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HEAT AND MASS TRANSFER OF MHD FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM WITH PERIODIC PERMEABILITY

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Abstract: The effect of transverse periodic variation of permeability on the heat and mass transfer and the free convective flow of viscous electrically conducting incompressible fluid through a highly porous medium bounded by an infinite vertical porous plate is investigated. Approximate analytical solutions are found to determine velocity, temperature and concentration distributions. The effects of various non-dimensional parameters Permeability Parameter (K_0), Buoyancy ratio (N), Reynolds number (Re) and Schmidt number (Sc) on velocity, temperature and concentration profiles, the skin friction, the Nusselt number and the Sherwood number are studied numerically.

Keywords: Porous flow, Periodic permeability, Heat and mass transfer.

1. INTRODUCTION

The magnetohydrodynamic flow has various applications in designing, cooling systems, petroleum industry, purification of crude oil, polymer solutions etc. The flow through porous media have numerous engineering and geophysical applications, for example, in the field of agriculture engineering to study the underground water resources, in purification processes in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs etc. The free convection flow through a porous medium bounded by a vertical porous plate with constant suction when the free stream velocity oscillates in time with a constant mean value was investigated by Raptis and Perdikis [3]. Nelson and Wood [2] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. Trevison and Bejan [6] have analyzed natural convection heat and mass transfer through a vertical porous layer subjected to uniform flow of heat and mass from the side. The flow is driven by the combined buoyancy effect due to temperature and concentration variation through the porous medium. Singh and Rakesh sharma [5] have investigated the transverse periodic variation of permeability on heat transfer and free convective flow of a viscous incompressible fluid through a highly porous medium bounded by a vertical porous plate. Atul Kumar Singh *et al.*, [4] discussed the effect of permeability variation and oscillatory suction velocity on free convective heat and mass transfer flow of viscous incompressible fluid past an infinite vertical porous plate. Chitti Babu and Prasada Rao [1] studied the effect of transverse periodic variation of the permeability on the heat and mass transfer and the free convective flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite vertical plate.

Motivated by various applications of flows through a porous medium, we have extended the work of Chitti babu and Prasada Rao [1] to study the flow of viscous incompressible electrically conducting fluid through a highly porous medium bounded by an infinite vertical porous medium and by an infinite vertical porous plate with constant suction. The permeability of the porous medium is assumed to be of the form

$$k^*(z) = \frac{k_0^*}{(1 + \varepsilon \cos \pi z^*/l)}$$

where k_0^* is the mean permeability of the medium, l is the wave length of the permeability distribution and ε (<< 1) is the amplitude of the permeability variation. The problem becomes three-dimensional due to such a permeability variation.

2. MATHEMATICAL FORMULATION

We consider the flow of viscous incompressible electrically conducting fluid through a highly porous medium bounded by an infinite vertical porous medium and by an infinite vertical porous plate with constant suction. The plate is lying vertically on the $x^* - z^*$ plane with x^*-axis taken along the plate in the upward direction. The y^*-axis is taken normal to flowing laminarly with a uniform free stream velocity u^*. The permeability of the porous medium is assumed to be of the form

$$k^*(z) = \frac{k_0^*}{(1 + \varepsilon \cos \pi z^*/l)} \tag{2.1}$$

where k_0^* is the mean permeability of the medium, l is the wave length of the permeability distribution and ε (<< 1) is the amplitude of the permeability variation. The problem becomes three-dimensional due to such a permeability variation. All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. Denoting velocity components u^*, v^*, w^* in the direction x^*, y^*, z^* directions respectively and the temperature by T^*, the flow through a highly porous medium is governed by the following equations:

The basic equations are

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{2.2}$$

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) + \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right)$$
$$\frac{\nu}{k^*}(u^* - U) - \frac{\sigma B_0^2}{\rho}(u^* - U) \tag{2.3}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\nu}{k^*} v^* - \frac{\sigma B_0^2}{\rho} v^* \tag{2.4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\nu}{k^*} w^* - \frac{\sigma B_0^2}{\rho} w^* \tag{2.5}$$

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) \tag{2.6}$$

$$v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D\left(\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}\right) \tag{2.7}$$

where $g, r, n, p^*, \beta, \mu, c_p, k$ denote respectively, acceleration due to gravity, density, kinematic viscosity, fluid pressure, coefficient of volume expansion, viscosity, specific heat at constant pressure , thermal conductivity.

The boundary conditions are

$$\left.\begin{array}{l} y^* = 0, \quad u^* = 0, \quad v^* = -v, \quad w^* = 0, \quad T^* = T_w^*, \quad C^* = C_w^* \\ y^* = \infty, \quad u^* = u, \quad w^* = 0, \quad p^* = p_\infty^*, \quad T^* = T_\infty^*, \quad C^* = C_\infty^* \end{array}\right\} \tag{2.8}$$

where T_w^*, T_∞^* are the temperatures of the plate and the temperature of the fluid far away from the plate, p_∞^* is a constant pressure in the free stream and $\nu > 0$ is a constant and the negative sign indicates that suction is towards the plate.

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{l}; \quad z = \frac{z^*}{l}; \quad u = \frac{u^*}{U}; \quad v = \frac{v^*}{U}; \quad w = \frac{w^*}{U}; \quad p = \frac{p^*}{\rho U^2}$$

$$\theta = \frac{(T^* - T_\infty^*)}{(T_w^* - T_\infty^*)}, \qquad C = \frac{(C^* - C_\infty^*)}{(C_w^* - C_\infty^*)} \tag{2.9}$$

Equations (2.2) to (2.7) reduce to the following forms

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.10}$$

$$v\frac{\partial \mathrm{u}}{\partial \mathrm{y}} + w\frac{\partial \mathrm{u}}{\partial \mathrm{z}} = G\,\mathrm{Re}(\theta + NC) + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{(u-1)(1+\varepsilon\cos\pi z)}{\mathrm{Re}\,k_0} - M(u-1) \tag{2.11}$$

$$v\frac{\partial \mathrm{v}}{\partial \mathrm{y}} + w\frac{\partial \mathrm{v}}{\partial \mathrm{z}} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)\,v}{\mathrm{Re}\,k_0} - Mv \tag{2.12}$$

$$v\frac{\partial \mathrm{w}}{\partial \mathrm{y}} + w\frac{\partial \mathrm{w}}{\partial \mathrm{z}} = -\frac{\partial p}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)\,w}{\mathrm{Re}\,k_0} - Mw \tag{2.13}$$

$$v\frac{\partial \theta}{\partial \mathrm{y}} + w\frac{\partial \theta}{\partial \mathrm{z}} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \tag{2.14}$$

$$v\frac{\partial C}{\partial \mathrm{y}} + w\frac{\partial C}{\partial \mathrm{z}} = \frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{2.15}$$

where $\mathrm{Re} = \dfrac{Ul}{\nu}$ Reynolds number, $\mathrm{Pr} = \dfrac{\mu c_p}{k}$ Prandtl number

$Sc = \dfrac{\nu}{D}$ Schmidt number, $k_0 = \dfrac{k_0^*}{l^2}$ Permeability parameter

$G = \dfrac{\nu g \beta (T_w^* - T_\infty^*)}{U^3}$ Grashoff number, $N = \dfrac{\beta^* \Delta C}{\beta \Delta T}$ Buoyancy ratio

$M = \dfrac{\sigma B_0^2 l}{\rho U}$ Magnetic parameter

The corresponding boundary conditions are

$$\left.\begin{aligned} &y = 0, \quad u = 0,\ v = -1,\ w = 0,\ \theta = 1,\ C = 1 \\ &y \to \infty,\ u = 1,\ w = 0,\ p = p_\infty,\ \theta = 0,\ C = 0 \end{aligned}\right\} \tag{2.16}$$

3. METHOD OF SOLUTION

Assuming the solutions to be of the following form

$$\left.\begin{aligned} u(y, z) &= u_0(y) + \in u_1(y, z) + \in^2 u_2(y, z) + \dots \\ v(y, z) &= v_0(y) + \in v_1(y, z) + \in^2 v_2(y, z) + \dots \\ w(y, z) &= w_0(y) + \in w_1(y, z) + \in^2 w_2(y, z) + \dots \\ p(y, z) &= p_0(y) + \in p_1(y, z) + \in^2 p_2(y, z) + \dots \\ \theta(y, z) &= \theta_0(y) + \in \theta_1(y, z) + \in^2 \theta_2(y, z) + \dots \\ C(y, z) &= C_0(y) + \in C_1(y, z) + \in^2 C_2(y, z) + \dots \end{aligned}\right\} \tag{3.1}$$

when $\in = 0$ the problem reduces to two-dimensional free convective flow through porous medium with constant permeability which is governed by the following equations. When we take terms of order 1, the equations are

$$\frac{dv_0}{dy} = 0 \tag{3.2}$$

$$\frac{d^2u_0}{dy^2} - v_0 \operatorname{Re} \frac{du_0}{dy} - \left(\frac{1}{k_0} + M \operatorname{Re}\right) u_0 = -G \operatorname{Re}^2 (\theta_0 + NC_0) - \left(\frac{1}{k_0} + M \operatorname{Re}\right) \tag{3.3}$$

$$\frac{d^2\theta_0}{dy^2} - v_0 \operatorname{Re} \Pr \frac{d\theta_0}{dy} = 0 \tag{3.4}$$

$$\frac{d^2C_0}{dy^2} - v_0 \operatorname{Re} Sc \frac{dC_0}{dy} = 0 \tag{3.5}$$

The boundary conditions are

$$\left.\begin{aligned} &u_0 = 0,\ v_0 = -1,\ \theta_0 = 1,\ C_0 = 1 \ \text{ at } \ y = 0 \\ &u_0 = 1,\ p_0 = p_\infty,\ \theta_0 = 0,\ C_0 = 0 \ \text{ as } \ y \to \infty \end{aligned}\right\} \tag{3.6}$$

The solution of the equations (3.3) to (3.5) is

$$u_0 = 1 - G\lambda_0 e^{-\operatorname{Re}\Pr y} - GN\lambda_0' e^{-\operatorname{Re} Sc y} + A_2 e^{-\bar{R}y} \tag{3.7}$$

$$\theta_0 = e^{-\operatorname{Re}\Pr y} \tag{3.8}$$

$$C_0 = e^{-\mathrm{Re}\,Scy} \tag{3.9}$$

with $$v_0 = -1,\ p_0 = p_\infty,\ w_0 = 0 \tag{3.10}$$

When we take coefficient of order $\in$, the equations are

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{3.11}$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = G\mathrm{Re}(\theta_1 + NC_1) + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{((u_0 - 1)\cos\pi z + u_1)}{\mathrm{Re}\,k_0} - M(u_1 - 1) \tag{3.12}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{(v_1 - \cos\pi z)}{\mathrm{Re}\,k_0} - Mv_1 \tag{3.13}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{\mathrm{Re}\,k_0} - Mw_1 \tag{3.14}$$

$$v_1 \frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) \tag{3.15}$$

$$v_1 \frac{\partial C_0}{\partial y} - \frac{\partial C_1}{\partial y} = \frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}\right) \tag{3.16}$$

The boundary conditions are

$$\left.\begin{array}{l} u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0,\ C_1 = 0 \ \text{at} \ \ y = 0 \\ u_1 = 0,\ w_1 = 0,\ p_1 = 0,\ \theta_1 = 0,\ C_1 = 0 \ \text{as} \ \ y \to \infty \end{array}\right\} \tag{3.17}$$

To solve, consider v_1, w_1, p_1 of the following form

$$v_1(y, z) = -v_{11}(y)\cos\pi z \tag{3.18}$$

$$w_1(y, z) = \frac{1}{\pi} v_{11}'(y)\sin\pi z \tag{3.19}$$

$$p_1(y, z) = p_{11}(y)\cos\pi z \tag{3.20}$$

Substituting the expressions (3.18) to (3.20) into (3.13) and (3.14) we get

$$v_1(y,z) = A_1(\pi e^{-\lambda y} - \lambda e^{-\pi y} - \pi + \lambda)\cos\pi z \tag{3.21}$$

$$w_1(y,z) = \lambda A_1(e^{-\pi y} - e^{-\lambda y})\sin\pi z \tag{3.22}$$

$$p_1(y,z) = \frac{\lambda A_1\left(\pi \mathrm{Re} + \dfrac{1}{k_0} + M\,\mathrm{Re}\right)}{\mathrm{Re}} e^{-\pi y}\cos\pi z \tag{3.23}$$

Let us consider the solutions of the form

$$u_1(y,z) = u_{11}(y)\cos\pi z \tag{3.24}$$

$$\theta_1(y,z) = \theta_{11}(y)\cos\pi z \tag{3.25}$$

$$C_1(y,z) = C_{11}(y)\cos\pi z \tag{3.26}$$

Substituting (3.24) to (3.26) into (3.12), (3.15) and (3.16) we get

$$u''_{11} + \mathrm{Re}\,u'_{11} - \left(\pi^2 + \frac{1}{k_0} + M\,\mathrm{Re}\right)u_{11} = -\mathrm{Re}\,v_{11}u'_0 - G\,\mathrm{Re}^2(\theta_{11} + NC_{11}) + \frac{(u_0 - 1)}{k_0} \tag{3.27}$$

$$\theta''_{11} + \mathrm{Re}\,\mathrm{Pr}\,\theta'_{11} - \pi^2\theta_{11} = -\mathrm{Re}\,\mathrm{Pr}\,v_{11}\theta'_0 \tag{3.28}$$

$$C''_{11} + \mathrm{Re}\,Sc\,C'_{11} - \pi^2 C_{11} = -\mathrm{Re}\,Sc\,v_{11}C'_0 \tag{3.29}$$

The boundary conditions are

$$\left.\begin{array}{l} u_{11} = 0,\ \theta_{11} = 0,\ C_{11} = 0 \ \text{ at } \ y = 0 \\ u_{11} = 0,\ \theta_{11} = 0,\ C_{11} = 0 \ \text{ as } \ y \to \infty \end{array}\right\} \tag{3.30}$$

Hence the solutions are

$$u_1(y,z) = \begin{pmatrix} A_{31}e^{-\lambda y} + Be^{-(\lambda+\mathrm{Re}\,\mathrm{Pr})y} + B_1e^{-(\pi+\mathrm{Re}\,\mathrm{Pr})y} + B_2e^{-(\mathrm{Re}\,\mathrm{Pr})y} + B_3e^{-(\lambda+\mathrm{Re}\,Sc)y} \\ +B_4e^{-(\pi+\mathrm{Re}\,Sc)y} + B_5e^{-\bar{R}y} + B_6e^{-(\mathrm{Re}\,Sc)y} + A_{17}e^{-(\lambda+\bar{R})y} + A_{18}e^{-(\pi+\bar{R})y} \\ +A_{20}e^{-\lambda_1 y} + A_{24}e^{-\lambda_2 y} \end{pmatrix}\cos\pi z \tag{3.31}$$

$$\theta_1(y,z) = (A_6e^{-\lambda_1 y} + A_3e^{-(\lambda+\mathrm{Re}\,\mathrm{Pr})y} + A_4e^{-(\pi+\mathrm{Re}\,\mathrm{Pr})y} + A_5e^{-(\mathrm{Re}\,\mathrm{Pr})y})\cos\pi z \tag{3.32}$$

$$C_1(y,z) = (A_7e^{-\lambda_2 y} + A_8e^{-(\lambda+\mathrm{Re}\,Sc)y} + A_9e^{-(\pi+\mathrm{Re}\,Sc)y} + A_{10}e^{-(\mathrm{Re}\,Sc)y})\cos\pi z \tag{3.33}$$

where the constants are given by

$$\lambda = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(\pi^2 + M\,\mathrm{Re} + \frac{1}{k_0}\right)}}{2}; \qquad \lambda_1 = \frac{\mathrm{Re}\,\mathrm{Pr} + \sqrt{\mathrm{Re}^2\,\mathrm{Pr}^2 + 4\pi^2}}{2};$$

$$\lambda_0 = \frac{\mathrm{Re}^2}{\mathrm{Re}^2\,\mathrm{Pr}\,(\mathrm{Pr}-1) - \left(\frac{1}{k_0} + M\,\mathrm{Re}\right)}; \qquad \lambda_0' = \frac{\mathrm{Re}^2}{\mathrm{Re}^2\,Sc\,(Sc-1) - \left(\frac{1}{k_0} + M\,\mathrm{Re}\right)};$$

$$\lambda_2 = \frac{\mathrm{Re}\,Sc + \sqrt{\mathrm{Re}^2\,Sc^2 + 4\pi^2}}{2}; \qquad \overline{R} = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(M\,\mathrm{Re} + \frac{1}{k_0}\right)}}{2};$$

$$A_1 = \frac{1}{(\pi-\lambda)\,((\pi^2 + M\,\mathrm{Re})\,k_0 + 1)}; \qquad A_2 = G\lambda_0 + G\lambda_0' N - 1;$$

$$A_3 = \frac{\mathrm{Re}^2\,\mathrm{Pr}^2\,A_1\pi}{\lambda^2 + \mathrm{Re}\,\mathrm{Pr}\,\lambda - \pi^2}; \qquad A_4 = \frac{-\mathrm{Re}\,\mathrm{Pr}\,A_1\lambda}{\pi}; \qquad A_5 = \frac{\mathrm{Re}^2\,\mathrm{Pr}^2\,A_1(\pi-\lambda)}{\pi^2};$$

$$A_6 = -(A_3 + A_4 + A_5); \qquad A_8 = \frac{\mathrm{Re}^2\,Sc^2 A_1\pi}{\lambda^2 + \mathrm{Re}\,Sc\lambda - \pi^2}; \qquad A_9 = \frac{-\mathrm{Re}\,Sc\,A_1\lambda}{\pi};$$

$$A_{10} = \frac{\mathrm{Re}^2\,Sc^2 A_1(\pi-\lambda)}{\pi^2}; \qquad A_7 = -(A_8 + A_9 + A_{10});$$

$$A_{11} = \frac{-G\lambda_0\,\mathrm{Re}^2\,\mathrm{Pr}\,A_1\pi}{(\lambda + \mathrm{Re}\,\mathrm{Pr})^2 - \mathrm{Re}\,(\lambda + \mathrm{Re}\,\mathrm{Pr}) - \left(\pi^2 + M\,\mathrm{Re} + \frac{1}{k_0}\right)};$$

$$A_{12} = \frac{G\lambda_0\,\mathrm{Re}^2\,\mathrm{Pr}\,A_1\lambda}{(\pi + \mathrm{Re}\,\mathrm{Pr})^2 - \mathrm{Re}\,(\pi + \mathrm{Re}\,\mathrm{Pr}) - \left(\pi^2 + M\,\mathrm{Re} + \frac{1}{k_0}\right)};$$

$$A_{13} = \frac{G\lambda_0 \operatorname{Re}^2 \Pr A_1(\pi - \lambda)}{(\operatorname{Re}\Pr)^2 - \operatorname{Re}(\operatorname{Re}\Pr) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{14} = \frac{-G\lambda_0' \operatorname{Re}^2 Sc A_1 N\pi}{(\lambda + \operatorname{Re}Sc)^2 - \operatorname{Re}(\lambda + \operatorname{Re}Sc) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{15} = \frac{G\lambda_0' \operatorname{Re}^2 Sc A_1 N\lambda}{(\pi + \operatorname{Re}Sc)^2 - \operatorname{Re}(\pi + \operatorname{Re}Sc) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{16} = \frac{G\lambda_0' \operatorname{Re}^2 Sc N A_1(\pi - \lambda)}{(\operatorname{Re}Sc)^2 - \operatorname{Re}(\operatorname{Re}Sc) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{17} = \frac{\bar{R}\operatorname{Re} A_1 A_2 \pi}{(\lambda + \bar{R})^2 - \operatorname{Re}(\lambda + \bar{R}) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{18} = \frac{-\bar{R}\operatorname{Re} A_1 A_2 \lambda}{(\pi + \bar{R})^2 - \operatorname{Re}(\pi + \bar{R}) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{19} = \frac{-\bar{R}\operatorname{Re} A_1 A_2(\pi - \lambda)}{(\bar{R})^2 - \operatorname{Re}(\bar{R}) - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)}; \quad A_{20} = \frac{-G\operatorname{Re}^2 A_7}{\lambda_1^2 - \operatorname{Re}\lambda_1 - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)};$$

$$A_{21} = \frac{A_4}{\lambda_0 \Pr A_1 \pi} A_{11}; \qquad A_{22} = \frac{-A_5}{\lambda_0 \Pr A_1 \lambda} A_{12}; \qquad A_{23} = \frac{-A_6}{\lambda_0 \Pr A_1(\pi - \lambda)} A_{13};$$

$$A_{24} = \frac{-G\operatorname{Re}^2 A_8 N}{\lambda_2^2 - \operatorname{Re}\lambda_2 - \left(\pi^2 + M\operatorname{Re} + \frac{1}{k_0}\right)}; \qquad A_{25} = \frac{A_9}{\lambda_0' Sc A_1 \pi} A_{14};$$

$$A_{26} = \frac{-A_{10}}{\lambda_0' sc A_1 \lambda} A_{15}; \qquad A_{27} = \frac{-A_{11}}{\lambda_0' Sc A_1(\pi - \lambda)} A_{16}; \qquad A_{28} = \frac{A_{13}}{\operatorname{Re}^2 A_1 \Pr(\pi - \lambda) k_0};$$

$$A_{29} = \frac{A_{16}}{\mathrm{Re}^2 A_1 Sc(\pi - \lambda) k_0}; \qquad A_{30} = \frac{A_{19}}{k_0 \,\mathrm{Re}\, \bar{R} A_1 (\pi - \lambda)};$$

$$A_{31} = -\begin{pmatrix} A_{11} + A_{12} + A_{13} + A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} \\ + A_{22} + A_{23} + A_{24} + A_{25} + A_{26} + A_{27} + A_{28} + A_{29} + A_{30} \end{pmatrix};$$

$$B = A_{11} + A_{21}; \quad B_1 = A_{12} + A_{22}; \quad B_2 = A_{13} + A_{23} + A_{28}; \quad B_3 = A_{14} + A_{25};$$

$$B_4 = A_{15} + A_{26}; \quad B_5 = A_{19} + A_{30}; \quad B_6 = A_{16} + A_{27} + A_{29}$$

Skin Friction

Skin friction component in the non-dimensional form is given by

$$\tau_x = \frac{\tau_x^*}{\rho U^2} = \frac{\nu}{Ul}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{\mathrm{Re}}\left[\frac{du_0}{dy} + \varepsilon \frac{du_{11}}{dy} \cos \pi z\right]_{y=0}$$

Nusselt Number

The rate of heat transfer coefficient in terms of Nusselt number (Nu) is given by

$$Nu = \frac{-q^*}{\rho U c_p (T_w^* - T_\infty^*)} = \frac{k}{\rho U c_p l}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{d\theta_0}{dy} + \varepsilon \frac{d\theta_{11}}{dy} \cos \pi z\right]_{y=0}$$

Sherwood Number

The rate of mass transfer coefficient in terms of Sherwood number (Sh) is given by

$$Sh = \frac{-q^*}{\rho U c_p (C_w^* - C_\infty^*)} = \frac{k}{\rho U c_p l}\left(\frac{\partial C}{\partial y}\right)_{y=0} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{dC_0}{dy} + \varepsilon \frac{dC_{11}}{dy} \cos \pi z\right]_{y=0}$$

4. RESULTS AND DISCUSSION

We have considered the flow of a viscous, incompressible electrically conducting fluid through a highly porous medium bounded by an infinite vertical porous medium and by an infinite vertical porous plate with constant suction. This work aims at studying the effect of magnetic field on the velocity, temperature and concentration distributions, skin friction, heat transfer and mass transfer for various governing parameters Reynolds number (Re) and Permeability parameter (k_0). The Prandtl number (Pr) is taken to be 0.71, the Schmidt number (Sc) is assumed to be 0.24 and ε is taken as 0.2.

Figures (1)-(3) depict the effect of Magnetic parameter M, Permeability parameter (k_0) and Reynolds number (Re) on velocity u. The velocity steeply rises due to increase in Magnetic parameter (M) and Reynolds number (Re). Increasing Permeabilty parameter (k_0) tends to decrease the velocity component.

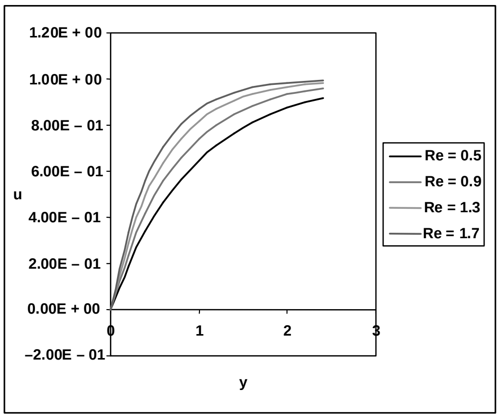

Figure 1: Effect of Reynolds Number Re on Velocity

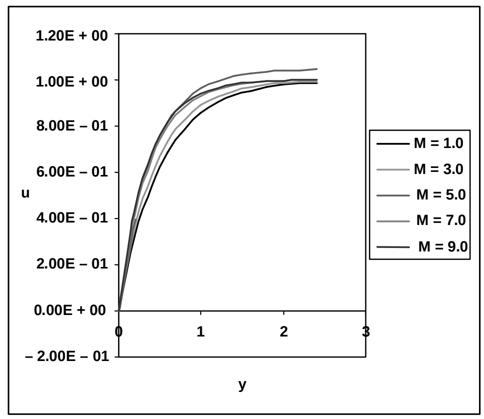

Figure 2: Effect of Magnetic Parameter *M* on Velocity

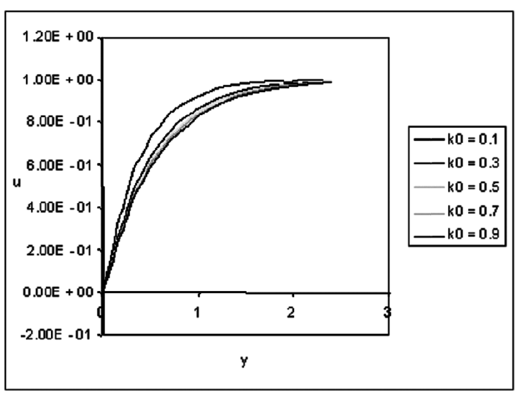

Figure 3: Effect of Permeability Parameter k_0 on Velocityi

Figures (4) and (5) exhibit the behaviour of temperature with respect to various parameters. The increase in Reynolds number (Re) and Prandtl number (Pr) decreases the temperature distribution.

Increase in Reynolds number (Re) and Schmidt number (Sc) tend to decrease the concentration distribution which can be seen from Figures (6) and (7).

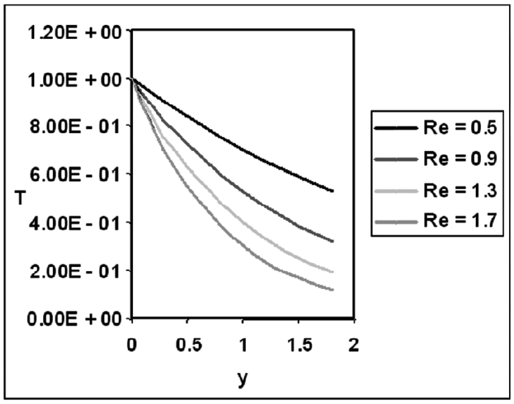

Figure 4: Effect of Reynolds Number on Temperature

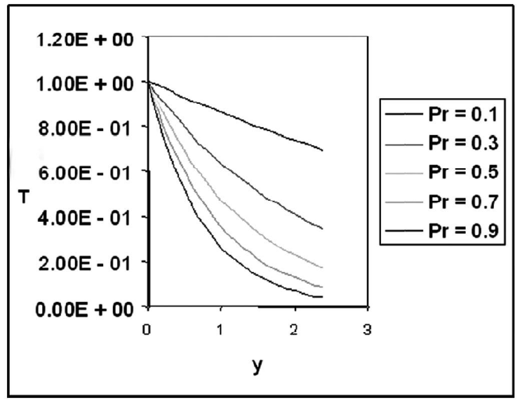

Figure 5: Effect of Prandtl Number on Temperature

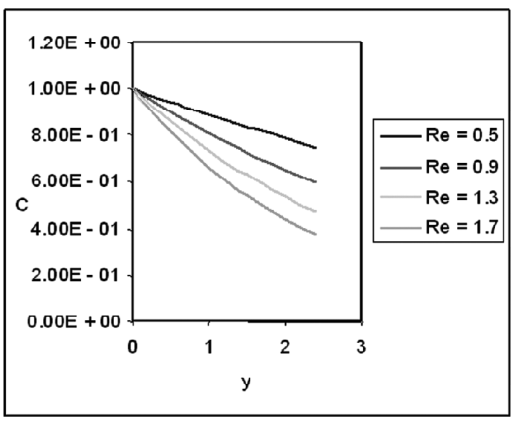

Figure 6: Effect of Reynolds Number Re on Concentration

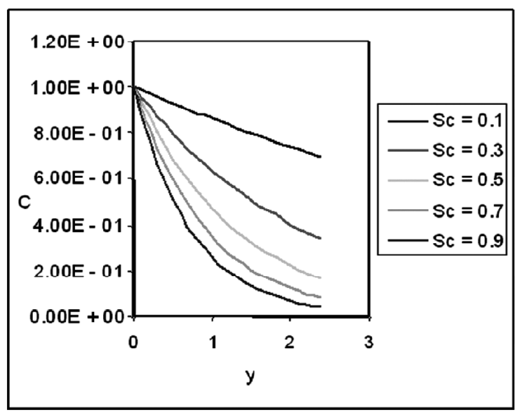

Figure 7: Effect of *Sc* on *C*

Figure (8)-(9) exhibit the effect of Magnetic parameter on Skin friction (t_x) at the vertical porous plate $y = 0$. As seen from these figures, increase in Reynolds number (Re) and Permeabilty parameter (k_0) decreases Skin friction (t_x). It may be deduced from these figures that increase in Magnetic parameter increases Skin friction at $y = 0$.

Figures (10) and (11) shows that increase in Prandtl number (Pr) decreases the rate of heat transfer whereas increase in Permeabilty parameter (k_0) decreases the same.

Figures (12) and (13) exhibit that increase in Prandtl number (Pr) and Permeabilty parameter (k_0) decreases the rate of mass transfer at $y = 0$. When the magnetic parameter is small, increase in the parameter enhances the mass transfer significantly. When the magnetic parameter is large, its effect in the mass transfer is negligible.

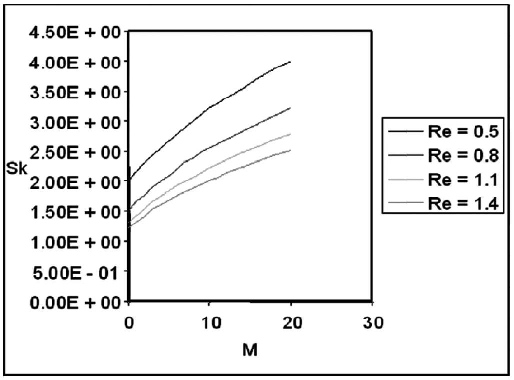

Figure 8: Effect of Reynolds Number Re on Skin Friction

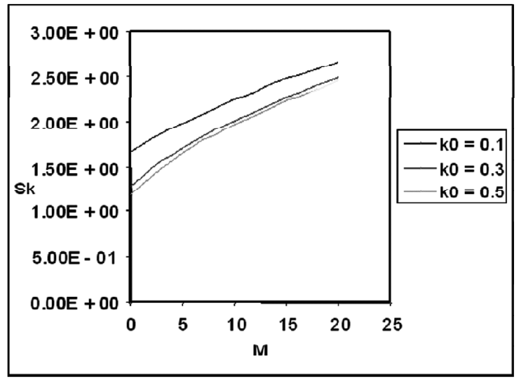

Figure 9: Effect of k_0 on Skin Friction

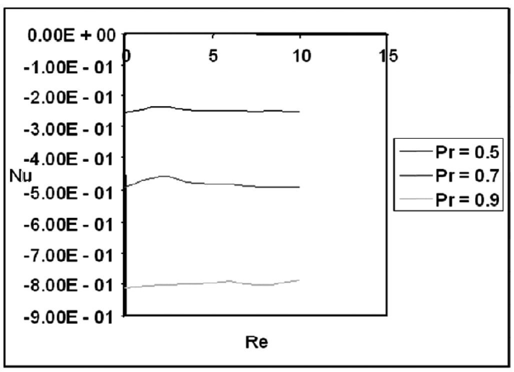

Figure 10: Effect of Prandtl Number Pr on Nusslet Number

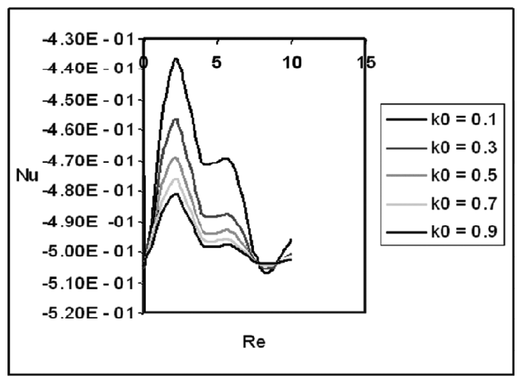

Figure 11: Effect of Permeability Parameter k_0 on Nusslet Number

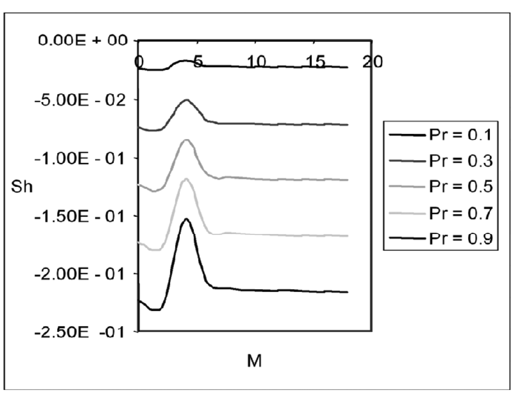

Figure 12: Effect of Prandtl Number Pr on Sherwood Number

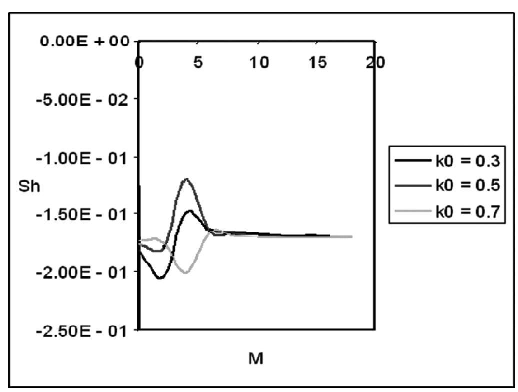

Figure 13: Effect of k_0 on *Sh*

5. CONCLUSION

Motivated by various applications of flows through a porous medium, we have extended the work of Chitti babu and Prasada Rao [1] to study the flow of viscous incompressible electrically conducting fluid through a highly porous medium bounded by an infinite vertical porous medium and by an infinite vertical porous plate with constant suction. Asymptotic solutions were found for velocity, temperature and concentration distributions. These results were computed numerically and graphs were drawn to study the effect of various non-dimensional parameters over these fields.

The velocity increases due to increase in Magnetic parameter (M) and Reynolds number (Re). The increase in Reynolds number (Re) and Prandtl number (Pr) decreases the temperature distribution. Increase in Reynolds number (Re) and Schmidt number (Sc) tend to decrease the concentration distribution.

Increase in Magnetic parameter increases Skin friction (t_x). Increase in Prandtl number (Pr) decreases the rate of heat transfer whereas increase in Permeabilty parameter (k_0) decreases the rate of heat transfer. Increase in Prandtl number (Pr) and Permeabilty parameter (k_0) decreases the rate of mass transfer.

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