countries falsely lower the coefficient attached to the income variable, hence, give the impression of better risk sharing in these countries. In contrast, relatively low

Table 3
Degrees of risk sharing

Country	*Degree of risk sharing*	*Country*	*Degree of risk sharing*
Switzerland	0.94	Brazil	0.75
Belgium	0.92	Thailand	0.70
Austria	0.91	Korea	0.80
Germany	0.94	Israel	0.76
Norway	0.87	South Africa	0.68
Sweden	0.89	Turkey	0.57
Japan	0.88	Morocco	0.66
Netherlands	0.87	Indonesia	0.71
Finland	0.90	Malaysia	0.57
Canada	0.83	Colombia	0.58
United Kingdom	0.82	India	0.74
Denmark	0.89	Mexico	0.57
United States	0.87	Mauritius	0.55
France	0.86	Senegal	0.59
Australia	0.84	Nicaragua	0.47
Spain	0.77	Tunisia	0.48
Italy	0.88	Jamaica	0.40
Greece	0.75	Costa Rica	0.37
Portugal	0.80		
New Zealand	0.81		

volatilities of income in more advanced countries hold the value of b being relatively closer to one, hence, understandably being interpreted as low international risk sharing among advanced countries.

3.2. Examining the correlation puzzle

In this section, we will extend the single-country model above to a multi-country model to shed light on consumption correlation puzzle that states that correlation of consumption growth is smaller than correlation of income growth in observed data. It is a puzzle because it is contradictory to theoretical results based on general equilibrium models.

The result from theory of international risk sharing shows that country-specific consumption fluctuation would be entirely due to aggregate shocks when countries share risk completely. Interpretation of aggregate shocks depends on the scope of sample of the study. Studies for advanced countries tend to consider fluctuations in US data or regional data as aggregate shocks. In contrast, studies whose samples including less-advanced countries tend to use fluctuations in weighted sum of consumption as aggregate shocks.

Practically, both global and country-specific aggregate shocks exist regardless of a degree of risk sharing of a country. In any time, possible sources of country-specific aggregate shocks include political risks, country-specific geographical conditions, technological knowledge stocks, entrepreneurial spirits, or demographic structures. Some examples of global aggregate shocks include nature and geographical conditions, or common political tensions among countries. These shocks are undiversifiable by any risk sharing activity.

Another feature of financial integration and globalization of economies is that the process facilitates transmission of income shocks across borders. It is usual to see impacts of income shock originated from one country to be felt in others. The case is immediately apparent when the shocks originate from a major economic hub. Notice that common income shocks may include permanent or transitory shocks or both. Examples of this possibility are technology breakthrough shared among a group of countries or a severe financial crisis that requires deep structural reform among shock-affected countries. The permanent shock would affect long-term outlook of countries while the transitory shock could create temporary positive or negative effects on national incomes. To simplify our presentation, we will assume that there is one common permanent income shock and one common transitory income shock each country has to deal with, in addition to its own permanent and transitory shocks.

We identify individual country by letters i and j in our extended model. The form of the extended model is similar to the single-country presented earlier.

$$\begin{bmatrix} y_{it} \\ c_{it} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_{it} + \begin{bmatrix} 1 & 0 \\ -\beta_i & 1 \end{bmatrix} \begin{bmatrix} \xi_{it}^{y} \\ \xi_{it}^{c} \end{bmatrix} \tag{5}$$

where the meaning of notations are similar to what were presented.

The new feature of the present model is that the unobserved stochastic trend now consists of a common global trend $\overline{\tau}_t$ and a country-specific stochastic trend τ_{it}^{*}. Both trends are modeled to follow unobservable random walk with deterministic drifts that capture positive long-run growth rates in both the world and individual economies

$$\begin{aligned}\tau_{it} &= \overline{\tau}_t + \tau_{it}^* \\ \overline{\tau}_t &= \overline{\mu} + \overline{\tau}_{t-1} + \overline{\eta}_t \\ \tau_{it}^* &= \mu_{it}^* + \tau_{it-1}^* + \eta_{it}^*\end{aligned} \tag{6}$$

where ξ_{it}^y and ξ_{it}^c are familiar stationary cyclical components of income and consumption for the country i at time t. β_i is country i's shock mitigation parameter and can be interpreted in a similar manner as one in the single-country model.

Another new feature of the current model compared with the previous one is the presence of common transitory income shock $\overline{\varepsilon}_t^y$. We again assume the unobservable transitory components follow finite-order autoregressive (*AR*) processes

$$\begin{bmatrix}\phi_{iy}(L)\xi_{it}^y \\ \phi_{ic}(L)\xi_{it}^c\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 1 & 1\end{bmatrix}\begin{bmatrix}\varepsilon_{it}^y \\ \varepsilon_{it}^c\end{bmatrix} \tag{7}$$

$$\varepsilon_{it}^y = \overline{\varepsilon}_t^y + \varepsilon_{it}^{*y} \tag{8}$$

where
$$\begin{bmatrix}\overline{\eta}_t \\ \eta_{it}^* \\ \overline{\varepsilon}_t^y \\ \varepsilon_{it}^y \\ \varepsilon_{it}^c\end{bmatrix} \sim \text{iid}\left(\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}\sigma_{\overline{\eta}}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_i}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\overline{\varepsilon}_y}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{iy}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ic}^2\end{bmatrix}\right)$$

To make the results tractable, we again assume transitory income and consumption shocks to follow *AR*(1) processes. We claim the followings (see proofs in appendix)

Proposition 3: Under one of the following assumptions:

1. Shock persistence are the same across countries: $\phi_{iy}(L) = \phi_{ic}(L) = \phi_{jy}(L) = \phi_{jc}(L) = 1 - \phi L$

2. Shock persistence are different across countries: $\phi_{iy}(L) = \phi_{ic}(L) = 1 - \phi_i L, \phi_{jy}(L) = \phi_{jc}(L) = 1 - \phi_j L$

when $\beta_i = \beta_j = 1$ consumption correlation is always smaller than income correlation

The economic intuition behind this proposition is seen by examining the formulas obtained in such cases. Under full risk sharing, comovement of consumption is driven by global income shocks that could be global permanent or global transitory income shocks or both. In contrast, comovement of income fluctuations is driven by the global income shocks *and* country-specific shock persistence. If we further assume that the impacts of country-specific transitory income shock on consumption are small, it is seen that country-specific consumption fluctuations would be affected by permanent shocks and transitory consumption shocks, e.g. preference shocks. Moreover, income fluctuation would be mainly driven by permanent shocks under the same assumption. In this case, the result is consistent with conclusion in Stockman and Tesar (1995) who found that both shocks to technology and preference shocks are needed to explain transmission of international business cycles.

3.3. More general cases

In more general cases, the relationship between consumption and income correlation is not entirely ambiguous when $\beta_i = \beta_j = 1$. However, simulations are needed to see the relationship clearer. We initially set $\beta_i = \beta_j = 1$, draw variances from diffuse gamma distributions and then calculate the correlations for income and consumption. The simulation results are presented in **Figure 3**. It is seen that income correlation is always larger than consumption correlation when $\beta_i = \beta_j = 1$ or when either $\beta_i = 1$ or $\beta_j = 1$. The reasons are that the component P_x^*, where $x = i, j$, of consumption variance is always positive while component K^* in the consumption covariance is always negative. This leads to smaller consumption covariance and larger consumption variance than those for income.

When we relax the assumption $\beta_i \neq \beta_j \neq 1$, the formulas indicate that both consumption and income correlation approaches 1 when the variance ratio $\sigma_{\bar{\eta}}^2 / \sigma_{\bar{\varepsilon}^y}^2$ dominates the effects of other parameters. Using simulations, it is observed that consumption correlation is always smaller than income correlation when $\sigma_{\bar{\eta}}^2 / \sigma_{\bar{\varepsilon}^y}^2$ is relatively smaller than other variance ratios. In other cases, there are always instances of having larger consumption correlation than income correlation, regardless of values of β.

Based on these results, we conjecture that the consumption correlation puzzle could be due to the relatively dominant role of the global transitory income shock in relation to the permanent income shock. In such case, consumption, in contrast with income, may not change due to the transitory nature of the shock, leading to the lower consumption correlation than income correlation.

In summary, in this section we constructed two models to examine factors affecting conventional measures of international risk sharing. The coefficient in the regression approach is subject to volatility and persistency of income and consumption shocks. Under some assumptions on dynamics of income and consumption, it can be shown that consumption correlation is always smaller than income correlation when countries are able to completely eliminate impacts of contemporaneous transitory shocks on consumption growth. In a more general case, we think that the consumption correlation puzzle could be due to the relative important role played by global income shock in consumption decisions.

4. ESTIMATING INTERNATIONAL RISK SHARING

4.1. Empirical model and data

This section will present empirical results for our new measure of international risk sharing indicated by the parameter β. The empirical model is the single-country model presented in the first subsection of section 2. We will use AR(2) processes for empirical purpose. We rewrite the transition equations of our estimated model as follows

$$\tau_t = \mu + \tau_{t-1} + \eta_t \tag{9}$$

$$\begin{bmatrix} \xi_t^y \\ \xi_{t-1}^y \\ \xi_t^c \\ \xi_{t-1}^c \end{bmatrix} = \begin{bmatrix} \phi_{1y} & \phi_{1y} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \phi_{1c} & \phi_{1c} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \xi_{t-1}^y \\ \xi_{t-2}^y \\ \xi_{t-1}^c \\ \xi_{t-2}^c \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ 0 \\ \varepsilon_t^c \\ 0 \end{bmatrix} \quad (10)$$

where all innovations are assumed to occur independently and follow normal distributions

$$\begin{bmatrix} \eta_{it} \\ \varepsilon_{it}^y \\ \varepsilon_{it}^c \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta_i}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon_{iy}}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon_{ic}}^2 \end{bmatrix} \right).$$

We use the country sample in Kose *et al.* (2009) that consists of 69 countries. The sample is divided into three country groups of advanced, emerging and developing countries. The data are extracted from Penn World Table version 7.0 that covers the period of 1950-2009. Per capita real GDP, real private consumption, and real public consumption represent measures of national output, private consumption and government consumption for country i at time t, respectively. The sum of per capita real private consumption and real public consumption is henceforth labeled consumption. All data are in constant (2005) international prices. Cointegration tests are done following Johansen's method (Søren Johansen, 1988; Soren Johansen & Juselius, 1990). The model is estimated using both Gibbs sampling method for a linear state-space model and maximum likelihood (Kim and Nelson (1999)). Priors for variances are set at $IG\left(6, 0.01^2\right)$. The autoregressive coefficient of the cyclical component is set to follow $\phi_{1c} \sim N(0,1), \phi_{2c} \sim N(0,0.5)$ and $\phi_{1y} \sim N(0,1), \phi_{2y} \sim N(0,0.5)$. The risk sharing parameter is set to follow $\beta \sim N(1,10)$.

The model requires a long-run equilibrium between consumption and income, a feature that not all countries in our sample could meet. A cointegration test is needed to select a sub sample of countries whose data can be fitted to the model. The Johansen cointegration rank test is a test of the rank of the product of two matrices in the error correction form. If, after inference, the rank is deemed to be (r), then there are (r) cointegrating relationships or vectors in the system. The cointegration test is a sequence of tests. We ran the test with and without time trend component in the specification. The results indicate that all advanced countries apparently exhibit a cointegration relationship between income and consumption with rank 1. Out of 21 and 27 emerging and developing countries, only 14 and 6 countries passed the test, respectively. Following these test outcomes and to facilitate comparison of results between the two models easily, the reported estimation outcomes are for countries that passed the test.

4.2. Empirical results

Having done testing for cointegration for each country, we estimate the single-country model using both Gibbs sampling and maximum likelihood approaches. The results are very similar, indicating that results are unaffected by settings of prior distributions.

The estimation results for the single-country model is presented in Table 3. The results follow our intuition that advanced countries tend to share more consumption risks than others due to their deeper integration to the world financial markets and better income prospects. The mean degree of risk sharing measured by the risk-sharing parameter β for advanced country group is .86. It is seen that among advanced countries Switzerland and Germany are able to share the most income risks while Portugal is relatively worse. The United States is ranked in the middle among advanced countries. It is also interesting to see that countries in the spotlight of European sovereign debt being ranked at the bottom of the advanced group. On the other hand, the values of the parameter β for emerging and developing countries are on average .68 and .48, respectively. For emerging country group, Brazil or Korea are doing a better job than others in their group. In contrast, countries in the developing country group share income risks quite similarly.

In sum, we present in this section the empirical results for our single-country international consumption risk sharing model. The results suggest that advanced countries have relatively high degrees of risk sharing while most emerging and developing countries still need to further improve their risk sharing abilities.

5. CONCLUSIONS

This paper makes three contributions to the literature. We was able to pin point factors affecting the coefficient in the regression approach. We also provide new insights into the consumption correlation puzzle. Lastly, we propose a new aproach to measuring degree of international risk sharing for individual countries.

The coefficient in the regression approach is affected by volatility of income shock and persistency of income and consumption shocks. We show that countries whose income fluctuations dominately driven by productivity shocks would be likely to have larger coefficients and vice versa.

Under assumptions of equal shock persistence between and within countries, we show that consumption correlation is always smaller than income correlation when countries are able to reduce transitory shocks completely. We also show that global shocks drive comovement of consumption correlation while the global shocks and country-specific shock persistence influence income correlation. In more general cases, simulations indicate that consumption correlation is larger than income correlation under condition that the variance ratio of global shocks is relatively large. We conjecture that the consumption correlation puzzle could be due to the dominant role played by global transitory income shocks because consumption only reacts to permanent shocks.

Finally, the estimates of our new measure indicate that international risk sharing is much better for several advanced and some emerging countries. The estimation results indicate that advanced countries are able to reduce 86 percent of transitory income shocks while emerging and developing countries elimiate 68 percent and 48 percent of the shocks, respectively.

The new approach proposed in this paper can be extended in other dimensions for future research. In particular, the single-country can be extended by allowing permenent and transitory components to be correlated. Morley *et al.* (2003) show that the correlation allows the cointegrated variables to have different speeds of adjustment in terms of restoring the long-run relationship. This feature may shed light on the mechanisms of risk sharing processes, hence, explaining the differences of degrees of risk sharing among countries. Another direction includes decomposing the risk sharing parameter β into several components called risk sharing channels used by countries to share income risks. Finally, the dynamic feature of international risk sharing process can be seen with a time-varying risk sharing parameter β. We will leave these extensions to future research.

NOTES

1. When $\beta = 0$, we will have $\Delta y_t = \mu + \eta_t + (1-\phi L)^{-1}\Delta\varepsilon_t^y$, $\Delta c_t = \mu + \eta_t + (1-\phi L)^{-1}(\Delta\varepsilon_t^y + \Delta\varepsilon_t^c)$. This implies that $\operatorname{var}(\Delta y_t) = \sigma_\eta^2 + \frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 \le \sigma_\eta^2 + \left[\frac{2}{1+\phi}\right](\sigma_{\varepsilon^y}^2 + \sigma_{\varepsilon^c}^2) = \operatorname{var}(\Delta c_t)$

2. When $\frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 \ge \left[\frac{2}{1+\phi}\right](\gamma\sigma_{\varepsilon^y}^2 + \sigma_{\varepsilon^c}^2)$ and $\beta = 0$, we will have $\operatorname{var}(\Delta y_t) = \sigma_\eta^2 + \frac{2}{1+\phi}\sigma_{\varepsilon^y}^2 \ge \sigma_\eta^2 + \left[\frac{2}{1+\phi}\right](\gamma\sigma_{\varepsilon^y}^2 + \sigma_{\varepsilon^c}^2) = \operatorname{var}(\Delta c_t)$ where $\gamma \in [0,1]$.

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Appendix

A1. Proofs of propositions

1. Proof of proposition 1

$$\begin{bmatrix}\Delta y_t\\ \Delta c_t\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}\mu+\begin{bmatrix}1\\1\end{bmatrix}\eta_t+\begin{bmatrix}1 & 0\\ -\beta & 1\end{bmatrix}\begin{bmatrix}\left(1-\phi_y L\right)^{-1} & 0\\ 0 & \left(1-\phi_c L\right)^{-1}\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\Delta\varepsilon_t^y\\ \Delta\varepsilon_t^c\end{bmatrix}$$

$$\operatorname{var}\begin{bmatrix}\Delta y_t\\ \Delta c_t\end{bmatrix}=\begin{bmatrix}\sigma_\eta^2+\left(1+\frac{\left(1-\phi_y\right)^2}{1-\phi_y^2}\right)\sigma_{\varepsilon^y}^2 & \sigma_\eta^2+\left\{\left(1+\frac{\left(1-\phi_y\right)\left(1-\phi_c\right)}{1-\phi_y\phi_c}\right)-\beta\left(1+\frac{\left(1-\phi_y\right)^2}{1-\phi_y^2}\right)\right\}\sigma_{\varepsilon^y}^2\\ \bullet & *\end{bmatrix}$$

$$*=\sigma_\eta^2+\beta^2\left(1+\frac{\left(1-\phi_y\right)^2}{1-\phi_y^2}\right)\sigma_{\varepsilon^y}^2+\left(1+\frac{\left(1-\phi_c\right)^2}{1-\phi_c^2}\right)\left(\sigma_{\varepsilon^y}^2+\sigma_{\varepsilon^c}^2\right)-\beta\left(1+\frac{\left(1-\phi_y\right)\left(1-\phi_c\right)}{1-\phi_y\phi_c}\right)\sigma_{\varepsilon^y}^2=\operatorname{var}\Delta y_t+T$$

where

$$T=\left[\frac{2}{1+\phi_c}\left(\sigma_{\varepsilon^y}^2+\sigma_{\varepsilon^c}^2\right)\right]-\left[\beta\frac{2-\left(\phi_y+\phi_c\right)}{1-\phi_y\phi_c}+\left(1-\beta^2\right)\frac{2}{1+\phi_y}\right]\sigma_{\varepsilon^y}^2$$

Under the assumption of $\phi_y=\phi_c=\phi$:

$$b=\frac{cov\left(\Delta y_t,\Delta c_t\right)}{var\left(\Delta y_t\right)}=\frac{\frac{\sigma_\eta^2}{\sigma_{\varepsilon^y}^2}+\left(1-\beta\right)\frac{2}{1+\phi_y}}{\frac{\sigma_\eta^2}{\sigma_{\varepsilon^y}^2}+\frac{2}{1+\phi_y}}$$

1. Proof of proposition 2

$$var\begin{bmatrix}\Delta y_{it}\\ \Delta y_{jt}\\ \Delta c_{it}\\ \Delta c_{jt}\end{bmatrix}=\begin{bmatrix}\sigma^2_{\bar\eta}+\sigma^2_{\eta_i}+\sigma^2_{\bar\varepsilon^y}\left[1+\frac{(1-\phi_{iy})^2}{1-\phi^2_{iy}}\right]+\sigma^2_{\varepsilon_{iy}}\left[1+\frac{(1-\phi_{iy})^2}{1-\phi^2_{iy}}\right] & \sigma^2_{\bar\eta}+\sigma^2_{\bar\varepsilon^y}\left[1+\frac{(1-\phi_{iy})(1-\phi_{jy})}{(1-\phi_{iy}\phi_{jy})}\right] & \dagger & \lozenge\\ * & ** & \circ & \bullet\\ \dagger & \circ & 3* & 4*\\ \lozenge & \bullet & 4* & 5*\end{bmatrix}$$

$$3*=\sigma^2_{\bar\eta}+\sigma^2_{\eta_i}+\left[1+\frac{(1-\phi_{ic})^2}{1-\phi^2_{ic}}\right]\sigma^2_{\bar\varepsilon^y}+\beta_i^2\left[1+\frac{(1-\phi_{iy})^2}{1-\phi^2_{iy}}\right]\sigma^2_{\bar\varepsilon^y}-\beta_i\left[1+\frac{(1-\phi_{ic})(1-\phi_{iy})}{(1-\phi_{ic}\phi_{iy})}\right]\sigma^2_{\bar\varepsilon^y}$$
$$+\left[1+\frac{(1-\phi_{ic})^2}{1-\phi^2_{ic}}\right]\sigma^2_{\varepsilon_{iy}}+\beta_i^2\left[1+\frac{(1-\phi_{iy})^2}{1-\phi^2_{iy}}\right]\sigma^2_{\varepsilon_{iy}}-\beta_i\left[1+\frac{(1-\phi_{ic})(1-\phi_{iy})}{(1-\phi_{ic}\phi_{iy})}\right]\sigma^2_{\varepsilon_{iy}}+\left[1+\frac{(1-\phi_{ic})^2}{1-\phi^2_{ic}}\right]\sigma^2_{\varepsilon_{ic}}$$

$$4*=\sigma^2_{\bar\eta}+\beta_i\beta_j\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]\sigma^2_{\bar\varepsilon^y}-\left[\beta_i\frac{2-(\phi_{ic}+\phi_{jy})}{1-\phi_{ic}\phi_{jy}}+\beta_j\frac{2-(\phi_{jc}+\phi_{iy})}{1-\phi_{jc}\phi_{iy}}\right]\sigma^2_{\bar\varepsilon^y}+\left[\frac{2-(\phi_{ic}+\phi_{jc})}{1-\phi_{ic}\phi_{jc}}\right]\sigma^2_{\bar\varepsilon^y}$$

$$5*=\sigma^2_{\bar\eta}+\sigma^2_{\eta_j}+\left[1+\frac{(1-\phi_{jc})^2}{1-\phi^2_{jc}}\right]\sigma^2_{\bar\varepsilon^y}+\beta_j^2\left[1+\frac{(1-\phi_{jy})^2}{1-\phi^2_{jy}}\right]\sigma^2_{\bar\varepsilon^y}-\beta_j\left[1+\frac{(1-\phi_{jc})(1-\phi_{jy})}{(1-\phi_{jc}\phi_{jy})}\right]\sigma^2_{\bar\varepsilon^y}$$
$$+\left[1+\frac{(1-\phi_{jc})^2}{1-\phi^2_{jc}}\right]\sigma^2_{\varepsilon_{jy}}+\beta_j^2\left[1+\frac{(1-\phi_{jy})^2}{1-\phi^2_{jy}}\right]\sigma^2_{\varepsilon_{jy}}-\beta_j\left[1+\frac{(1-\phi_{jc})(1-\phi_{jy})}{(1-\phi_{jc}\phi_{jy})}\right]\sigma^2_{\varepsilon_{jy}}+\left[1+\frac{(1-\phi_{jc})^2}{1-\phi^2_{jc}}\right]\sigma^2_{\varepsilon_{jc}}$$

Hence, we have

$$\mathrm{cov}(\Delta c_{it},\Delta c_{jt})=\left\{\frac{\sigma^2_{\bar\eta}}{\sigma^2_{\bar\varepsilon^y}}+\beta_i\beta_j\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]-\left[\beta_i\frac{2-(\phi_{ic}+\phi_{jy})}{1-\phi_{ic}\phi_{jy}}+\beta_j\frac{2-(\phi_{jc}+\phi_{iy})}{1-\phi_{jc}\phi_{iy}}\right]+\left[\frac{2-(\phi_{ic}+\phi_{jc})}{1-\phi_{ic}\phi_{jc}}\right]\right\}\sigma^2_{\bar\varepsilon^y}$$
$$=\left[\frac{\sigma^2_{\bar\eta}}{\sigma^2_{\bar\varepsilon^y}}+\beta_i\beta_j\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]+K\right]\sigma^2_{\bar\varepsilon^y},\ K=-\left[\beta_i\frac{2-(\phi_{ic}+\phi_{jy})}{1-\phi_{ic}\phi_{jy}}+\beta_j\frac{2-(\phi_{jc}+\phi_{iy})}{1-\phi_{jc}\phi_{iy}}\right]+\left[\frac{2-(\phi_{ic}+\phi_{jc})}{1-\phi_{ic}\phi_{jc}}\right]$$

$$\mathrm{cov}(\Delta y_{it},\Delta y_{jt})=\sigma^2_{\bar\eta}+\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]\sigma^2_{\bar\varepsilon^y}=\left\{\frac{\sigma^2_{\bar\eta}}{\sigma^2_{\bar\varepsilon^y}}+\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]\right\}\sigma^2_{\bar\varepsilon^y}$$

When $\beta_i=\beta_j=1$:

$$\mathrm{cov}(\Delta c_{it},\Delta c_{jt})=\left\{\frac{\sigma^2_{\bar\eta}}{\sigma^2_{\bar\varepsilon^y}}+\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]-\left[\frac{2-(\phi_{ic}+\phi_{jy})}{1-\phi_{ic}\phi_{jy}}+\frac{2-(\phi_{jc}+\phi_{iy})}{1-\phi_{jc}\phi_{iy}}\right]+\left[\frac{2-(\phi_{ic}+\phi_{jc})}{1-\phi_{ic}\phi_{jc}}\right]\right\}\sigma^2_{\bar\varepsilon^y}$$
$$=\left[\frac{\sigma^2_{\bar\eta}}{\sigma^2_{\bar\varepsilon^y}}+\left[\frac{2-(\phi_{iy}+\phi_{jy})}{1-\phi_{iy}\phi_{jy}}\right]+K^*\right]\sigma^2_{\bar\varepsilon^y},\ K^*=-\left[\frac{2-(\phi_{ic}+\phi_{jy})}{1-\phi_{ic}\phi_{jy}}+\frac{2-(\phi_{jc}+\phi_{iy})}{1-\phi_{jc}\phi_{iy}}\right]+\left[\frac{2-(\phi_{ic}+\phi_{jc})}{1-\phi_{ic}\phi_{jc}}\right]$$

$$\operatorname{cov}\left(\Delta y_{it},\Delta y_{jt}\right)=\sigma_{\bar{\eta}}^{2}+\left[\frac{2-\left(\phi_{iy}+\phi_{jy}\right)}{1-\phi_{iy}\phi_{jy}}\right]\sigma_{\bar{\varepsilon}^{y}}^{2}=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[\frac{2-\left(\phi_{iy}+\phi_{jy}\right)}{1-\phi_{iy}\phi_{jy}}\right]\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$$

Under condition $\phi_{iy}=\phi_{ic}=\phi_{jy}=\phi_{jc}=\phi$, we will have $\operatorname{cov}\left(\Delta y_{it},\Delta y_{jt}\right)=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$

$$\operatorname{cov}\left(\Delta c_{it},\Delta c_{jt}\right)=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left(\beta_{i}\beta_{j}-\beta_{i}-\beta_{j}+1\right)\frac{2}{1+\phi}\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left(\left[\beta_{j}-1\right]\beta_{i}-\left[\beta_{j}-1\right]\right)\frac{2}{1+\phi}\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$$

$$=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[\beta_{j}-1\right]\left[\beta_{i}-1\right]\frac{2}{1+\phi}\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$$

Under full risk sharing $\beta_i=\beta_j=1$:

$$corr\left(\Delta c_{it},\Delta c_{jt}\right)=\frac{\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right\}}{\sqrt{\left[\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_i}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\left[1+\frac{\sigma_{\varepsilon_{iy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]+\frac{2}{1+\phi}\frac{\sigma_{\varepsilon_{ic}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]}\sqrt{\left[\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_j}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\left[1+\frac{\sigma_{\varepsilon_{iy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]+\frac{2}{1+\phi}\frac{\sigma_{\varepsilon_{jc}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]}}$$

$$<\frac{\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\right\}}{\sqrt{\left[\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_i}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\left[1+\frac{\sigma_{\varepsilon_{iy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]\right]}\sqrt{\left[\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_j}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\frac{2}{1+\phi}\left[1+\frac{\sigma_{\varepsilon_{jy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]\right]}}=corr\left(\Delta y_{it},\Delta y_{jt}\right)$$

Under assumption $\phi_{iy}=\phi_{ic}=\phi_i;\phi_{jy}=\phi_{jc}=\phi_j$, we will have

$$\operatorname{cov}\left(\Delta c_{it},\Delta c_{jt}\right)=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left(1+\beta_i\beta_j-\beta_i-\beta_j\right)\left[\frac{2-\left(\phi_i+\phi_j\right)}{1-\phi_i\phi_j}\right]\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$$

$$\operatorname{cov}\left(\Delta y_{it},\Delta y_{jt}\right)=\left\{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[\frac{2-\left(\phi_i+\phi_j\right)}{1-\phi_i\phi_j}\right]\right\}\sigma_{\bar{\varepsilon}^{y}}^{2}$$

When $\beta_i=\beta_j=1$:

$$corr\left(\Delta c_{it},\Delta c_{jt}\right)=\frac{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}}{\sqrt{\operatorname{var}\Delta y_{it}+\frac{2}{1+\phi_i}\frac{\sigma_{\varepsilon_{ic}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}}\sqrt{\operatorname{var}\Delta y_{jt}+\frac{2}{1+\phi_j}\frac{\sigma_{\varepsilon_{jc}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}}}<\frac{\frac{\sigma_{\bar{\eta}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[\frac{2-\left(\phi_i+\phi_j\right)}{1-\phi_i\phi_j}\right]}{\sqrt{\operatorname{var}\Delta y_{it}}\sqrt{\operatorname{var}\Delta y_{jt}}}=corr\left(\Delta y_{it},\Delta y_{jt}\right)$$

Where $\operatorname{var}\Delta y_{it}=\left\{\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_i}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[1+\frac{\sigma_{\varepsilon_{iy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]\left[\frac{2}{1+\phi_i}\right]\right\}$, $\operatorname{var}\Delta y_{jt}=\left\{\frac{\sigma_{\bar{\eta}}^{2}+\sigma_{\eta_j}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}+\left[1+\frac{\sigma_{\varepsilon_{jy}}^{2}}{\sigma_{\bar{\varepsilon}^{y}}^{2}}\right]\left[\frac{2}{1+\phi_j}\right]\right\}$