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Exponential-Splines in Compressed Sensing ECG Reconstruction

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ABSTRACT

Use of Splines is not yet investigated for compressed sensing (CS). We tried exponentialsplines (1E-splines) and double sided exponential splines (2E-splines, proposed here)in compressed sensing scenario for ECG reconstruction and found that both outperform commonly used wavelets based approaches.Using these splines, peak root mean Square deviation (PRD) values were much lower for the same amount of measurements when compared to bi-orthogonal wavelets. Also, the variation of PRD when analyzed againstthe number of random samples taken, the relationship is almost linear up to a value of 50% of the samples. The frequency domain representations of these 2E-Splines were investigated and the differences from E-splines are discussed. Comparisons of results are presented for E-Splines,2E-Splines and bi-orthogonal wavelets.The interpolatingproperties of these splines were also investigated,for various orders, for a comparison. Filtering of the ECG signal and thresholding during reconstruction were not attempted, since a realistic comparison of the two analytical methods were found necessary.

Keywords: Compressed Sensing (CS), electrocardiography (ECG), PRD, Double exponential Splines (2E Splines)

1. INTRODUCTION

In approximation problems, the elementary fragments are to be selected in a way that their description should be simple,universally accepted and geometrically understandable. The benefit of simplicity is adequatelyunderstandable. The universality helps to reduce puzzlement and extended calculations, geometric clarity helps to conjecture subsequent fragments and inanalyzing the resulting curve. Meeting all the above requirements at one instanceisnot easy. Using polynomials or piece wise functions full fill a part of these requirements but in each case of its own.Splines [1] were introduced to combine the advantage of both polynomials and piece wise functions. Even though splines are polynomial functions connected together they will not show as much as oscillatory behavior as compared to higher order polynomials. Local polynomial approximation produces discontinuities in the connecting regions where splines surfaces are smooth everywhere.

B-spline interpolation yields best cost quality trade off when compared to other interpolation methods [2]. In problems where continuous model of signal or images are required,spline representation gives a better option than other techniques and it was already shown that [1] interpolation and approximation using splines can be performed by digitals filters.Multiresolution properties of splines make them a principal contender in the construction of wavelet bases [3]. Having all these advantages polynomial splines aresparingly used in continuous signal processing, the main reason behind thismay bepiece-wise polynomials appears only marginal in continuous time processing systems. Exponential functions are more significant in continuous system than piece wisepolynomials, for example exponentials corresponds to the responses of filtersand circuits.Exponential-Splines (E-splines) proposed by Michael Unser through his series of papers [4-5] narrowed the gap between spline fitting techniques and continuous system theory.

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As smoother functions gives better frequency resolution we tried splines in Compressed Sensing [6] scenario. Exponential splines are attempted instead of polynomial splines as it is more significant because of the stated reasons above.Ourexperimental results were very encouraging. During our experiments we found that the splines obtained from basic exponential splines(through convolution) are asymmetric in nature for first few orders. Due to this fact, symmetric exponential splines, a new type of exponential splines, that are named here as double exponential splines or 2E splineswere formulated. Wavelets are commonly employed to get better reconstruction in CS based ECG reconstruction applications.Our experimental results shows that 1E splines and 2E splines basisperformed better than thewavelet based approaches.

Even though proposed byMichael Unser [4-5] in 2005, the potential of exponentials plines are not explored much. Some recent research reveals the application of exponential splines in diverse domains, for example Moreno *et.al* [7] recently (2015) proposed a new method for radar target identification. The method uses E-pulses (extinction-pulses) which are built using exponential-splines. Like this, the aim of this paper is to give a brief introductionto 2E splines to the benefit of engineering researcher as an alternative.The improvement in CS based ECG reconstruction werealso found worth noting. Mathematical properties of the 2E space, (space spanned by the double sided exponentials) are not analyzed as we aim to give an introduction to 2E splines. We had analyzed the frequency response of 2E splines. Variations in interpolation for both splines were also investigated.

The main advantage of CS is that it reduce the amount of data to be captured, CS theory states that we can recover the signals with less number of samples [8].In situations where there is a limitation in available recourses for data capturing, like sensors or in situations where these resources are extremely expensive CS plays a vital role.It also helps when we have to capture very large amount of data like 24 hour ECG monitoring. For this purpose of 24 hour monitoringHolter monitors [9]were generally employed to record ECG of patients with the help two,three or twelve electrodes. For continuous monitoring for longer periods this method causes discomfort in patients.The wires between the electrode and the recorder may alter because of patient's movements, which may cause variation in the measurements. Wireless sensors can be used as an alternative solution [10].These sensors sense the biomedical signals and transmit the data. Compared to wired network wireless sensors are more attractive and provides more comforts to the patients but the main drawback is increased power consumption, whichis propositional to the amount of data transmitted.

The major share of power is consumed by the radio frequency (RF) power amplifier. For example, the power amplifier of the body sensor network transmitter developed for the Medical Implant Communications Service,uses around 74% of the total power consumed in the system [11]. As the power required is propositional to data transmission, reduction in the amount data transmitted will drastically improve the power saving. In these scenarios CS based methods provides an attractive solution which aims to reduce the samples and tries to reconstruct the data with lesser samples but reconstructing the original data from limited samples is always a challenging problem, always there is a tradeoff between the number of samples chosen and the reconstruction quality. Once we reduce the number of measurements the quality of the reconstructed signal will also get reduced.

Applying 1E and 2E splines in CS based ECG reconstruction is a move forward in that direction. CS techniques are predominantly applied in image applications as compared to signal processing applications, part of the reason may be the lack of proper hardware techniques available for taking the random input measurements, but Random Modulation [12-13] based analog to information converters made significant impact in this area.

Random demodulation(RM) provides a hardware-compact architecture for realizing compressive sensing systems especially in signal processing. Modulated Wideband Converters (MWC) [14] are also used for accessing random samples. The structure is somewhat same as to that of parallel random demodulator. Here input signal is applied parallely to an array of analog mixers.The most notable contribution in compressed sensing hardware technology was proposed by Ravelomanantsoa *et al*, the CS encoder proposed by them saves up to 82.9% and 75% of power in electrocardiogram and electroencephalogram applications [15].Some recent research in CS hardware technology shows that the method can be used as a compression technique with extremely low computational complexity [16].

Recently M. Bortolotti. *et.al* proposed an ultra dual mode ECG monitor based on Compressed Sensing which is 70% energy efficient than the base lines systems [17]. All these research shows that CS is an ideal candidateagainst the bottle neck caused by energy crisis.Our research shows that introduction of splines in CS will increase the reconstruction quality further. We were able to get the desired quality with less number of measurements as compared to wavelet based approach. For example by using our approach with splines we got the PRD (Percentage Root mean square Difference) of around 35% with 280 measurements, but by using the commonly used bi-orthogonal wavelet approach same PRD is obtained onlyafter taking580 measurements.As noted the power consumption is directly propositional to the number of measurements, it is eminent that spline basis will help in energy saving also.

Even though CS has advantages in energy saving, practical obstruction in the adaptation of CS technique is the reconstruction delay. This causes bottleneck between real time processing and CS.But the use of GPU(Graphics Processing Unit) bought a break through in this area. For exampleDavid S. Smith *et.al* performed real time CS basedMRI reconstruction using split Bregman solver combined with a GPU computing platform which shows that larger amount of data can be used in real time computations [18] Other significant contributions in this area are noted in [19-25]. In CS based ECG reconstruction scenario, Kanoun *et.al* proposed a method for real time ECG monitoring system, which uses an iPhone as a decoder forreconstructing the ECG data [26].

CS theory states that any S-sparse signal can be recovered correctly by ensuringnumber of random measurements to be greater than or equal to four times the number of non zero coefficients in the signal [27]. i.e., $m \geq 4S$, where ‘m’ is the number of random measurements and ‘S’, the number of non zero coefficients in the signal. As the number of coefficients increases number of random measurements for acceptable reconstruction quality will also get increased. Thresholding in time or frequency domain are usually employed in order to increase the sparsity in data (to reduce the number of non zerocoefficients), but in practical, ideal CS reconstructionrequires to reconstruct the original signal from few random samples, that is few randomly projected samples (m) will be available and CS algorithms is expected to reconstruct the data from it, it means that thresholding is not possible in actual CS. So in our experiments we didn’t threshold or filtered the data. In CS based ECG reconstruction biorthogonal wavelets are usually employed, as biorthogonal wavelets seems to yield better results [28-29], so we had considered biorthogonal wavelets as the baseline approach to compare our results.

The algorithm performance is analyzed on the basis of (PRD).PRD can be found out by taking the ration between l_2 norm of the difference in the input and reconstructed signal and the input signal.PRD formula is defined as

$$PRD = \sqrt{\frac{\sum_{n=1}^{N}\left(Xi(n) - Xr(n)\right)^2}{\sum_{n=1}^{N} Xi(n)^2}} \times 100 \tag{1}$$

Where $Xi(n)$ be theoriginal signal, $Xr(n)$ be the reconstructed signal and N be its length

The paper is arranged as follows. In Section II we are giving a brief introductionof compressed sensing. Section II details one of the main contributions of this paper. In this section we describeabout the formulation of exponential splines and our newly found double exponentials. In subsections we investigate the frequency response characteristic of 2E splines and analyze the variation in interpolation for both 1E and 2E splines. In Section III,we show our results when 1E spline and 2E spline are applied in compressed sensing scenario and we prove that both splines perform better than wavelet based approaches, we conclude this paper in section IV.

2. COMPRESSIVE SENSING (CS)

CS is an emerging area in the field of signal and image processing. CS ideally shows how to reconstruct the signals with lesser samples.CS theory asserts that one can recover certain signals and images from limited number of randomly projected samples. Here we explore the possibility of CS theory in Electro Cardio Gram (ECG).

Compressed Sensing can be simply explained as, if '*x*' is the input vector of size N × 1 and if it has alternative notation in another domain say ψ, where it is sparse. i.e. $x = \psi\alpha$. Then CS theory states that x can be recovered from few randomly projected samples of x on another basis φ by solving the optimization problem

$$\min_{s.t\ y=\Phi_\alpha} \|\alpha\|_{11}$$

α is the coefficients of '*x*' in ψ and Φ = ψ

The detailed mathematical explanation can be found in [30-31].

3. EXPONENTIAL AND DOUBLE EXPONENTIAL SPLINES

Exponential splines are made by connecting exponential segments in smooth fashion. The connecting points are called as knots. The splines we considering are cardinal in nature, which means knots are placed at integer points.

3.1. Construction of Basic Exponential (IE)Splines

In [4] Michael Unserelaborates the equation for generating the exponential splines, for example consider if $f_a(t)$ is an one sided exponential extending from 0 to + ∞, with 'a' as the co efficient of the power

$$f_a(t) = e^{at}, 0 < t < \infty \ f_a(t)$$

Then first order Basis Exponential spline with parameter 'a' can be obtained as

$$B_a(t) = f_a(t) - e^a f_a(t-1) \quad (2)$$

The whole process for a = -0.5 is illustrated in Fig. 1

Higher order Splines can be obtained by the convolution of lower order splines i.e.

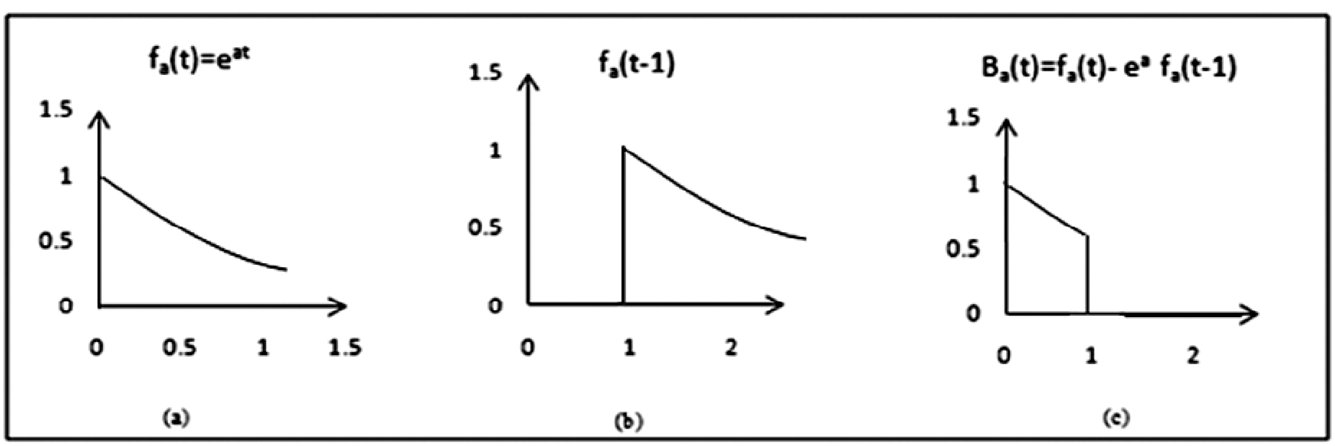

Figure 1: From causal exponent to B-Spline. (a) Causal exponent $f_a(t)$. (b) $f_a(t)$ Shifted by one unit. (c) Basis spline

$$B(t) = (B_{a1} * B_{a2} * B_{a3} \dots \dots \dots B_{aN})t$$

The first order, second order, third order and fourth order exponential splines are shown in Fig. 2. From the figure it is eminent that the E-Splines obtained are not symmetric in nature.

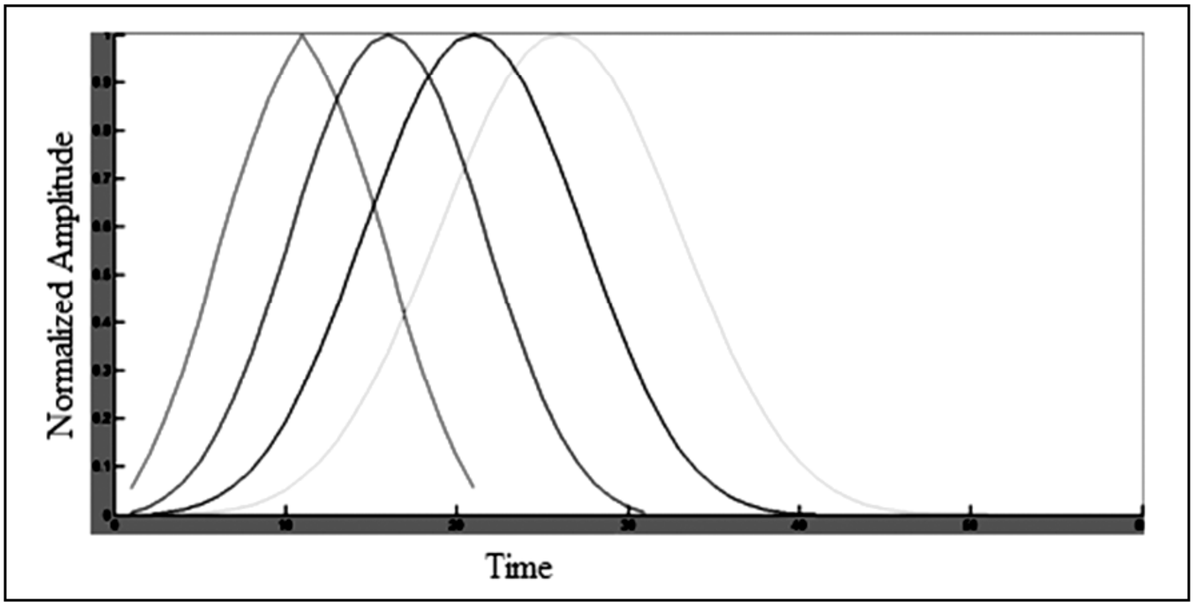

Figure 2: Normalized 1st, 2nd,3rd and 4th order one sided exponential(1E or E) splines.

3.2. Generation of Double sidedExponential Symmetric Splines

The splines obtained from one sided (1E) exponentials are not symmetric in nature, for generating an exponential spline that is symmetrical we had used the B-spline equation as

$$B(t) = Ae^{-a|t-b|};\ for\ all\ t$$

where ‘a’ is the scaling parameter and the parameter ‘b’ decides the center of spline (Maximum value). By changing the values of ‘a’ and ‘b’ in all possible ways (from ∞ to +∞) a space can be formulated which contains all the double exponentials , we can use the equation (1) to span this space, which can be called as double sided exponential space(or 2E space). Putting, b = 0 will give a spline which is centered at 0.

So, B-spline which is centered at zero is

$$B(t) = Ae^{-a|t|}$$

i.e. $B(t) = Ae^{+at}; fort = -\infty\ to\ 0$

$= Ae^{-at}; fort = 0\ to\ \infty$

We have truncated the exponential in the interval form -0.5 to +0.5

The truncated double sided exponential function in the interval (-0.5, 0.5) is

$$B(t) = Ae^{-a|t|};\ -0.5 < t < 0.5$$

$$B(t) = Ae^{+at};\ fort = -0.5\ to\ 0$$

$$= Ae^{-at};\ fort = 0\ to\ 0.5$$

= 0; else where

The truncated exponential in the interval (-0.5, 0.5) is shown in Fig. 3.

The higher order splines can be obtained by the successiveconvolution of lower order splines

$$B(t) = (B_1 * B_2 * B_3 \; \dots \dots \dots \; B_N)t$$

1st order, 2nd order, 3rd order and 4th order normalized double exponential splines are shownin Fig. 4. Splines are symmetric in nature

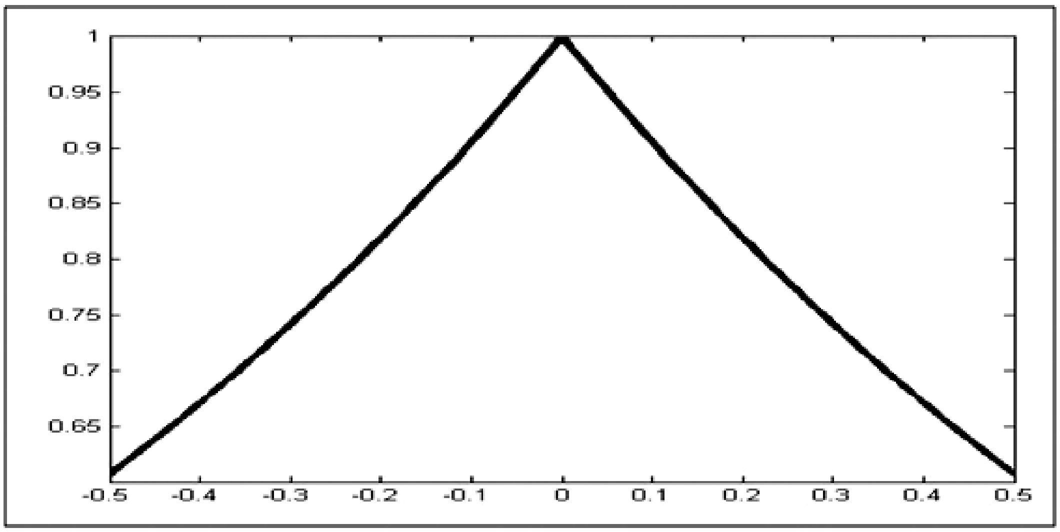

Figure 3: Double sided exponential in the interval –0.5 to 0.5

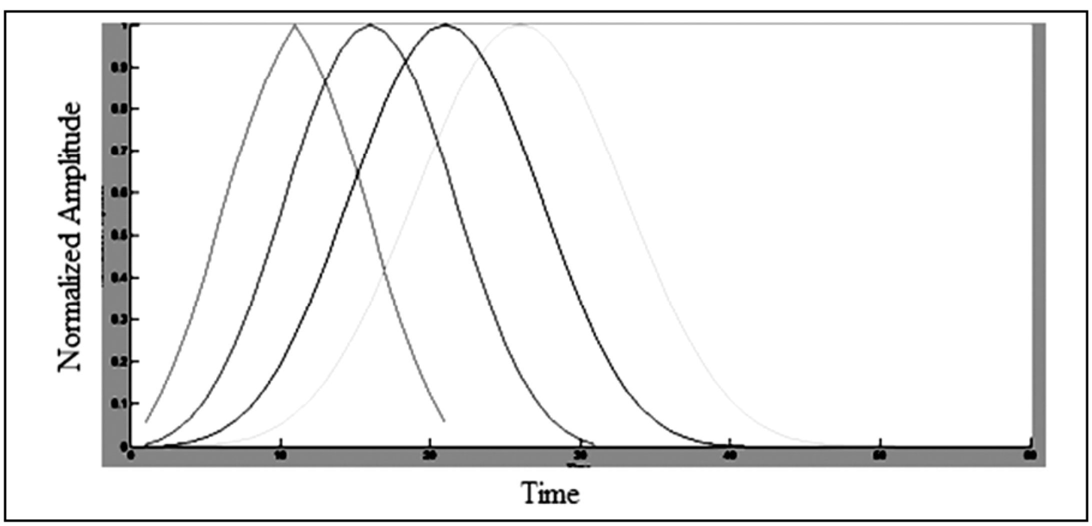

Figure 4: Splines obtained from double sided exponential

3.3. Frequency Response of Double Sided Exponential Splines

Fourier transform of a function is

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

Fourier transform of the truncated double sided exponential function

$$X(j\Omega) = \int_{-0.5}^{0.5} A e^{-a|t|} e^{-j\Omega t} dt$$

$$= \int_{-0.5}^{0} Ae^{at} e^{-j\Omega t} dt + \int_{0}^{0.5} Ae^{-at} e^{-j\Omega t} dt + 0$$

On integrating and applying limits

$$\left[\frac{Ae^{0}}{a - j\Omega} - \frac{Ae^{-0.5(a - j\Omega)}}{a - j\Omega} \right] + \left[\frac{Ae^{-0.5(a + j\Omega)}}{-(a + j\Omega)} - \frac{Ae^{0}}{-(a + j\Omega)} \right]$$

On further simplification

$$\frac{A - Ae^{-0.5(a - j\Omega)}}{(a - j\Omega)} + \frac{A - Ae^{-0.5(a + j\Omega)}}{(a + j\Omega)}$$

$$\frac{2Aa - Ae^{-0.5(a - j\Omega)}(a + j\Omega) - Ae^{-0.5(a + j\Omega)}(a - j\Omega)}{(a^2 + \Omega^2)}$$

$$\frac{2Aa - Ae^{\frac{-a}{2}} * e^{\frac{j\Omega}{2}}(a + j\Omega) - Ae^{\frac{-a}{2}} * e^{\frac{-j\Omega}{2}}(a - j\Omega)}{(a^2 + \Omega^2)}$$

$$= \frac{2Aa - Ae^{\frac{-a}{2}} \left\{ e^{\frac{j\Omega}{2}}(a + j\Omega) + e^{\frac{-j\Omega}{2}}(a - j\Omega) \right\}}{(a^2 + \Omega^2)}$$

$$= \frac{2Aa - Ae^{\frac{-a}{2}} \left\{ a\, e^{\frac{j\Omega}{2}} + j\Omega\, e^{\frac{j\Omega}{2}} + ae^{\frac{-j\Omega}{2}} - j\Omega e^{\frac{-j\Omega}{2}} \right\}}{(a^2 + \Omega^2)}$$

$$= \frac{2Aa - Ae^{\frac{-a}{2}} \left\{ a\left(e^{\frac{j\Omega}{2}} + e^{\frac{-j\Omega}{2}} \right) + j\Omega\left(e^{\frac{j\Omega}{2}} - e^{\frac{-j\Omega}{2}} \right) \right\}}{(a^2 + \Omega^2)}$$

By Euler's formula

$2\ cos\ x = e^{ix} + e^{-ix}$

$2\ isin\ x = e^{ix} - e^{-ix}$

Applying Euler's formula in (10)

$$\frac{2Aa - Ae^{\frac{-a}{2}} \left\{ 2a\cos\left(\frac{\Omega}{2}\right) + 2\Omega j * i * \sin\left(\frac{\Omega}{2}\right) \right\}}{(a^2 + \Omega^2)}$$

$$i = j = \sqrt{-1}, i * j = -1$$

$$= \frac{2Aa - 2Ae^{\frac{-a}{2}} \left\{ a\cos\left(\frac{\Omega}{2}\right) - \Omega\sin\left(\frac{\Omega}{2}\right) \right\}}{(a^2 + \Omega^2)}$$

That is the Fourier transform of the truncated exponential in the interval (-0.5, 0.5) is

$$X\left(j\Omega\right)=\frac{2A\left[a-e^{\frac{-a}{2}}\left(a\cos\left(\frac{\Omega}{2}\right)-\Omega\sin\left(\frac{\Omega}{2}\right)\right)\right]}{\left(a^{2}+\Omega^{2}\right)} \quad (3)$$

For a function A = a = 1; that is

$$x\left(t\right)=e^{-|t|};\ -0.5<t<0.5$$

$$X\left(j\Omega\right)=\frac{2\left[1-e^{\frac{-1}{2}}\left(\cos\left(\frac{\Omega}{2}\right)-\Omega\sin\left(\frac{\Omega}{2}\right)\right)\right]}{\left(1+\Omega^{2}\right)}$$

$$e^{\frac{-1}{2}}=0.6065$$

$$X\left(j\Omega\right)=\frac{\left[2-1.213\left(\cos\left(\frac{\Omega}{2}\right)-\Omega\sin\left(\frac{\Omega}{2}\right)\right)\right]}{\left(1+\Omega^{2}\right)}$$

Normalized Frequency Response for the function for different values of a, a = 0, 1, 3 and 5

The cross over's for sidelobes ideally has to occur at multiplies of 2π. However deviations can be noticed from the frequency plot for a>1. Even though we haveexpected a better results for 2E splines in ECG reconstruction the performance would have been affected by the difference in cross over's. Our experiments show that the reconstruction quality of ECG data lies marginally less for 2E splines than 1E.Frequency response of 2E splines response broaden out with increasing values of 'a' and smoothening out of the loops on either side of the central loop.

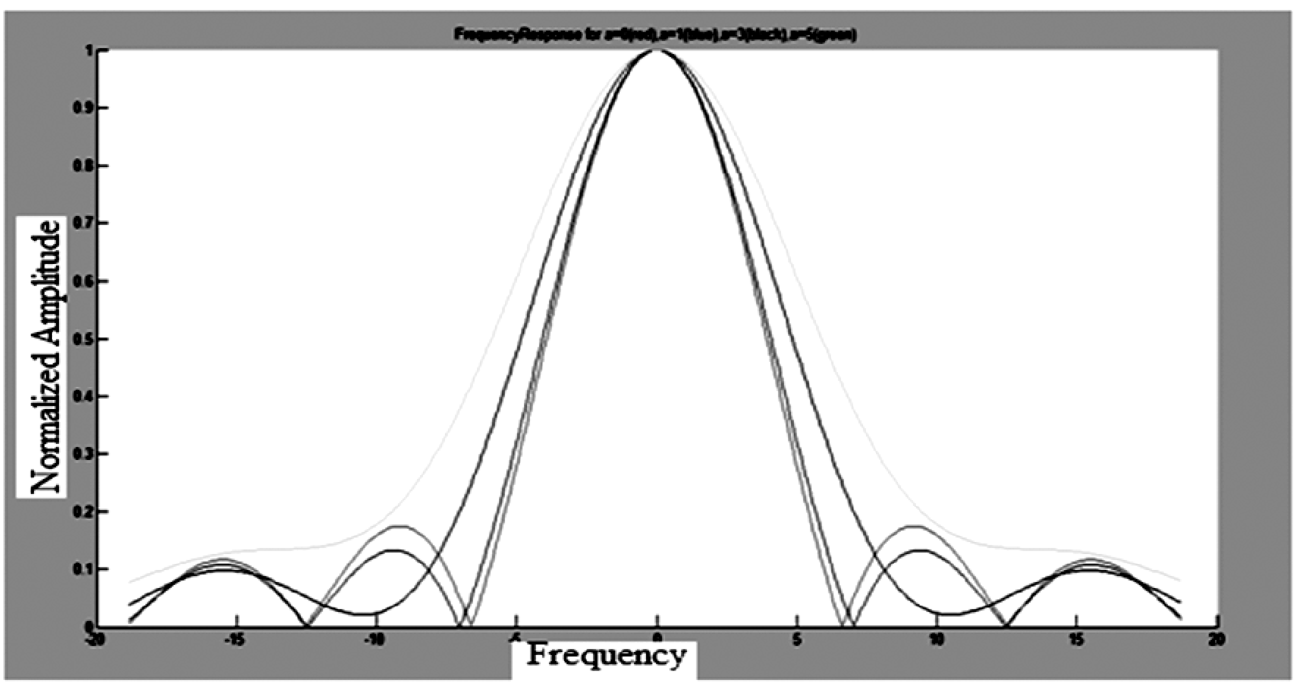

Figure 5: Frequency response for the values a =0, 1, 3 and 5

3.4. Comparison between 1E Splines and 2E Splines

3.4.1. Percentage variation (PV)for 1E splines

To find the PV, each spline and its two translates are added together.First translate is obtained by shifting the spline by one unit and second translate is obtained by shifting the first translate by one unit, then all the shifted versions are added together. For an example Fig. 6 shows the addition of 1st order splines. The variation in boundary and variation in the top are separately found out and are tabled below.

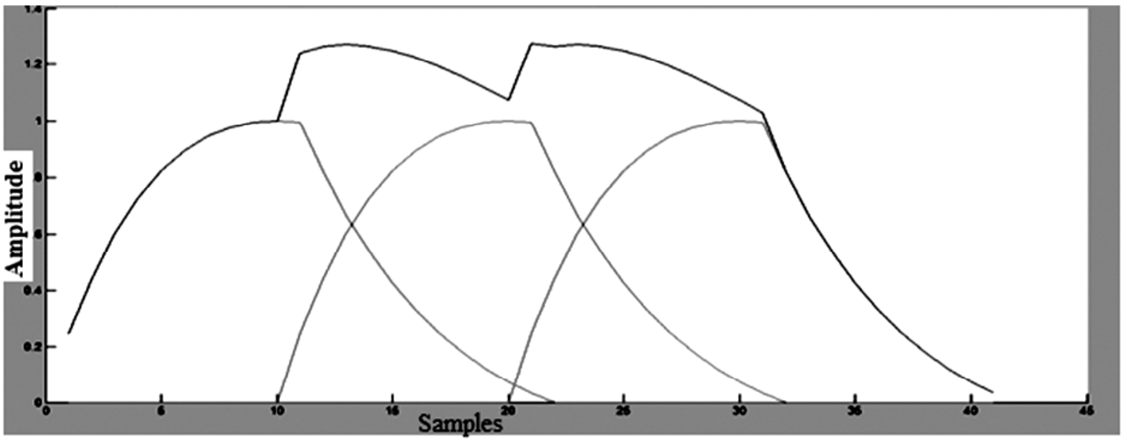

Figure 6: Shows the addition of 1storder E-spline and it'stranslates.PV in boundary and in top for 1st order, 2nd order and 3rd order 1E splines are tabulated and shown in Table 1.

Table 1
Percentage of variation

Spline	*1E1*	*1E2*	*1E3*
P.V. in boundary (%)	21.32	20.39	18.56
Variation in top	12.25	3.45	2.81

1E1 - First order one sidedexponential spline

1E2 - Second orderone sided exponential spline.

1E3 - Thirdorderone sided exponential spline.

3.4.2. Percentage variation (PV) for 2E splines

The same process as in case of 1E is repeated here for 2E. Fig. 7. shows the addition of 2nd order douvle exponential spline and its translates.

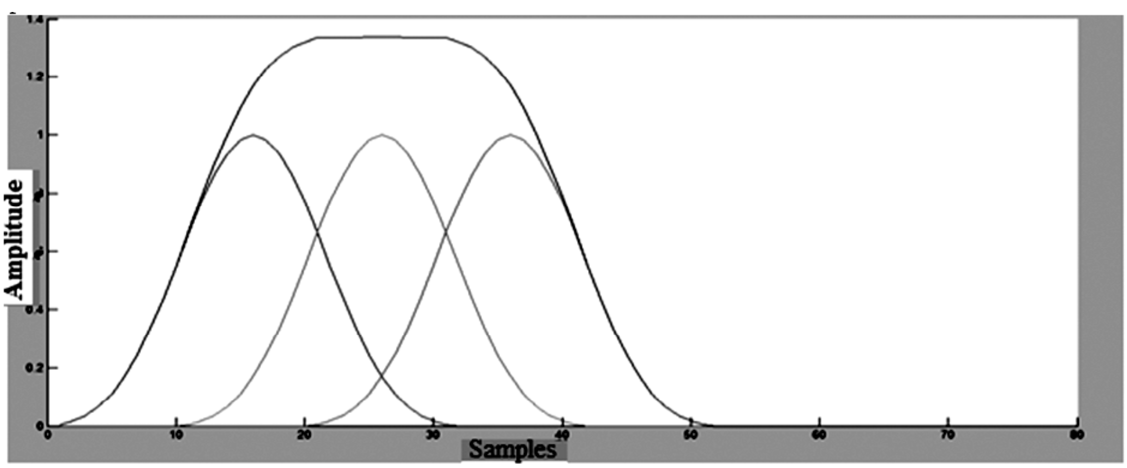

Figure 7: Addition of 2E2 and its translates.

PV in boundary and in top for 1st order, 2nd order and 3rd order2E splines are tabulated and shown in Table 2.

Table 2
Percentage of variation in 2E splines

Table 4 Spline	*2E1*	*2E2*	*2E3*
P.V. in boundary in %	4.95	10.25	17.13
Variation in top in %	3.42	0.15	3.94

4. EXPONENTIAL-SPLINES IN CS

Bi-orthogonal wavelets are commonly employed in ECG based CS reconstruction as they seems to give better results [28-29]. So we had compared our results against it. Forone sided and two sided exponential spline based CS, representative ordinates are selected from first order, second order and third order splines. These values are used to make transform matrices for CS instead of bi-orthogonal wavelet basis. Each spline behavior is analyzed in the context of CS for a scaling factor of a = 1. The input ECG signal is taken from MITArrhythmia data base, which is shown below in Fig. 8 [32]. MIT data base contain ECG recording of 47 subjects taken between 1975 and 1979.

The recordings were digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range.Input data we consideredis of size of 800 × 1(N = 800).

Random samples are obtained by projecting the data using a random matrix. For each experiment input data is projected onto a random matrix (of size ‘m × 800’) to take random measurements. We had varied ‘m’ from 10% of the data size i.e. 10 % of 800(80) to 75% of 800 (600).

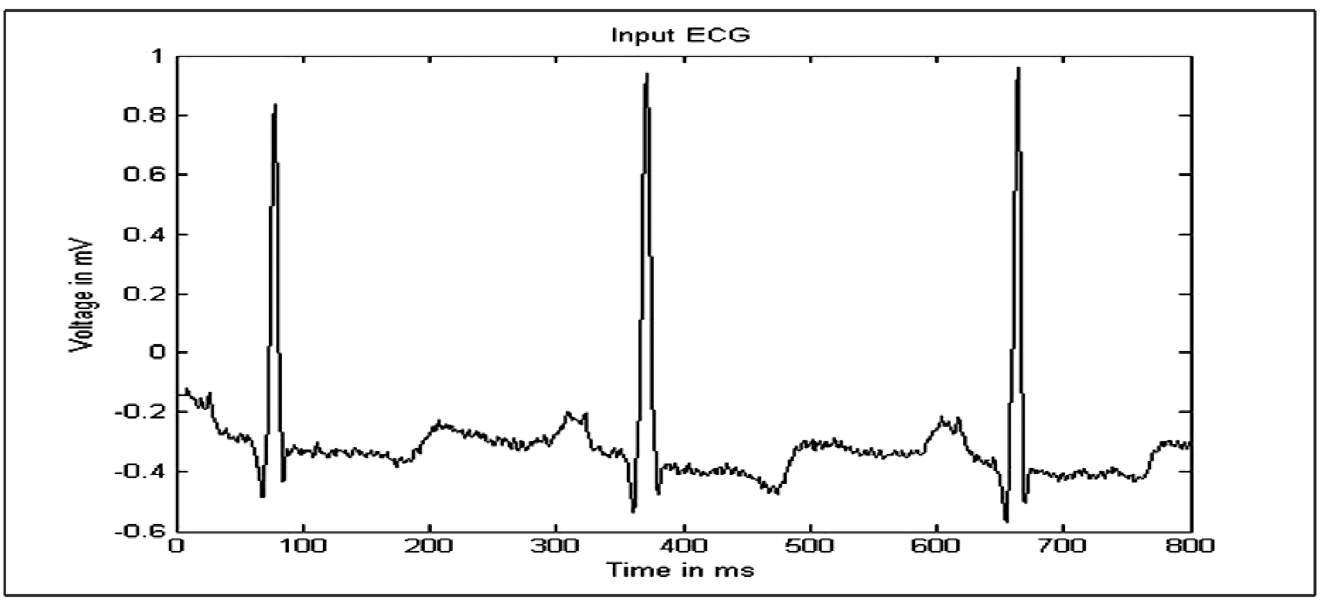

Figure 8: Input ECG from MIT data base [32]

4.1. Compressed sensing using one sided exponentials

PRD obtained for each one sided exponential splines are plotted against the number of random samples (measurement) taken in Table.3.

1E1 denotes 1st order one sided exponential spline.1E2 denotes 2nd orderone sided exponential spline and 1E3 indicate 3rd order one sided exponential spline.PRD at each of these splines are plotted against the number of measurements as shown in Fig. 9.

Table 3
Measurements vs PRD for 1Esplines

Measurement in%	*PRD-1E1*	*PRD-1E2*	*PRD-1E3*	*Bior 4.4*
10.00	88.81	93.32	92.38	108.35
15.00	74.16	78.79	74.02	86.88
20.00	67.88	69.24	66.25	81.47
25.00	53.81	57.50	51.79	72.52
30.00	43.51	45.07	41.86	73.84
35.00	35.93	33.09	30.34	71.56
40.00	26.32	24.90	20.39	71.22
45.00	18.56	16.08	14.72	59.23
50.00	16.62	10.60	11.65	61.78
55.00	10.82	6.19	7.31	54.67
60.00	8.83	4.26	6.37	45.11
65.00	7.03	3.90	5.70	45.26
70.00	4.57	2.99	4.25	36.93
75.00	3.35	2.63	3.26	28.25

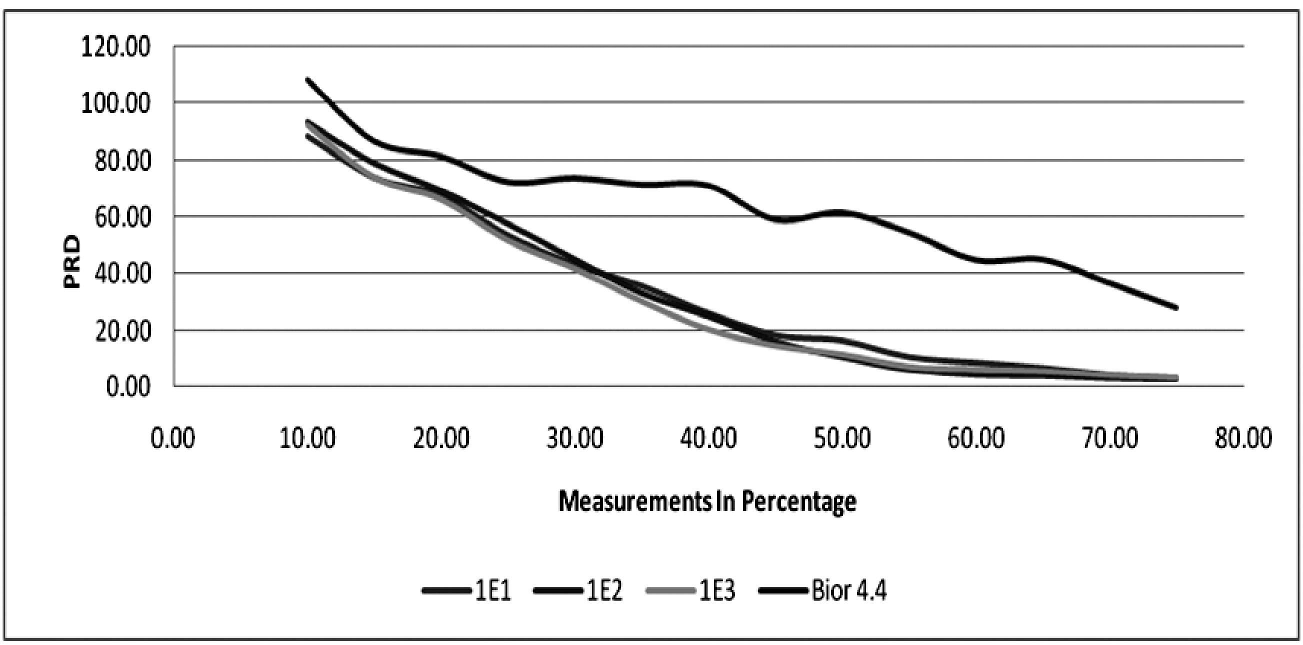

Figure 9: Measurements vs PRD for exponential Splines and bior 4.4

We had repeated the experiments for 56 times (112 including double exponentials) for different values of ‘m’, with ‘m’ ranging from 80 to 600 and we observed that PRD is coming under 20% when the measurements are above 400(50%). There was a sustainable improvement compared to conventional bi orthogonal wavelet basis, which is shown in violet color.

4.2. Compressed Sensing Using 2E Splines

Table 4 shows the PRD obtained when 2E1 (1st order 2E spline), 2E2 (2nd order 2E spline) and 2E3 (3rd order 2E spline) where used to reconstruct the ECG signal using CS approach, and the same is graphically shown in Fig. 10 which illustrate the improvement over bi orthogonal wavelets based approach.

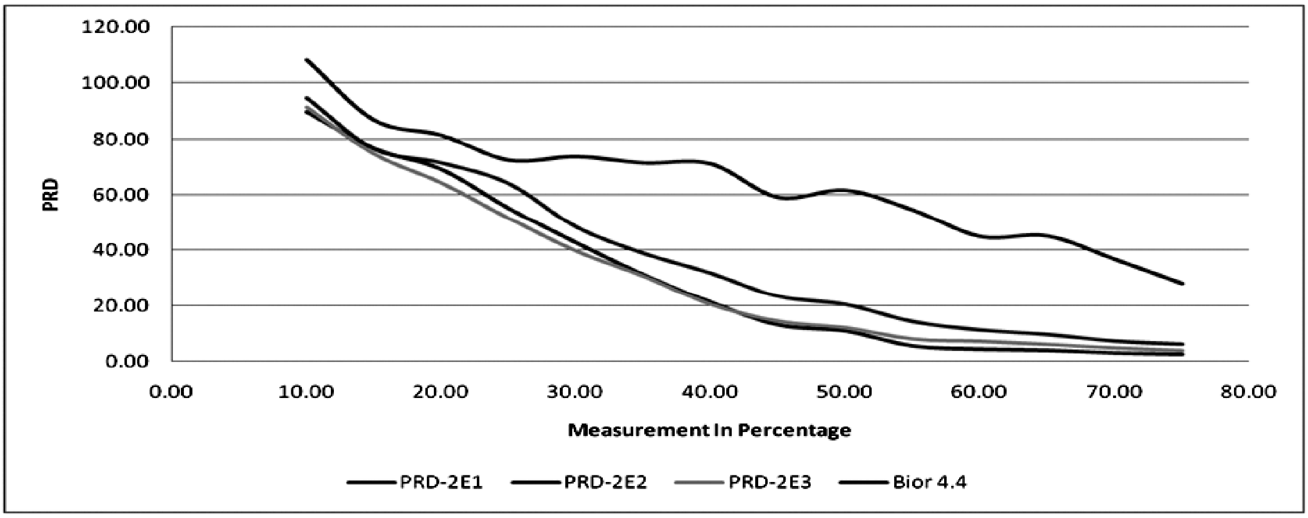

Figure 10: PRD for 2E1, 2E2, 2E3 and bi orthogonal 4.4 against the number of measurements (In percentage)

Table 4
Measurements vs PRD for 2Esplines

Measurement Percentage	*PRD-2E1*	*PRD-2E2*	*PRD-2E3*	*Bior 4.4*
10.00	94.89	89.80	91.49	108.35
15.00	76.56	76.68	74.71	86.88
20.00	71.55	69.16	64.29	81.47
25.00	64.09	55.14	51.52	72.52
30.00	48.42	42.72	39.67	73.84
35.00	38.89	31.19	30.64	71.56
40.00	31.69	21.17	20.37	71.22
45.00	23.28	13.04	14.26	59.23
50.00	20.34	10.85	11.91	61.78
55.00	14.15	5.49	7.89	54.67
60.00	11.14	4.33	7.06	45.11
65.00	9.50	3.93	5.97	45.26
70.00	7.15	2.97	4.66	36.93
75.00	6.03	2.47	3.71	28.25

4.3. PRD E1 vs. 2EI

Fig. 11 compares the PRD obtained when one sided exponential spline bases (1E) and two sided exponential spline bases (2E) are used for reconstructing the ECG. From the graph we can interfere that 2E splines and 1E splines give almost similar values, 2E values slightly on lower side This may be because of the Cross over's as shown in frequency response of 2E splines.Both splines perform far better than conventional bi - orthogonal basis (which is shown in light blue color).

5. CONCLUSIONS

In this paper we proposed a compressed sensing technique using symmetrical and asymmetrical E-splines. We had applied exponential splinesin CS for the first time and developed a new type of symmetrical splines from double sided exponentials .The frequency domain representation of these splines were investigated and discussed. As expected 2E spline reconstructed signal did not perform better than E spline based one. The frequency spectrum answers this, where frequency shifts are noticed at low level of signals. However,

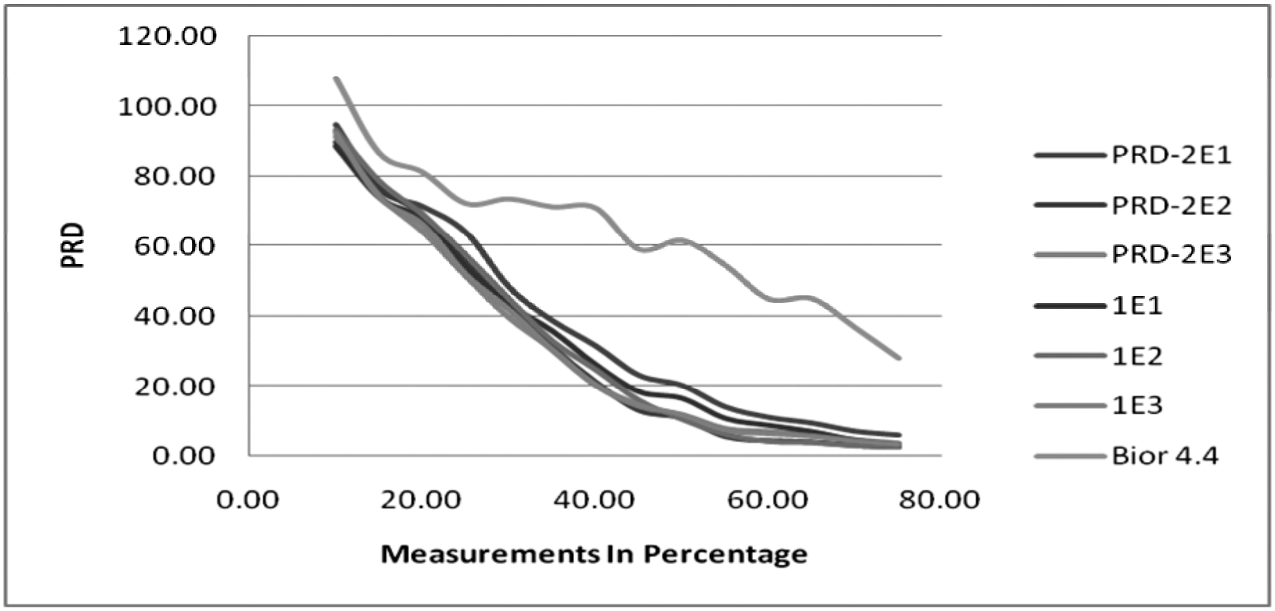

Figure 11: Comparing 1E and 2E PRD with Bi orthogonal wavelet basis

the results were better as compared to the state of art methods usingbiorthogonalwavelet basis. The variation in interpolation also studied and detailed for both 2E and E-Splines.

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