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A 2 Dimensional Direction of Arrival Estimation with Pair Matching Algorithm for Adaptive Array Antenna

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ABSTRACT

It is known that smart antenna is adequate for 3G/ 4G mobile communication. Smart antenna is used in mobile communication to describe adaptive processes designed to improve the capacity and bandwidth[1] . It employs adaptive beamforming techniques. DOA (direction of arrival) estimation is an essential part of smart antenna. DOA estimation parameters are used for source localization and it provides the desired signal locations.

In this paper we consider a L shaped antenna array. L shaped array provides high estimation accuracy, low computational burden, and easy analysis. The most popular techniques in DOA estimation are MUSIC and ESPRIT. These algorithms are based on Eigen value decomposition (EVD) of cross spectral matrix .The computational complexities are very high and costly. The no of sources and antenna elements are large.

This paper will employ a novel algorithm which is implemented on the L shaped antenna array. The computational method is very simple. The conventional Pair matching method is adapted. We have compared our result with MUSIC and ESPRIT.

Key words: DOA, L shaped array, MUSIC, ESPRIT, Pair matching.

1. INTRODUCTION

The problem is to estimate elevation and azimuth angle of received signals. It has been shown an L shaped antenna array provides a better performance than any other shaped array (ULA, planner array, circular array, and parallel shaped array. For accurate and fast estimation of the direction of arrival of the transmitted signals a L shaped array antennas is considered. The detection and estimation of parameters of multiple waves are discussed.

Many DOA estimation algorithms have been developed using beam former methods like MUSIC [2], ESPRIT [3, 4]. A lot of work has been done on ULA and the1D DOA azimuth angle is estimated [8-11]. For 2 D DOA estimation any type of planner array is required.

Table 1
CRB $\cos(\alpha_1) = \cos(\beta_1)$

Octagon array	$57/\delta N^3$
L shaped Array	$60/\delta N^3$
Cross Array	$96/\delta N^3$
Square Array	$96/\delta N^3$
Right Triangle Array	$108/\delta N^3$
Generalized cross Array	$192/\delta N^3$

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In this paper we have presented MUSIC and ESPRIT. We propose a new algorithm 2 D DOA estimation. We compare our results with MUSIC and ESPRIT. A L shaped array consists of two ULA connected orthogonally at one end. The CRB (Cramer-Rao Bound) estimation of different array are given in table 1. [12]

$\delta = 2SNR1(2\pi\varepsilon/\lambda)^2$, $SNR1=|a|^2/2\delta^2$ where $2\delta2$ is the variance of the white noise. As the array are symmetry in nature $CRB(\alpha_1)= CRB(\beta_1)$. $CRB(\alpha_1)= CRB\cos(\alpha_1)/\sin^2(\alpha_1)$ and $CRB(\beta_1)=CRB\cos(\beta_1)/\sin^2(\beta_1)$

From the table only octagon array is 5% less CRB than L shaped array. CRB of L shaped array is 37% smaller than cross array. L shaped array has higher accuracy level than cross arrays and many other simple arrays.

2. DATA MODEL

Two uniform linear orthogonal arrays considered and these form a L shaped array configuration. Each array contains M no of antenna elements and the spacing between each elements is d. Let us say there are K narrow band far field sources impinging on the antenna array from different directions. The signal wavelength be λ and the k^{th} source has elevation angle be θ_k and azimuth angle be ϕ_k, $k = 1, 2, \ldots, K$. The angle of k^{th} source with respect to X axes is represented in the fig 1.

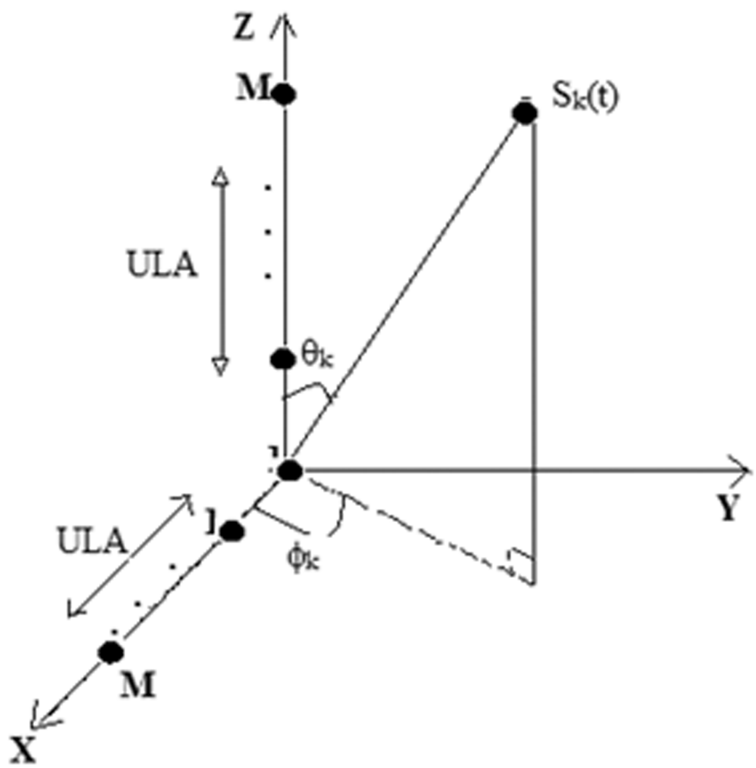


Figure 1: L shaped array configuration for 2-D AOA estimation.

The observed signal along X axes sub array and Z axes sub array be

$$x(t)= A(\phi)\, s(t) + n_x(t) \tag{1}$$

$$z(t)= A(\theta)\, s(t) + n_z(t) \tag{2}$$

The matrices and the vectors in (1) and (2) have the following forms.

$$z(t) =[z_1(t), z_2(t), \ldots, z_M(t)]^T$$

$$x(t)= [x_1(t), x_2(t), \ldots, x_M(t)]^T$$

$$A(\theta) = [a(\theta_1),a(\theta_2), \ldots, a(\theta_k)]$$

$$A(\phi) = [a(\phi_1), a(\phi_2), \ldots, a(\phi_k)]$$

$$A(\theta_k) = [1, e^{-j\Phi_k}, \ldots, e^{-j(M-1)\Phi_k}]^T$$
$$A(\phi_k) = [1, e^{-j\beta_k}, \ldots, e^{-j(M-1)\beta_k}]^T$$
$$\Phi_k = \pi \cos \theta_k,\ \beta_k = \pi \cos \phi_k$$
$$s(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$$
$$n_z(t) = [n_{z1}(t), n_{z2}(t), \ldots, n_{zM}(t)]^T$$
$$n_x(t) = [n_{x1}(t), n_{x2}(t), \ldots, n_{xM}(t)]^T \tag{3}$$

The subscripts T denote transpose. $Z_m(t)$ and $x_m(t)$ denote m^{th} element of received data in Z axes and X axes sub array respectively. S(t) is the K X 1 vector of source signal. $n_z(t)$ and $n_x(t)$ are additive noise and are independent of signal samples. The elements of { $n_z(t)$ and $n_x(t)$} are white Gaussian random processes w]ith zero mean and variance σ^2. $A(\theta)(a(\theta_k))$ and $A(\phi)(a(\phi_k))$ are the M X K (M X 1) array response matrices in Z axes and X axes sub array.

3. DOAALGORITHMS

3.1. MUSIC

MUSIC is the most defacto algorithm to estimate multiple source parameters like elevation angle, azimuth angle, range, polarization etc. MUSIC requires a priori knowledge of spatial background noise and interferences. It says The desired signal array response is orthogonal to noise subspace [2A] . The signal and noise subspace are indentified by using Eigen value decomposition of the received signal covariance matrix. MUSIC spatial spectrum is calculated and DOA is estimated. In general an array is set in the region of interest in the DOA space. The region of θ is extracted from the region of A(θ). A(θ) is the array response vector. The subspace estimation is achieved by eigen decomposition of the auto-covariance matrix of the received data R_{xx}.

It is assumed that the spatial whiteness $E\{n_x(t)n_x^H(t)\} = \sigma_n^2 I$. The eigen value

$$\lambda_n = \lambda_1 > \lambda_2 > \ldots > \lambda_k > \lambda_{k+1} = \sigma_n^2$$

The eigenvectors $e_n \in C^N, n = 1, 2, \ldots N$, for R_{xx}

$$E = [E_s, E_n] \tag{2}$$

$$= E_s \Lambda_s E_s^H + \sigma_n^2 E_n E_n^H = E_s \bar{\Lambda}_s E_s^H + \sigma_n^2 I \quad ([]3)$$

Where $E = [e_1, e_2, \ldots e_N]$,

$E_s = [e_1, e_2, \ldots e_k]$,

$E_n = [e_{k+1}, e_{k+2}, \ldots e_N]$,

$\Lambda = \text{diag}\ \{\lambda_1, \lambda_2, \ldots \lambda_N\}$,

$\Lambda_s = \text{diag}\ \{\lambda_1, \lambda_2, \ldots \lambda_K\}$,

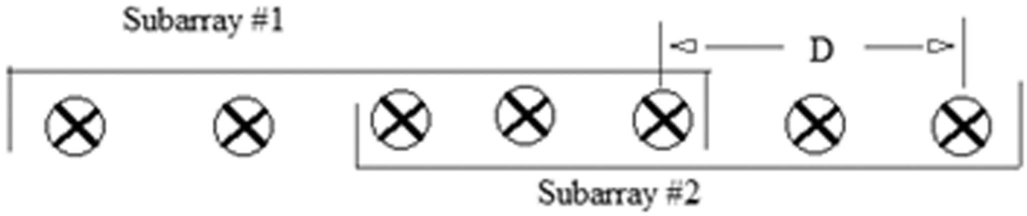
$\Lambda_n = \text{diag}\ \{\lambda_{k+1}, \lambda_{k+2}, \ldots \lambda_N\}$, and

$\Lambda_s = \Lambda_s - \sigma_n^2 I$. The eigen vector $E = [E_s, E_n]$ can be assumed to form an orthogonal basis. The span of K vector E_s defines the signal subspace and E_n defines the noise subspace. After determining the subspaces the DOA of the desired signal can be calculated through MUSIC algorithms [3]

$$P_{MUSIC}(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)E_n E_n^H a(\theta)} \tag{4}$$

3.2. ESPRIT

Estimation of Signal Parameters via Rotational Invariance Techniques are as follows. An antenna comprise of two identical sub arrays. Some antenna array elements may be member of both sub arrays[4]. Let an array contains M elements and m elements which are member of both sub array (so that M < 2m). The individual elements of sub array can have arbitrary polarization, directional gain, phase response.



Let "d" *signals* impinging onto the array. Let x1(*t*) and x2(*t*) are signal received by the two sub arrays and let the received signals are corrupted by additive noise n1(*t*) and n2(*t*). Each of the sub-arrays has *m* elements. The elements are separated by a fixed displacement vector D. The received signals may be expressed as

$$x_1(t) = [a(\mu_1),...a(\mu_d)]s(t)] + n_1(t)$$
$$= A_1\Theta s(t) + n_1(t) \tag{5}$$

$$x_2(t) = [a(\mu_1)e^{j\mu 1},...a(\mu_d)e^{j\mu d}]s(t)] + n_2(t)$$
$$= A_2\Theta s(t) + n_2(t) \tag{6}$$

$x_1(t)$ and $x_2(t)$ are the $m \times 1$ vectors represents received data of ist and 2nd sub array. $n_1(t)$ and $n_2(t)$ are m × 1 noise vectors. $A_1\Theta$ and $A_2\Theta$ belongs to $C^{m \times k}$. This indicates the manifold of each sub array is unitary diagonal matrix

Let J1 and J2 represent the M x m selection matrix .

$$J_1 = \left[0_M \times (m - M) \vdots I_m\right] \tag{7}$$

$$J_2 = \left[0_M \times (m - M) \vdots I_m\right] \tag{8}$$

I_m is the M x M identity matrix and $0_{M \times (m-M)}$ is the M × (m-M) matrix of zeros. The two identical sub arrays satisfies

$$\theta_K = \sin^{-1}\{\frac{\arg\{\}}{\frac{2\pi}{\lambda}D}\},\ i = 0, 1...K \quad JA(\Theta) = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} A(\Theta) = \begin{bmatrix} A_1(\Theta) \\ A_1(\Theta)\Phi \end{bmatrix} \tag{9}$$

Where Φ is the diagonal matrix and

$$\Phi_i = \exp\{-j\beta_i^T.D\}, i = 1, 2..K \tag{10}$$

β_i = vector wave number of incident plane from ith narrow band source

D = vector displacement between two sub array.

E_s = eigenvector corresponding to K largest eigen values of received signal

$$E_s \triangleq \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} A_1(\Theta) \\ A_1(\Theta)\Phi \end{bmatrix} T \tag{11}$$

T is full rank matrix and $T \in C^{K \times K}$.

Solving the equation (11) we can get

$$E_2=E_1T^{-1}\Phi T=E_1\Psi \tag{12}$$

Where $\Psi = T^{-1}\Phi T$ or $\Phi=T\Psi T^{-1}$. So eigen values of Ψ must be equal to diagonal elements of Φ. This is the fundamental relation in the properties of ESPRIT. If M > K and $D = |D| < \frac{\lambda}{2}$. From the eigen values of operator, Ψ E_1 and E_2, DOA can be determined .

$$\theta_K=\sin^{-1}\{\frac{\arg\{\Psi_i\}}{\frac{2\pi}{\lambda}D}\},i=0,1...K \tag{13}$$

Ψ_i is the each eigen value of Ψ matrix.

3. DIRECTION OF ARRIVAL ESTIMATION BY ADAPTIVE ARRAY ANTENNA PLANE

A new technique for 2 D DOA estimation is proposed. Consider a L shaped array has antenna elements in X and Z axes. Each ULA contains N no of elements and the distance between each element is d. A narrow band signal $\{S_k(t)\}_{k=1}^{K}$ impinge on the array in an elevation angle $\{\theta_k\}_{k=1}^{K}$ and azimuth angle $\{\phi_k\}_{k1}^{K}$. Both elevation angle and azimuth angle can be found out independent. It is presumed both ULA are synchronized. The pair matching method can be exploited independently. The baseband signals of the t^{th} snapshot of the array along Z axes is expressed as

$$z(t)=\sum_{k=1}^{K}a(\theta_k)S_k(t)+n_z(t) \tag{14}$$

$a(\theta_k)$ is called steering vector.

$$a(\theta_k)=[a_1(\theta_k)\cdots a_m(\theta_k)]^T \tag{15}$$

The signal component arriving on nth antenna element. For an adaptive antenna system, if p users transmit signals from different locations, and each user's signal arrives at the array through multiple paths. Let LMi denote the number of multipath components of i[th] user. We have $\sum_{i=1}^{p}L_{Mi}=p$. Let's further assume that all of the multi path components for a particular user arrive within a time window which is much less than the channel symbol period for that user, then the input data vector could be expressed as-

$$x(t)=\sum_{i=1}^{p}\sum_{k=1}^{L_{Mi}}\alpha_{i,k},a(\theta_{i,k})s_i(t)+n(t) \tag{14}$$

or we can write

$$x(t)=\sum_{i=1}^{p}G_is_i(t)+n(t) \tag{15}$$

where $\theta_{i;k}$ is the DOA of the k-th multi path component for the i-th user, $a(\theta_{i;k})$ is the steering vector corresponding to $\theta_{i;k}$, $\theta_{i;k}$ is the complex amplitude of the k-th multipath component for the i-th user, and G_i is the spatial signature for the i-th user and is given by

$$G_i = \sum_{k=1}^{LMi} \alpha_{i,k} a(\theta_{i,k}) \tag{16}$$

The signal component arriving on *n*th antenna element at a particular instance of time is given by

$$X_n = A\exp(j2\pi\, \mathrm{ndsin}\,\theta\cos\phi/\lambda) \tag{17}$$

$$Y_n = A\exp(j2\pi\, \mathrm{ndsin}\,\theta\sin\phi/\lambda) \tag{18}$$

Where A = complex amplitude of the signal, φ = Direction of Arrival (DOA) of the signal (Azimuth Angle) (unknown), θ = Direction of Arrival (DOA) of the signal (Elevation Angle) (unknown), d = spacing between antenna elements and λ = wavelength.

Now one can view (4) & (5) as-

$$X_n = A\exp[j2\pi f(n\,\mathrm{dsin}\,\theta\cos\phi/\mathrm{c})] \tag{19}$$

$$Y_n = A\exp[j2\pi f(n\,\mathrm{dsin}\,\theta\sin\phi/\mathrm{c})] \tag{20}$$

Where f = frequency of the signal and c = velocity of wave.

Now if we mechanically steer the antenna plane by $\delta\varphi$ & $\delta\theta$, then (19) & (20) becomes –

$$X^1{}_n = A\exp[j2\pi f(n\,\mathrm{dsin}\,\theta\cos(\phi+\delta\phi)/\mathrm{c})] \tag{21}$$

$$Y^1{}_n = A\exp[j2\pi f(n\,\mathrm{dsin}\,\theta\sin(\phi+\delta\phi)/\mathrm{c})] \tag{22}$$

$$X^2{}_n = A\exp[j2\pi f(n\,\mathrm{dsin}(\theta+\delta\theta)\cos\phi/\mathrm{c})] \tag{23}$$

$$Y^2{}_n = A\exp[j2\pi f(n\,\mathrm{dsin}(\theta+\delta\theta)\sin\phi/\mathrm{c})] \tag{24}$$

Now taking the frequencies (which can be known by seeing the spectra of the signal) of the signal from (19) and (21), and taking their ratio one could get-

$$\frac{frequency \to X_n}{frequency \to X^1{}_n} = \frac{\cos\phi}{\cos(\phi+\delta\varphi)} = \frac{1}{k} \text{ (k is known)}$$

Hence

$$\phi = \tan^{-1}\left[\frac{\cos\delta\phi - k}{\sin\delta\phi}\right] \tag{25}$$

And from (20) & (24), we could get

$$\frac{frequency \to Y_n}{frequency \to Y^1{}_n} = \frac{\sin\theta}{\sin(\theta+\delta\theta)} = \frac{1}{k}\ \theta = \cot^{-1}\left[\frac{k-\sin\theta}{\cos\delta\theta}\right] \tag{26}$$

Now using the simple relation given in (25) & (26) one can determine the unknown DOA (θ & φ) of all incoming signal impinging on the array with suitable algorithm based on (19), (20), (21), (22), (23), (24), (25) and (26).

3.1. Pair Matching

The pair matching algorithm utilize cross correlation matrix of received signals on both ULAs. The pair matching done separately for 2D estimation.

The baseband signal of t-th snapshot of the array output measured along Z axis is expressed as [13].

The observed signal along X axes sub array and Z axes sub array be

$$x(t) = A\ (\phi)\ s(t) + n_x(t)$$
$$z(t) = A\ (\theta)\ s(t) + n_z(t)$$

The cross correlation matrices R_{zx} between $z(t)$ and $x(t)$.

$$R_{zx} = E\{z(t)x^H(t) \tag{27}$$

$$= \sum a(\theta_k) E\left\{s_k(t)s_k{}^H(t)\right\} a^H(\phi_k) + E\left\{n_z(t)n_x{}^H(t)\right\}$$

$$= A(\theta) S A^H(\phi) + N_{zx} \tag{28}$$

The matrix $A(\theta) = [a(\theta_1), \cdots a(\theta_k)]$

$$A(\phi) = [a(\phi_1), \cdots a(\phi_k)]$$

$S = diag\left\{r_{1,\ldots} r_k\right\}$, is the power of k^{th} signal.

$$N_{zx} = E\left\{n_z(t), \cdots n_x{}^H(t)\right\}$$

The noise vector $n_z(t)$ and $n_x{}^H(t)$ are independent and uncorrelated. So $N_{zx} = 0$

The cross-correlation matrix

$$R_{zx} = A(\theta) S A^H(\phi) \tag{29}$$

Whose (p, q) element is

$$[R_{zx}]_{p,q} = \sum_{k=1}^{K} r_k \exp[-j\mu\{(p-1)\cos\theta_k - (q-1)\cos\phi_k\}]$$

Where $\mu \triangleq 2\pi d / \lambda$.

Based on the diagonal elements of R_{zx}

$$r_{zx} = [\sum_{k=1}^{K} r_{k,} \sum_{k=1}^{K} r_k e^{-j\mu w_{k,}} \cdots, \sum_{k=1}^{K} r_k e^{-j\mu(M-1)w_k}]^T$$

Where

$w_k = \cos\theta_k - \cos\phi_k$, k = 1, . . ., K

The relation between elevation angle and azimuth angle emerge in vector r_{zx}.

R_{cc} is the toeplitz matrix and its first column and row are r_{zx} and r_{zx}^H.

$$R_{cc} = \begin{bmatrix} r_{zx}(1) & r_{zx}^*(2) & \cdots & r_{zx}^*(M) \\ r_{zx}(1) & r_{zx}(1) & \ddots & \vdots \\ \vdots & \cdots & \ddots & r_{zx}^*(2) \\ r_{zx}(M) & \cdots & \cdots & r_{zx}(1) \end{bmatrix} \tag{30}$$

The angle of arrival techniques using covariance matrix are applicable to R_{cc} to obtain $\left\{w_k\right\}_{k=1}^{K}$.

The pair matching method is used to achieve a computationally efficient estimation . We did the estimation and compared with our novel algorithm.

3.2. Simulation

The simulation for DOA based on MUSIC, ESPRIT and our novel algorithm for 2 D are simulated through MATLAB 13.

Pattern due to MUSIC

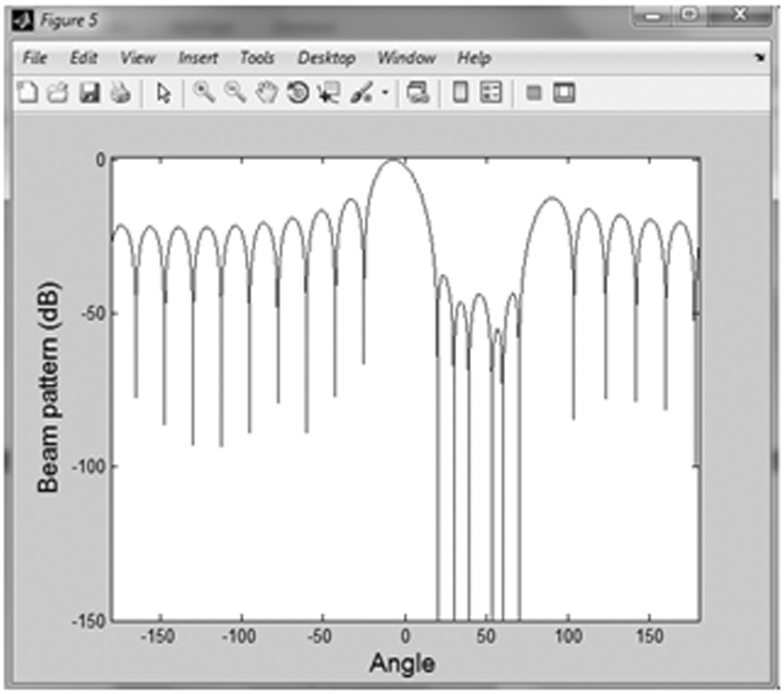


Figure 1: Rectangular pattern due to MUSIC

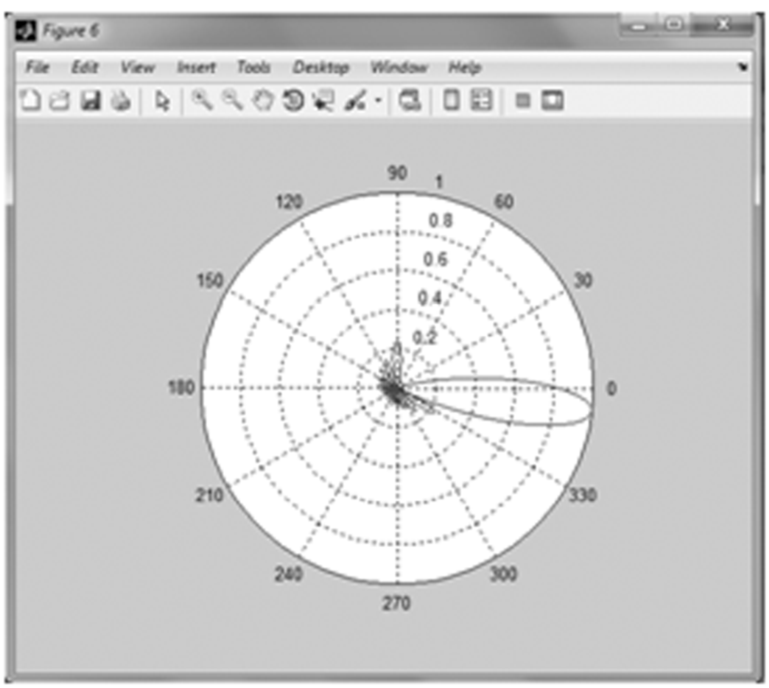


Figure 2: Polar pattern due to MUSIC

Radiation pattern based on ESPRIT algorithm

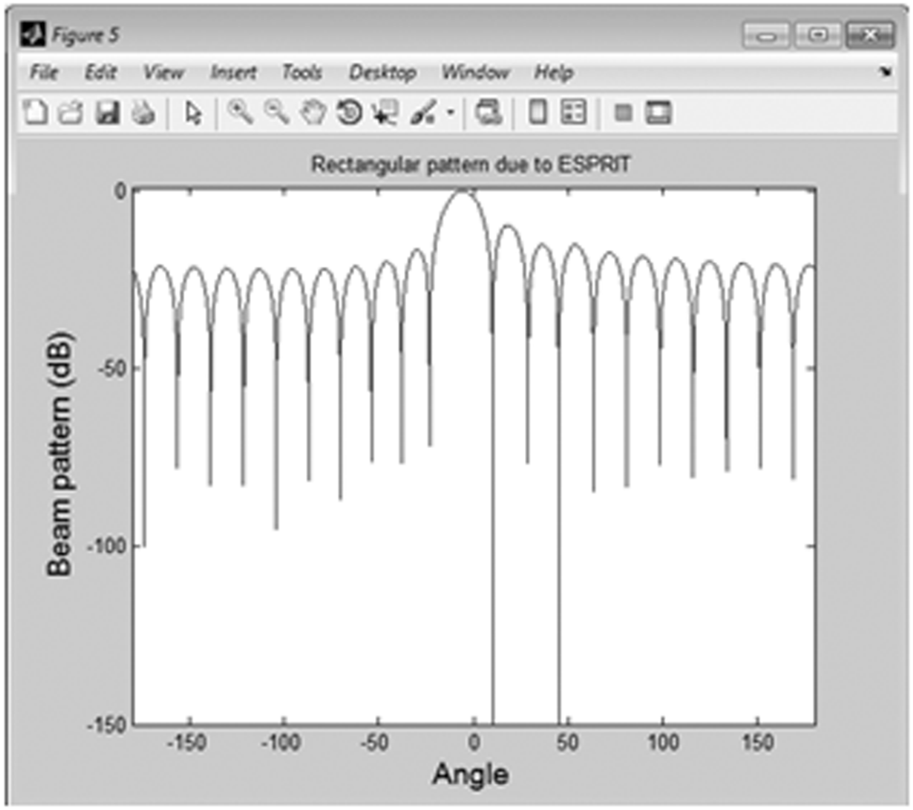


Figure 3: Rectangular pattern due to ESPRIT

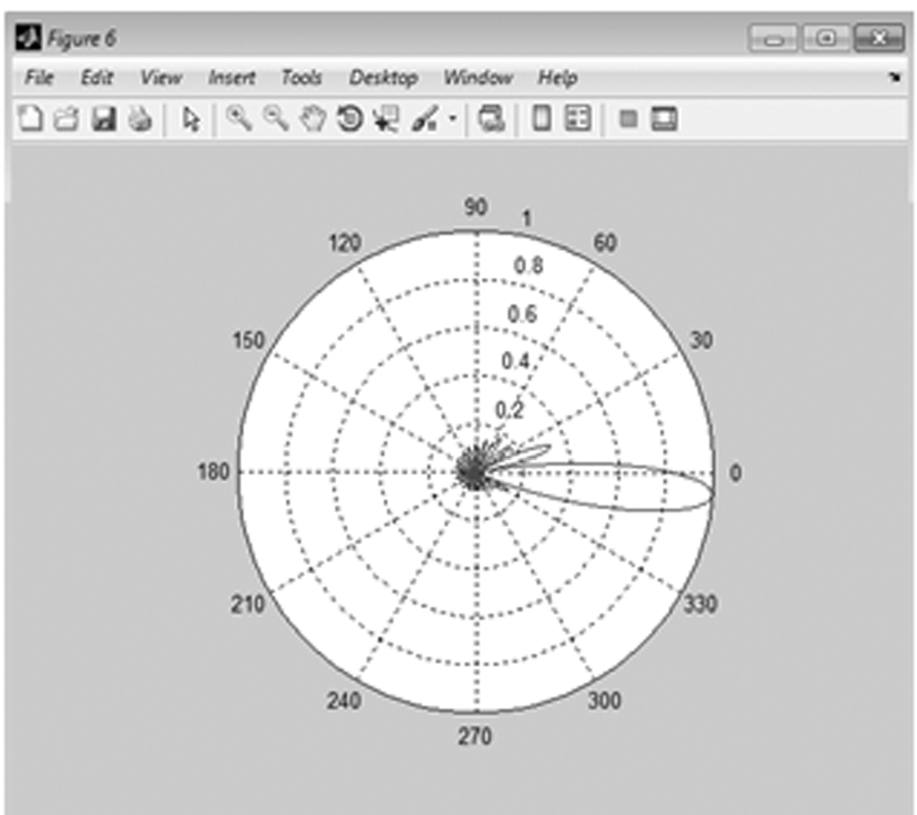


Figure 4: Polar pattern due to ESPRIT

Patterns due to our novel algorithm

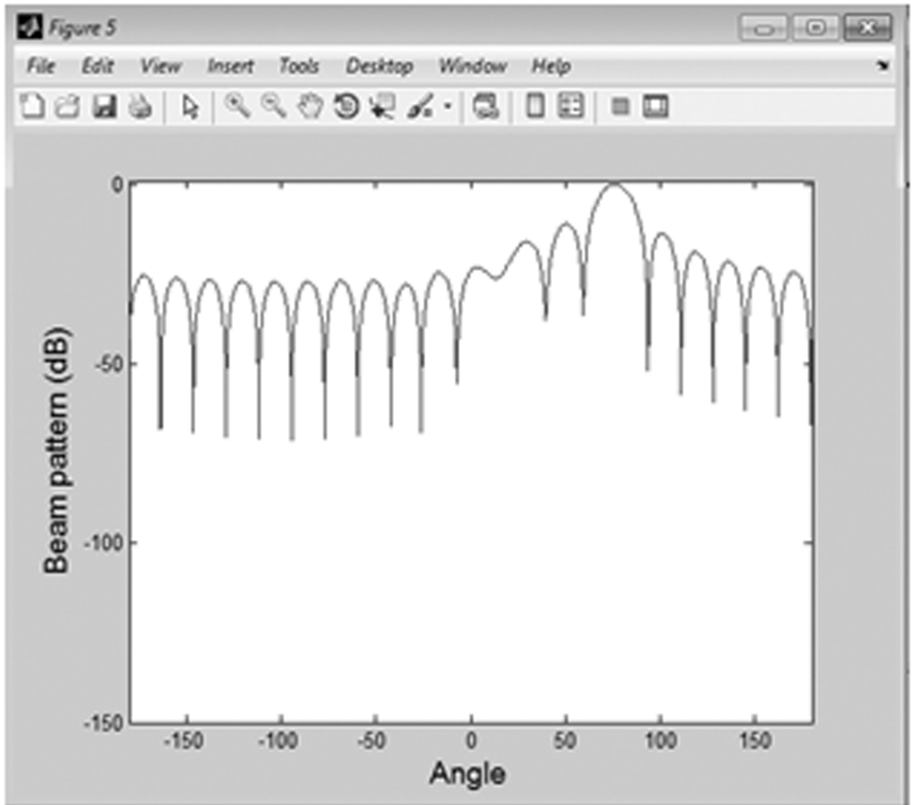


Figure 5: Rectangular pattern due to Novel Algorithm

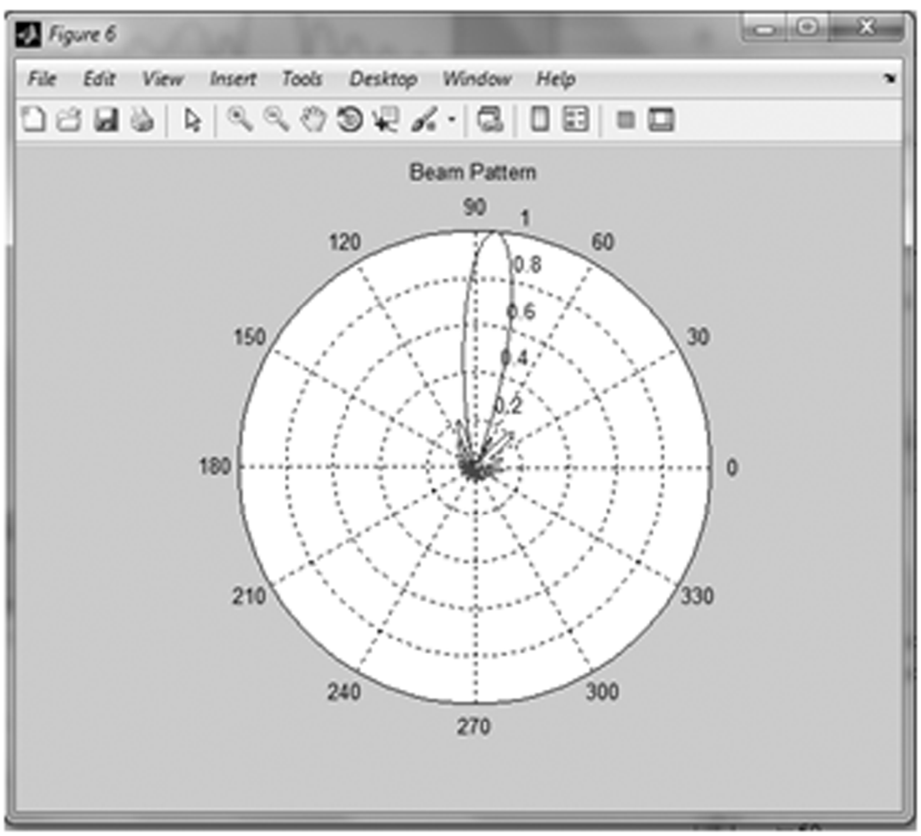


Figure 6: Polar pattern due to Novel Algorithm

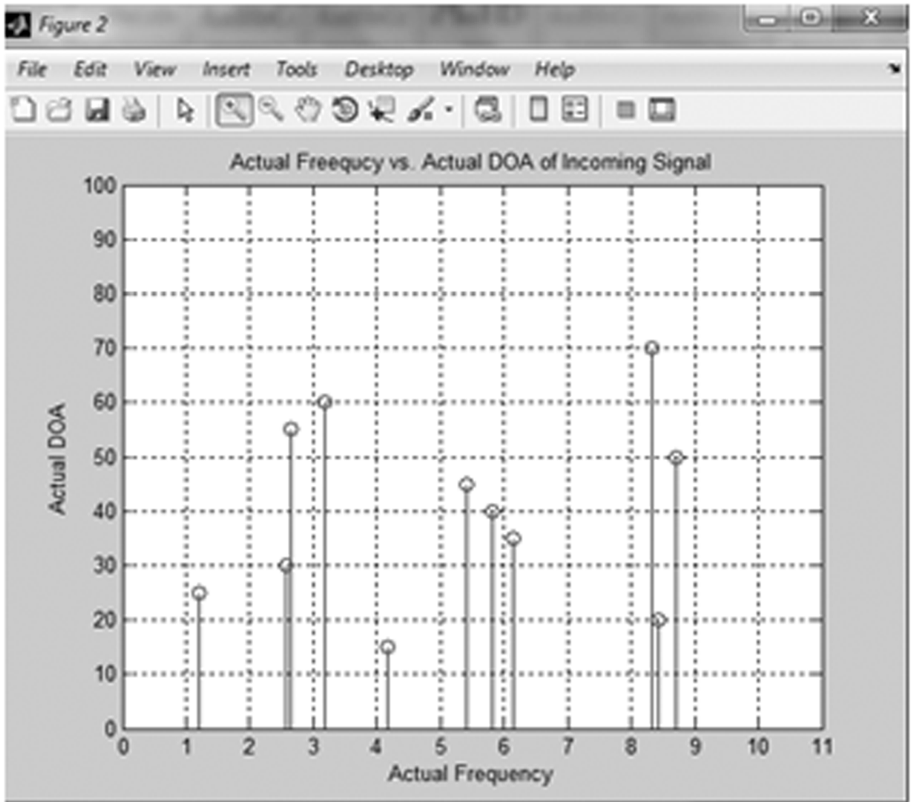


Figure 7: Actual freq. Vs Actual DOA

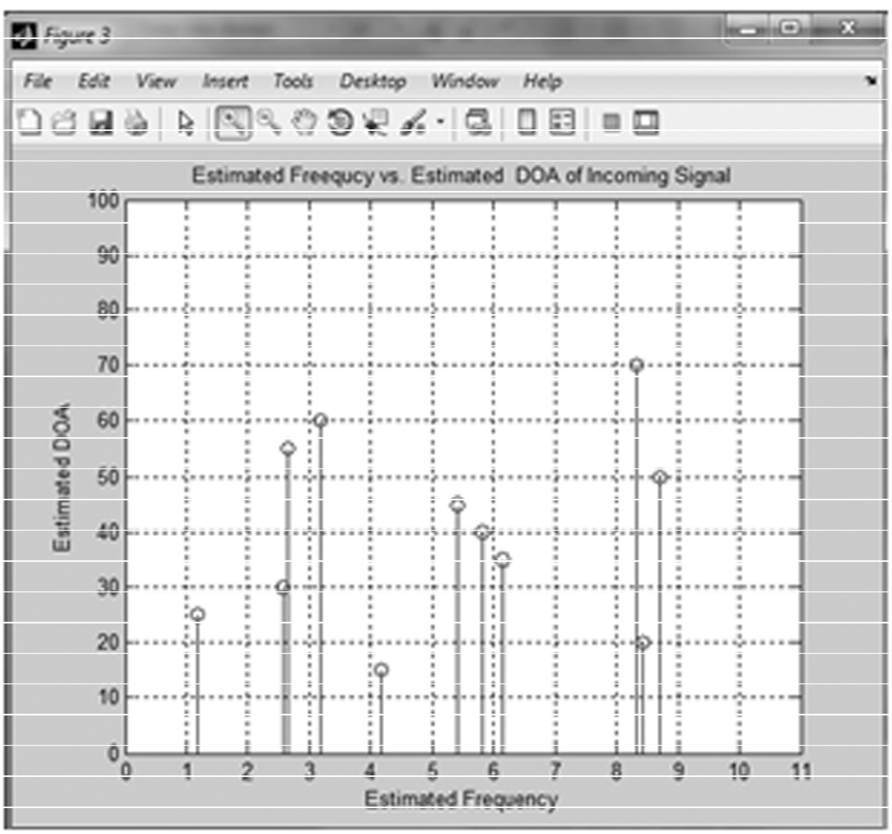


Figure 8: Estimated Freq. Vs Estimated DOA

4. OBSERVATION

We did simulation of DOA using MUSIC and our new algorithm and compare them. From the Polar plot it is observed that the minor lobe is substantially cancelled and the major lobe is more directional towards the desired angle.

The DOA estimation of both algorithms are presented in tabular form. There are 6 pair of angles are considered between $0 < x_i < 90^0$.

Table 2

ANGLE in DEGREE		DOA BASED	DOA BASED ON NOVEL
ELIVATION	AZIMUH	ON MUSIC	ALGORITHM
20	45	19.9983	20.1067
30	55	29.9967	29.9850
40	48	39.1880	40.0276
50	37	53.5397	49.9955
60	25	60.0032	60.0051
70	15	70.0125	70.0009

At x_1, x_2, x_3, x_4, x_5, x_6, are 20 45;30 55;40 48;50 37;60 25;70 15.

The comparative DOA based on MUSIC and our proposed algorithm are presented in the above table.

5. CONCLUSIONS

For DOA estimation through MUSIC, it require a priori knowledge of spatial background noise and interferences. The performance increased if we increase the no of elements of array. In the case of ESPRIT is required minimum 2 sub arrays and very complex computation.

Our proposed algorithm is very simple & it neither required a priori knowledge of background noise nor it needs complex calculations. It consumes minimum time for calculation the DOA. A adequate hardware can be designed for use. The main lobe is more directional and side lobes are cancelled and more reduced w.r.t that of MUSIC and ESPRIT.

REFERENCES

[1] Constantine A. Balanis, Panayiotis I Ioannides ,"Introduction to smart antennas, Synthesis Lecturers on Antennas," #5.

[2] Schmidt R.O. 1986. Multiple emitter location & signal parameter estimation 'IEEE Trans Antenna propag" , AP-34, pp 276-280.

[3] Paluraj A, Roy R Kailath T. 1986. 'A subspace rotation to signal parameter estimation'. Proc 'IEEE , pp 1044-1045.

[4] Roy R, Kailath T. 1989 "ESPRITestimation of signal parameter via rotational invariance technique' IEEE Trans, Acoust speech signal process, 1989,37,(7), pp 984-995

[5] Yanxing Z, Qinye Y. 2000. 'Closed space t.ime blind estimation for space time coded MC-CDMA systems , Septemeber 2000, pp 786 – 794.

[6] Vertatschitsch EJ haykin S. 1991. 'Impact of linear array geometry on direction of arrival estimation for a single source', IEEE Trans antenna Propag .1991 39 pp 576 – 586.

[7] M. Mukhopadhyay, B.K.Sarkar, and A. Chakraborty. 2007. Augmentation of anti-jam gps system using smart antenna with a simple doa estimation algorithm progress in electromagnetics research, Pier 67, 231–249.

[8] Zhizhang Chen Gopal Gokeda Yiqiang Yu "Introduction to Direction -of- Arrival Estimation" ch 5 pp-82-85.

[9] Yanxing Z, Qinye Y. Closed space time blind estimation for space time coded MC-CDMA system IEICE Trans Comm., 2005, 88(7) pp.3050-3056.