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Extended Formalization of the Description of Networks and Network Conflicts

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Abstract : We propose an extended formalization of the description of networks, taking into account the weights of vertices and edges, as well as the value of the filler, circulating in the network. The set of network metrics is expanded on the basis of weighing the vertices and arcs, as well as calculating the potential of the network. The criteria for the network conflict assessment are substantiated based on the analysis of the dynamics of the network potential, taking into account the capacity of the filler stored and processed at the vertices, as well as the carrying capacity of the arcs transporting the filler. The possible strategies and tactics of the network conflict resolution are considered. We suggest developing the proposed evaluations and methods for the case of virus attacks in the conditions of network confrontation.

Keywords : Network, filler, capacity, vertices, edges, virus attacks.

1. INTRODUCTION

The modern information space is inconceivable without network structures [16-20]. Their extensive dissemination has found its application in the social [38] and other networks, which have become the subject of comprehensive study. In particular, the analytical expressions have been obtained for the network metrics such as the distribution of degrees, cohesion and diameter, the clustering coefficient and others. In addition, there have been identified and studied the Poisson [8-10], exponential [12], scaleless [4, 15] networks and the networks of the "small worlds" type [39-40]. A particular attention has been paid to the epidemic resistance of these networks [23-34]. The paradigmatic models of the analog nature have been the basis of this analysis. At the same time, some discrete models [36] have appeared, more adequately describing such processes.

However, the confrontation observed in the network structures is by no means limited to mutual virus attacks and subsequent epidemics. Of the theoretical and practical interest is the study of strategies and tactics of this process, which has now acquired a global character, so that some analysts have started to talk even about "network wars".

Therefore, in the present paper we attempt to formalize the network confrontation. In this connection, it is desirable not only to describe the networks from the viewpoint of topology, as this has been done hitherto [16-20], but also to take into account the filler circulating in them. The formalization of the network conflicts, taking into account the dynamics of the potential of the parties, also appears to be quite important.

Topological definition of a network

Oftentimes, the network structures are described by topological means. For example, to describe networks, two disjoint sets can be identified, so that the elements of one of them are interconnected via the elements of another according to a certain law [1-3, 5-7, 11, 13-14, 22]. In particular, this could be a set of variables and a set of functionals, which establish relationship between the variables of the mathematical model of the system under study. In the theory of electronic networks, such sets are, for example, the set of node potentials and the set of

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transmissions from a node of the circuit to another node [1-3, 5-7, 11, 13-14, 22]. The set of the elements being connected is called the set of vertices, whereas the set of connecting elements is called the set of edges.

Therefore, to define a network topologically, one should define the sets of vertices and edges (Table 1), as well as a law (predicate), establishing the mutual belonging (incidence) of the elements of the sets. It is considered that topologically a network

$$G = G(X, A, \Gamma) \tag{1}$$

is defined, if there are given a nonempty set of vertices $X \neq \varnothing$, a disjoint with it set of edges $A(A \cap X = \varnothing)$ and a predicate (incidentor) Γ.

Table 1. The kinds of vertices and edges for the types of networks.

Network type	***Network vertices***	***Network edges***
1	2	3
Electrical network	Electric power plants and electric substations	Electric power lines
Gas transmission network	Gas fields and storages, gas pumping stations, gas-filling hubs	Gas pipelines and logistics of sea transportation of liquefied natural gas
Oil-transportation network	Oil fields and storages, oil pumping stations, oil-filling hubs	Oil pipelines and logistics of sea transportation of oil
Internet	Computers	Cable and wireless channels of communication
Cellular networks	Cellular phone, base stations of the network	Wireless channels
Railroad networks	Railroad stations	Train routes
Retail networks	Supermarkets, shops, retail outlets, online stores	Logistics of the retail articles delivery
Citation networks	Papers	Quotations
Neural networks	Neuron	Synapse
Wholesale trade networks	Wholesale warehouses	Logistics of the merchandise delivery
Exchange trade networks	Brokerage firms and clients	Communication channels for bidders
Social networks	Users, portal, websites, blogs, accounts and so on	Requests for content
Postal networks	Post offices, centers of postal service	Routes of mail transportation

Usually, Γ is a triadic predicate [41], *i.e*. a predicate defined on all ordered triples x_i, x_j and a_k, for which $x_i, x_j \in X$ and $a_k \in A$. Analytically, the predicate is described [41] by a logical statement of the following type:

$$\Gamma(x_i, a_k, x_j), \tag{2}$$

which means that edge a_k joins the vertices x_i and x_j. The vertices x_i and x_j are called incident, whereas the edge a_k, incident to these vertices.

Geometrically, a network is usually represented by a graph, *i.e*. a collection of points, which are in one-to-one correspondence with the elements of the set of vertices X, and the lines connecting them, which are in one-to-one correspondence with the elements of the set of edges A.

Elements and parts of the network topology

On the basis of the definition (1) of a network, for each element $a_k \in \mathrm{A}$ there holds [41] one and only one of the following statements:

$$\exists\, x_i x_j \left[x_i \neq x_j \;\&\; \Gamma\,(x_i, a_k, x_j) \;\&\; \overline{\Gamma\,(x_i, a_k, x_1)} \right]; \tag{3}$$

$$\exists\, x_i \left[\Gamma\,(x_i, a_k, x_j) \right]; \tag{4}$$

$$\exists\, x_i x_j \left[x_i \neq x_j \;\&\; \Gamma\,(x_i, a_k, x_j) \;\&\; \Gamma\,(x_i, a_k, x_1) \right]; \tag{5}$$

Table 2. The weights of vertices and edges for various types of networks.

Network type	***Weights of the network vertices***	***Weights of the network edges***
1	2	3
Electrical network	Volume of the generated or accumulated electrical energy	Losses in the transmission of electrical energy
Gas transmission network	Volume of the extracted and accumulated gas	Carrying capacity of the gas pipeline, tonnage of the liquefied gas tanker
Oil-transportation network	Volume of the extracted and accumulated oil	Carrying capacity of the oil pipeline, tonnage of the tanker
Internet	Volume and value of the generated content	Capacity of the internet-connection
Cellular networks	Personal data of subscriber	Availability and reliability of the communication channel
Railroad networks	Volume of the accommodated trains	Freight traffic rate of the moving trains
Retail networks	Volume of revenue for each element of the network	Commodity circulation along the supply chains
Citation networks	Authorship and the amount of information	Citedness of the source
Neural networks	State of the neuron	Speed of response
Wholesale trade networks	Volume of accumulated and sold batches of goods	Volume of goods supplied along the network edges
Exchange trade networks	Exchange rates and share prices, stock exchanges indices	Volume of trade over the communication channels
Social networks	Quantity of users and network resources	Intensity of requesting the resources

The logical statements (3) - (5) allow classifying edges as the oriented (directed) edges, arcs (3), loops (4) and non-oriented (non-directed) edges, links (5). It is appropriate to note that, in a number of practical cases (we have in mind unistor graphs), the links are represented, according to the statement (5), by an aggregate of two (merged into one) arcs. In studying such properties of the network that do not depend on the direction of its arcs, it is convenient to use the predicate

$$\widetilde{\Gamma}\,(x_i, a_k, x_j) \;\leftrightarrow\; \Gamma(x_i, a_k, x_j) \vee \Gamma(x_j, a_k, x_i),$$

which is called a semi-incidentor; it is applied in the social networks.

In the description of physical networks, a weight $\delta(a_k)$ corresponds to each edge, which is called the weight of the edge a_k and which is equal to a specific physical quantity (Table 2).

As for the network vertices, they are often identified with the variables that describe the state of the object. For example, in solving a number of technical problems (generalized signal graph), the concept of a weighted vertex (weight of the vertex) is used, which can be interpreted as the weight of the loop, incident to the given vertex.

As the kinds of elements in the network, we may distinguish chains and cycles. By a chain we mean a sequence $x_0a_1x_1a_2x_2\ldots x_{N-1}a_N x_N$ of the elements of a graph, for which the statement $\&_{i=0}^{N-1}\Gamma(x_i, a_{i+1}, x_{i+1})$ is true. A cycle is a closed chain $x_0 = x_N$.

Such elements of graphs as path and loop play a very important role in the description of networks with the help of graphs. A path $P_{0,N}$ from the vertex x_0 to the vertex x_N is a finite chain $x_0a_1\, x_1\, a_2\, x_2\ldots x_{N-1}\, a_N\, x_N$, for which the statement $\&_{i=0}^{N-1}\Gamma(x_i, a_{i+1}, x_{i+1})$ is true.

As a qualitative characteristics of the path P, one uses such concepts as the path length l(P) and the path weight δ(P).

The number of edges that form the path is called the length of the path. For the path $x_0a_1x_1a_2x_2\ldots x_{N-1}\, a_N\, x_N$, its length $l(P_{0,N}) = N$. The weight of the path can be defined as the product of weights of the edges that form it; for the path above, the weight

$$\delta(P_{0,N}) = \prod_{i=1}^{N}\delta(a_i).$$

A path may be finite or infinite; it is called simple if no edge occurs in it twice. The path P, in which none of the vertices occurs twice, is called elementary. In the description of a network, simple elementary paths are usually used. Therefore, for brevity, the "path" term will refer, in what follows, to a simple elementary path.

A closed path is called a contour L, for the contour $x_0 = x_N$. The definitions of the length l(L) and the weight δ(L) of the contour are analogous to the corresponding definitions, formulated above for a path. A contour L is called simple, if all its edges are different, or composite (complex), otherwise. The contour L is called elementary, if all its vertices are different.

To describe the properties of a network, the elementary contours are applied, which in the sequel will be called just contours. For such contours, as for the paths (excluding the initial and end vertices), the statement is true that each vertex is incident to two arcs, while for one of them it is the end vertex, whereas for the other, the initial one, *i.e.*

$$\forall x_i \in L[s^+(x_i) = s^-(x_i) = 1],$$

$$\forall x_i \in P \;\&\; i \neq 0 \;\&\; i \neq N[s^+(x_i) = s^-(x_i) = 1],$$

where $s^+(x^i)$ is the number of arcs, issuing from the vertex x_i; $s^-(x_i)$ is the number of arcs, entering the vertex x_i (degree of the vertex) [22].

Trees and pre-trees are the elements of graphs which are rather widely used in practice. A tree is a graph, not containing cycles. The edges, complementing a tree, are called chords. A pre-tree T is a tree, in which each vertex (with the exception of one of them, x_0, called a root of the pre-tree) is the end one only for a single arc, *i. e.*

$$\forall x_i \in T[s^+(x_i) = 1 \;\&\; i \neq 0].$$

The weight of a pre-tree is defined as the product of the weights of all arcs included in it.

To describe graphs, one often uses the notion of a k-tree; a k-tree (k-pre-tree) is the union of k non-touching trees (pre-trees); the weight of a k-tree (k-pre-tree) is equal to the product of the weights of all the constituent trees (pre-trees). A dual to the notion of a k-tree is the notion of a k-chord, which is a collection of k non-touching chords of a tree (pre-tree). A particular case of pre-tree is a star, a collection of simple paths with a common end or initial vertex; this vertex will be called the center x_c of the star. If the center of the star S is the initial vertex of the paths $P_{c,i}$, *i.e.*

$$S = \&_{i=1}^{m} P_{c,i},$$

where m is the number of paths that form the star S, then such star will be called divergent. Otherwise, if the center of the star S is the end vertex of the paths $P_{j,c}$ that constitute S, *i.e.*

$$S = \&_{i=1}^{m} P_{j,c},$$

then the star will be called convergent. Besides, we will call a star simple, if every its path has the length equal to one.

A characteristic feature of the star is that, to any vertex that is not the star center, only one outgoing and one incoming arc are incident.

Star matrix of the network

To describe graphs, the matrices of neighborliness (adjacency), sections and contours are used. However, a shortcoming of these matrices is that they reflect only the presence or absence of incidence between the vertices and edges and do not take into account the weights of the edges, which are oftentimes assigned to each of these elements in describing real objects (in particular, networks). In this regard, one should consider to be more convenient the matrix S, the elements of which are determined by the following relations:

$$\Gamma(x_i, a_{ij}, x_j) \rightarrow \mu_{ij} = \delta(a_{ij});$$
$$\Gamma(x_k, a_{jk}, x_j) \rightarrow \mu_{ik} = \delta(a_{ik});$$

where $i, j, k = 1(1)n$; $G(X, A, \Gamma)$ is the unigraph, described by the matrix S; $X=\{x_1, x_2,\dots, x_n\}$.

Let us call S the star matrix of the network. It is easy to notice that S is a square matrix of the dimension $n \times n$. Moreover, the diagonal elements of the star matrix correspond to the weights of the loops a_{ii} at the nodes x_i, whereas the non-diagonal elements, situated at the intersection of the j-th row and the k-th column, correspond to the weights of the arcs a_{jk}, connecting the vertex x_k with the vertex x_j. In the general case, the row j (Figure 1) of the matrix S contains the weights of the loop and the arcs entering the vertex j:

μ_{j1}		μ_{jk}		μ_{jj}		μ_{jn}

Fig. 1. The row *j* of the star matrix of the network.

To this row, there corresponds a convergent simple star with a loop at its center x_j (Figure 2*a*). Therefore, the method of constructing the network graph can be reduced to a sequence of construction of convergent simple stars and loops at each node with their subsequent uniting. Similarly, in the general case, to the k-th column (Figure 3) of the matrix S, there corresponds a divergent simple star with a loop at its center x_k (Figure 2*b*).

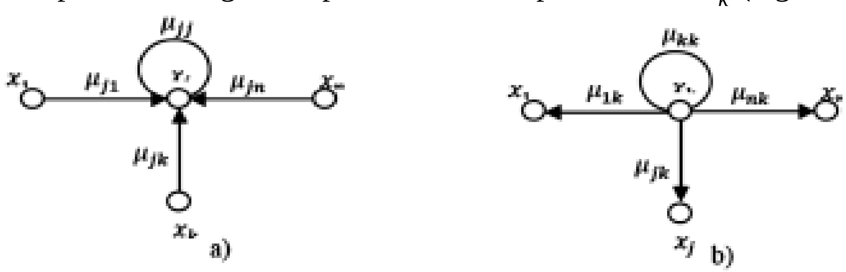

Fig. 2. The parts of the network graph, corresponding to a row (*a*) and a column (*b*) of the S-matrix.

μ_{1k}
………
μ_{kk}
………
μ_{jk}
………
μ_{nk}

Fig. 3. The *k*-th column of the star matrix of the network.

Hence, any digraph of the network can be constructed by uniting the divergent simple stars and loops, formed for each of its vertex separately.

An example of a star matrix can be the matrix, constructed for the graph (Figure 4).

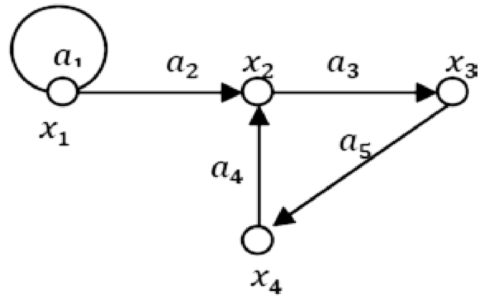

Fig. 4. An example of a network digraph.

The given example testifies to the simplicity of forming the star matrix. An advantage of the matrix S is also its conformity with the system of homogeneous linear equations of the network

$$\sum_{j=1}^{n} a_{ij} x_j = 0, \; i = 1(1)\, n \tag{6}$$

or, in the matrix form, $$SX = 0,$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$ is the vector of state variables of the network vertices.

Among the merits of the matrix S is its convenience for the formation of the contour and tree subgraphs, necessary in the calculation of symmetrical and non-symmetrical algebraic complements of the determinants of the described linear systems [22].

Fillers of the networks

No matter how diverse [23-35] the networks are, they are characterized by the presence of filler (Table 3). It is this filler that is collected and processed at the vertices (nodes) of the network and transported along its edges.

The concentration and transfer of the filler is the main function of any network, from electronic to the biological ones. The nodes and transfer channels are necessarily adapted to the network type, *i.e.*, to the kind of its filler (Table 3). This filler is usually stored, processed and filtered at the network vertices. The processing of the filler is diverse. Clearly, the block diagram of the implementing it network node (Figure 5) only generally reflects the set of its operators (in practice, it may be shorter and more detailed). For example, in the oil transport networks, a new oil product, gasoline, can be obtained from oil.

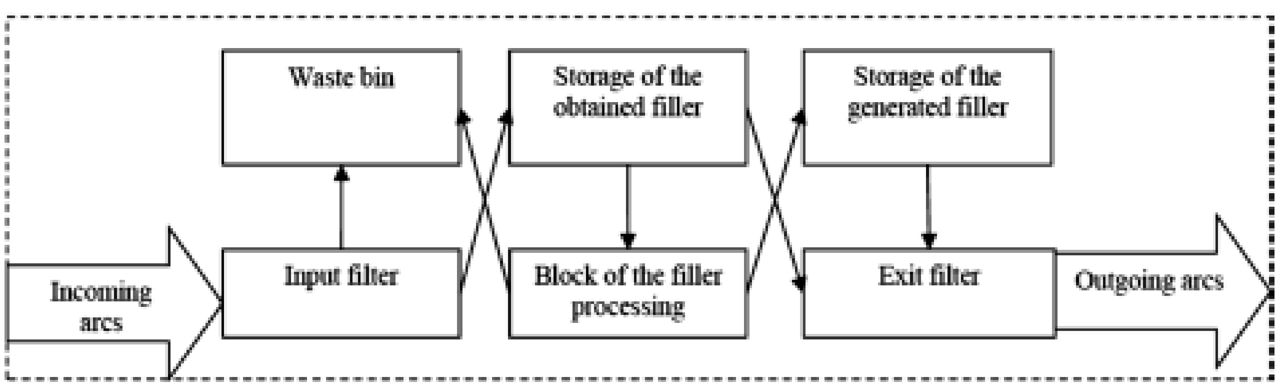

Fig. 5. Generalized block diagram of the node (vertex) of the network, taking into account the processing of its filler.

In the gas transmission networks, the processing may include producing LNG for its subsequent loading through the terminal to tankers. In the banking networks, the carrier may undergo a currency conversion, etc. However, regardless of the form, the filler is necessarily present in each network, as the blood in any living organism.

In the above context, the definition of a network will require not only specifying its graph G (1), but also clarifying the issue of the network filler I. Thus, it is possible to define a network N by setting the graph defining its topology and the filler flows circulating over its structure

$$N_{et} = N_{et}(I,G), \tag{7}$$

where I is the set of filler flows; G is the structure (topology) of the network, defined by a graph or its matrix.

In (7), the network has a less abstract than in (1) representation, because not only the nature of connections of its vertices is taken into account, but also the parameters of the filler transmitted along its edges. This is the fundamental distinctive feature of the definition of network proposed in (7).

Table 3. Kinds of filler according to the network type.

Network type	***Filler of the network***
1	2
Electrical network	Electric current
Gas transmission network	Natural gas
Oil transportation network	Oil
Internet	Internet-content
Wireless network	Radio-signal
Railroad networks	Cargo and passengers
Trade network	Commodities
Neural network	Neural impulse
Social network	Social information
Spider's net	Vibration signal
Exchange trade networks	Quotations
Network of military bases	Arms, military personnel, ammunition
Blood circulatory system	Venous and arterial blood
Bank network	Money
Subway network	Trains with the subway passengers

It should be noted that I and G are disjoint ($I \cap G = \varnothing$) sets, but, obviously, between their elements should exist some one-to-one correspondences of routing (which flow, along which edge and to which vertex goes).

From this perspective, the last expression (7) will assume the form

$$N_{et} = N_{et}(I, G, M), \tag{8}$$

where M is the predicate (schedule) of routing the flows I in the structure G.

The obtained expression (8), combined with (1), should be considered as the most complete definition of a network, taking into account its topology G and filler I, as well as the routing M of the filler in its structure.

In addition, it should be noted that all sets listed in (8) are functions of the time, after which the carrier flows and their routes may change and even the network topology may change. The schedule M is responsible for their synchronization. A typical example of this is social networks [23-35], pulsating with their connections, the number of users, popularity of contents, etc. In contrast to the corporate networks, the schedule M has an obvious stochastic character.

The development of topological variety of networks is generally related to the structures with the information filler [23-35].

A fundamentally different idea from all of the above is a proposal to introduce a filler matrix (Figure 6), which takes into account not only the topology of the storage and transfer of filler, but also the parameters of these operations from the viewpoint of the filler. The elements of this matrix are obviously functions of time, so it can help to monitor the dynamics of the network. At the same time, the elements of the matrix (Figure 6) offer the prospect of weighing the components (vertices and edges) of the network, which is quite essential for the description of heterogeneous networks, which form the basis of modern information, economic and other spaces of the world order.

To consider the features of the filler matrix, we mark in Figure 7 arbitrary vertices x_i and x_k, as well as the arcs a_{jk} and a_{kj} which connect them.

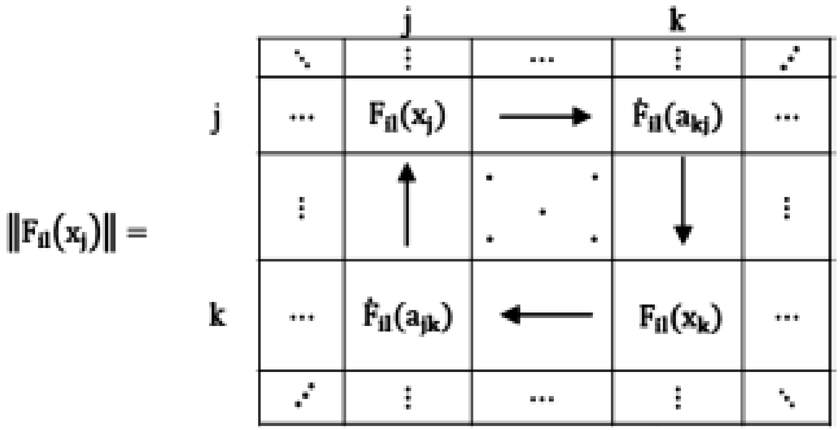

Fig. 6. Filler matrix of the network, where = $\dot{F}_{il} = \partial F_{il}/\partial t$

As we can see (Figure 6), the matrix $\|F_{il}(x_j)\|$ retains all the properties of the network topology, reflected in the star matrix (Figure 7). The only difference is that its cells contain the parameters of the filler: in the diagonal cells, the parameters of the filler stored in the network vertices, whereas in the non-diagonal elements, the carrying capacity of the filler circulating through the corresponding arcs of the network.

$$S = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{|c|c|c|c|} \hline a_1 & 0 & 0 & 0 \\ \hline a_2 & 0 & 0 & a_4 \\ \hline 0 & a_3 & 0 & 0 \\ \hline 0 & 0 & a_5 & 0 \\ \hline \end{array} \end{array} \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}$$

Fig. 7. Star matrix of the network.

Metrics of the weighted networks

Using unweighted graphs [21-22] significantly limits the possibilities of adequate description of network structures. This shortcoming is especially manifested in addressing the problems of the network security assessment tasks where the risk analysis includes an assessment of potential damage. Without the weighing δ(.) of the vertices and edges of the network graph, it is virtually impossible. We introduce the necessary notation: V[.] is the operation of weighing the filler volume; C[.] is the operation of weighing the value of the unit volume of the filler; P_{ot} [.] is the operation of weighing the network potential or its element with respect to the filler; F_{il} is the network filler; R_{eal} [.] is the operation of weighing the real filling of the network or its element; Net is the network; $F_{il(in)}$ is the filler entering the network; $F_{il(out)}$ is the filler coming out of the network.

The question arises: what and how to weigh? Here the most important factor is the network filler. Its volume accumulated at a vertex or transported along an arc becomes the result of weighing. Thus, the concept of carrying capacity has started to be used for the network arcs; this concept refers to the maximum possible volume of filler transmitted along the arc per unit time. Obviously, this is an extreme (potential) assessment, which determines the potential of the arc. Similarly, for a network vertex, it is possible to find the maximum admissible volume

$$\mathrm{P_{ot}}[x_i] = \max\{\delta(x_i)\} = \max\{\mathrm{V}[\mathrm{F_{il}}(x_i)]\}, \tag{9}$$

the maximum of its weight with respect to V, the filler volume, stored at the vertex x_i. In turn, the potential of a network arc is

$$\mathrm{P_{ot}}\,[a_{jk}] = \max\{\delta(a_{jk})\} = \max\left\{\frac{\partial \mathrm{V}[\mathrm{F_{il}}(a_{jk})]}{\partial t}\right\}, \tag{10}$$

the maximum of the (time) derivative of the volume V of the filler, pumped through the arc a_{jk}.

In actual circulation of the filler over the network, the instant values of weights of its vertices and edges are substantially different from their potentials. Here some averaged (over the period of study) estimations are appropriate.

However, it is not so important whether the average or extreme estimation is realized. It is important to find the potential of the network as a sort of integral (with regard to the limiting filling) characteristic. To do this, one should find the total weight of all vertices of the network

$$\mathrm{P_{ot}}[\mathrm{X}] = \sum_i \mathrm{P_{ot}}[x_j] \tag{11}$$

and all its arcs

$$\mathrm{P_{ot}}[\mathrm{A}] = \sum_i \mathrm{P_{ot}}[\mathrm{a}_{jk}]. \tag{12}$$

From (11) and (12), it is possible to propose the following potential metric of the network

$$\mathrm{P_{ot}}[\mathrm{Net}] = \mathrm{P_{ot}}[\mathrm{X}] \times \mathrm{P_{ot}}[\mathrm{A}]. \tag{13}$$

For a non-weighted network, where the weights of the vertex and the arc are identical, we have

$$\mathrm{P_{ot}}\,[\mathrm{Net}] = \mathrm{N} \times \mathrm{M},$$

where N is the number of vertices and M is the number of arcs in the networks.

These estimates do not take into account the most important (in terms of safety) parameter. This is the value of the unit volume of the filler, which, generally speaking, has its own value for each vertex and arc of the network. In this context, the expressions (9) and (10) take the form

$$\mathrm{P_{ot}}[x_i] = \max\{\mathrm{C}[\mathrm{F_{il}}(x_i)]\mathrm{V}[\mathrm{F_{il}}(x_i)]\} \tag{14}$$

and

$$\mathrm{P_{ot}}[a_{jk}] = \max\left\{\mathrm{C}[\mathrm{F_{il}}(a_{jk})]\frac{\partial \mathrm{V}[\mathrm{F_{il}}(a_{jk})]}{\partial t}\right\}. \tag{15}$$

In the case of immensity of the problem, its simplification is possible due to categorizing the network elements. So, for inhomogeneous networks, a layer-by-layer consideration is possible, where the layer is formed by the degree of the vertices. The value of the filler can be considered roughly equal and then the expression (13), which takes into account (14) and (15), is substantially simplified.

At the same time, the real content of the network is of practical interest

$$\mathrm{P_{eal}}[\mathrm{Net}] = \mathrm{P_{eal}}\,[\mathrm{X}] \times \mathrm{P_{eal}}[\mathrm{A}], \tag{16}$$

where:

$$\mathrm{P_{eal}}[\mathrm{X}] = \sum_i \{\mathrm{C}[\mathrm{F_{il}}(x_j)]\,\mathrm{V}\,[\mathrm{F_{il}}(x_i)]\},$$

$$\mathrm{P_{eal}}[\mathrm{A}] = \sum_{jk} \mathrm{C}[\mathrm{F_{il}}(a_{jk})]\frac{\partial \mathrm{V}[\mathrm{F_{il}}(a_{jk})]}{\partial t}. \tag{17}$$

as well as the maximum admissible filling of the simplest piece of the network (two vertices connected by an arc), from a collection of which the network itself is formed.

Besides, it is possible to find the balance of the network filler

$$P_{eal}\,[\mathrm{Net}] \;=\; P_{eal}\,[F_{il(in)}] - P_{eal}[F_{il(out)}], \tag{18}$$

which is, obviously, a function of time. In the information-telecommunication networks, F_{il} is the content.

Formalization of the strategies of network confrontation

A network [22] is a system (structure-parametric) formation

$$\mathrm{Net}\;(X, A, F_{il}) \tag{19}$$

on the set of vertices (concentrators) X, the disjoint with it set A of arcs (transporters) $X \cap A = \varnothing$, the filler F_{il}, which circulates, is accumulated and processed in the given network N_{et}.

Accordingly, it is possible to evaluate the network capabilities through the cardinality of the above sets |X| and |A|. In particular, the structural potential of the network (19) can be defined as follows:

$$P_{ot}\,(X, A) \;=\; |X|\,|A|. \tag{20}$$

One of the strategies of network confrontation is to limit the structural capabilities of the enemy network. This approach can be implemented in two ways. **First**, one can impede the development of the network structure and its potential, when one seeks to reduce the positive derivative of the growth of powers of the sets of concentrators and network transporters

$$\frac{\partial \ln |X|}{\partial t}\downarrow \text{and}\ \frac{\partial \ln |A|}{\partial t}\downarrow .$$

For example, with regard to the European energy supply, the restrictions have been introduced to the Russian gas transmission network, concerning building the "South Stream" and developing the "Nord Stream".

Without any doubt, these decisions of the European Union were lobbied by the players interested in the development in Europe of their energy networks and weakening of Europe's dependence on the energy resources produced in Russia. A key player in the implementation of this strategy is the United States, under the pressure from which Bulgaria has refused to allow the "South Stream" gas pipeline passing through its territory.

It should be noted that all these alternatives have arisen because of problems with the gas pipeline passing through Ukraine to Europe, which serves as an illustration of implementation of the **second strategy** of the network confrontation. The thing is that it is also possible to reduce the potential of the enemy network by means of attacking the stem (with the highest carrying capacity) transporters A_M, which leads to reducing the filler flow

$$\frac{\partial \ln |F_{il}(A_M)|}{\partial t}\downarrow$$

due to various reasons. For Ukraine, these are political factors and its economic difficulties, leading to "extortion by the pipe". The unauthorized gas takeoff, violations of the contractual obligations, and also extortion of money from the European Union to pay for gas are observed there.

The Ukrainian center of political instability, organized by the United States, in the absence of alternative network capacity to supply gas to Europe, remains a significant obstacle to establishing normal economic Russian-European relations.

It is appropriate to note that such crisis phenomena have a multi-network character. Under the pretext of "exporting democracy", the Ukrainian coup in 2014 was carried out in the context of the slogan "Ukraine is not Russia", which contained a seed of severance of the critical network links with the Russian Federation. As the subsequent events showed, these ties started to be cut with respect to the military-industrial complex, transport and information communication, where the potential (20) of the network cooperation of the countries significantly diminishes

$$\frac{\partial \ln P_{ot}(X, A)}{\partial t}\downarrow .$$

The attack on the network may also be carried out by reducing the value of its filler. In fact, in this way the potential of the network (16) - (17) diminishes. Such an operation was carried out by the United States and Saudi Arabia by causing to fall the oil prices and thus undermining the economy of the USSR in the late twentieth century.

The third strategy should be especially mentioned, when not so much the structural "sequesters" are imposed on the network but the filling of the network is restricted. In relation to Russia, this attack is carried out by means of sanctions, restricting the influx into the domestic network structures of the filler in the form of credit resources, advanced technologies and investments.

Recall that these sanctions were imposed under the pretext of settling the Ukrainian crisis, that is, Ukraine was used as a tool in the network war against Russia.

Today, the most important task of the manufacturer is not so much to make the product, but to sell it. Even the consumer society, formed by the Western civilization, cannot help in solving this problem without additional measures. In connection with this, a true network economic war has started in the global economic space.

The states that are the leading producers seek to expand their trade networks to the new economic spaces, restricting the access of their competitors to these markets. For example, the USA is preparing a project of market integration with Europe. They intend to implement something similar in the Asia-Pacific region, driving China out of this market. Creating the most favored regime for its own multinational companies, obviously having a network structure, such alliances provide an opportunity for the rapid development of their networks and prevent a similar development of the network economy of the countries which are not the members of these unions. In fact, these are the measures of economic globalization of the world and getting more opportunities to control the planet through the control over the markets of the North-Atlantic and Asia-Pacific areas primarily in the interests of the United States.

This is the **fourth strategy** of the network confrontation, when one network tries to increase its potential

$$\frac{\partial \ln \mathrm{Pot}_1(\mathrm{X}, \mathrm{A})}{\partial t} \uparrow . \tag{21}$$

At the same time, there are created the conditions for the other network

$$\frac{\partial \ln \mathrm{Pot}_2(\mathrm{X}, \mathrm{A})}{\partial t} \uparrow ,$$

which reduces its potential. These conditions may concern the restrictions on the movement of goods and so on.

Criteria for evaluation of network conflict, taking into account the metrics of the weighted networks

To classify conflicts, the modern conflict studies use the dynamics of efficiencies E_1 and E_2 of the conflicting parties on the basis of the differential sensitivity [37] in the form of $\partial E_2 / \partial E_1$. First, correct assessment of the effectiveness of the network is very difficult. Second, the most objective are the relative rather than differential assessments, *i.e.* one should calculate the relative sensitivities of the analyzed parameters of the conflicting networks. This is confirmed by the fact that, for the same absolute deviations of efficiency, the advantage in relative terms belongs to the party with the lesser current value of efficiency. Let us try to correct this shortcoming in the conflict assessments.

Suppose that the metrics of some conflicting networks Net_1 and Net_2 are known; in particular, their potentials $P_{ot}[\mathrm{Net}_1]$ and $P_{ot}[\mathrm{Net}_2]$ have been evaluated. Assuming that the confrontation is aimed at changing the capabilities of the conflicting parties, let us assess the conflict according to these extreme estimates. In the differential form, the assessment criterion will be the following

$$\mathrm{K} = \frac{\partial P_{ot}[\mathrm{Net}_1]}{\partial P_{ot}[\mathrm{Net}_2]}. \tag{22}$$

However, as has been shown above, this assessment (13) does not quite satisfy the researcher. So we move on to the relative sensitivity

$$K = \frac{\partial \ln P_{ot}[Net_1]}{\partial \ln P_{ot}[Net_2]} = \frac{\partial \ln P_{ot}[Net_1]/dt}{\partial \ln P_{ot}[Net_2]/dt}, \tag{23}$$

where, taking into account the metrics on the set of vertices $P_{ot}[X]$ and $P_{ot}[A]$ of the network, we have

$$K = \frac{\partial \ln P_{ot}[X_1]/\partial t}{\partial \ln P_{ot}[X_2]/\partial t} = \frac{\partial \ln P_{ot}[A_1]/\partial t}{\partial \ln P_{ot}[A_2]/\partial t}. \tag{24}$$

In the expression (24) we have $P_{ot}[X_1] = \sum_i \max\left\{C[F_{il_1}(a_{jk})]\frac{\partial V[F_{il_1}(a_{jk})]}{\partial t}\right\};$ (25)

where the operators C[.] and V[.] respectively assess the value and the volume of the filler F_{il} in the elements (vertices x_i and arcs a_{jk}) of the network Net_1. The expressions analogous to (25) can be written also for the second network Net_2

$$P_{ot}[X_2] = \sum_i \max\{C[F_{il_2}(x_i)]\, V[F_{il_2}(x_i)]\};$$

$$P_{ot}[A_2] = \sum_{jk} \max\left\{C[F_{il_2}(a_{jk})]\frac{\partial V[F_{il_2}(a_{jk})]}{\partial t}\right\}; \tag{26}$$

In the expressions (25) and (26), the summation is performed over the entire set of vertices and arcs of each network separately.

In the case, when the confrontation aims at changing the real parameters of the networks, it is appropriate to assess the conflict according to the following criterion

$$K = \frac{\partial \ln R_{eal}[X_1]/\partial t}{\partial \ln R_{eal}[X_2]/\partial t} + \frac{\partial \ln R_{eal}[A_1]/\partial t}{\partial \ln R_{eal}[A_2]/\partial t}, \tag{27}$$

where the operator R_{eal} [.] realizes weighing of the real (not extremal) filler in the elements of the network Net_1

$$P_{eal}[X_1] = \sum_i \{C[F_{il_1}(x_i)]\, V[F_{il_2}(x_i)]\}; \tag{28}$$

$$P_{eal}[A_2] = \sum_{jk} C[F_{il_1}(a_{jk})]\frac{\partial V\,[F_{il_1}(a_{jk})]}{\partial t}; \tag{29}$$

and the network Net_2 $P_{eal}[X_2] = \sum_i \{C[F_{il_1}(x_i)]\, V[F_{il_2}(x_i)]\};$ (30)

$$P_{eal}[A_2] = \sum_{jk} C[F_{il2}(a_{jk})]\frac{\partial V\,[F_{il_2}(a_{jk})]}{\partial t}. \tag{31}$$

As can be seen, the expressions (28) and (29) differ from the expressions (25) and (26) by the absence of searching the maximum admissible values.

In each specific case of confrontation, a conflict can be classified by the values of K, determined by the above expressions (24) – (29).

As for measuring the depth ρ of the conflict, the following analytical assessments can be offered. In particular, we have for the set of vertices

$$\rho[X] = |(\partial P_{eal}\,[X_1])/\partial t - \partial P_{eal}\,[X_2])/\partial t|. \tag{30}$$

Correspondingly, for the set of arcs we can write

$$\rho[A] = |(\partial P_{eal}\,[A_1])/\partial t - \partial P_{eal}\,[A_2])/\partial t|. \tag{31}$$

Expressions (30) and (31) reflect the relative dynamics of confrontation between Net_1 and Net_2 in the context of expansion (contraction) of real possibilities of the concentrators (30) and the transporters (31) of the filler.

For the extremal assessments (in the context of the struggle just for them), it is possible to obtain similar expressions:

$$\rho[X] = |(\partial P_{ot}[X_1])/\partial t - \partial P_{ot}[X_2])/\partial t| ; \tag{32}$$
$$\rho[A] = |(\partial P_{ot}[A_1])/\partial t - \partial P_{ot}[A_2])/\partial t| ;$$

Accordingly, for the integral assessment of the depth of the conflict, one can write

$$\rho[Net] = |(\partial P_{ot}[Net_1])/\partial t - - \partial P_{ot}[Net_2])/\partial t|. \tag{33}$$

In conclusion, it should be noted that the obtained expressions (24) - (33), as well as other characteristics of the conflict, are functions of time and must be calculated separately for each specific situation of confrontation.

2. CONCLUSION AND THE DEVELOPMENT DIRECTION

The obtained above mathematical models may be useful for describing the distributed systems [22] and similar network structures which conflict in space [21]. One of the forms of their confrontation is mutual virus attacks. The epidemics arising in this case cause significant damage to the opponent. Therefore, in terms of further development of this research work, it is expedient to examine the dynamics of the potentials of the opposing sides in case of failure of the elements under virus attacks.

For example, for homogeneous networks, their potential, based on the results of this paper, can be estimated initially as follows

$$P_{ot}[Net] = \frac{k}{2}[X]^2,$$

where k is the degree of the network vertices; |X| is the number of the network vertices;

Taking into account the number $|X_R|$ of the vertices, destroyed by the virus, the relative dynamics of the network potential will be

$$\frac{\Delta P_{ot}}{P_{ot}} = \frac{\frac{k}{2}(|X| - |X_R|)^2}{\frac{k}{2}|X|^2} = N_X^2,$$

where NX = $\frac{|X_R|}{|X|}$ is the epidemic resistance [36] of the network vertices during a virus attack on the network.

To assess the conflict at the considered stage, it is appropriate to compare the relative deviations of the conflicting party potentials

$$\frac{\Delta P_{ot\,1}}{P_{ot\,1}} - \frac{\Delta P_{ot\,2}}{P_{ot\,2}} = (N_{X2} - N_{X1})(N_{X2} + N_{X1}),$$

where N_{X1} and N_{X2} are the corresponding epidemic resistances of the mutually attacked sides of the conflict.

The presented expressions show that it is the party with a larger epidemic resistance, *i.e.* with a smaller N_X, that has an advantage.

The development of such estimates is of real practical and theoretical interest for further studies of network confrontation with the usage of virus attacks.

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