

PERFORMANCE COMPARISON OF DIFFERENT ADAPTIVE FILTERING ALGORITHMS

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Abstract: This paper describes the comparison between Least Mean Square (LMS) and its variants based on different performance parameters like Mean Square Error (MSE), Computational complexity etc. The performance of various adaptive filtering algorithms has been analyzed for stochastic input signals. LMS algorithm along with its variants like sign-sign LMS, sign-data LMS etc has also been compared with RLS algorithm. As compared to fixed filtering, adaptive filtering is more suited for random processes. It has many applications like adaptive echo cancellation, adaptive line enhancement, adaptive noise cancellation, system identification and inverse modeling.

Key Words: Least mean Square(LMS), Recursive Least Square(RLS), optimum filter weights, LMS convergence.

1. INTRODUCTION

An adaptive filter is a filtering device that finds the relationship between the input and output signals of the filter. An adaptive filter self-adjusts the filter coefficients according to an adaptive algorithm. It is different from conventional fixed coefficients FIR filters and IIR filters in a way that it adapts to the changing behavior of the input signal.

Large number of advantages is associated with fixed coefficient filters. But in cases where the input is random, a fixed set of filter weights do not provide optimum output. In such cases, algorithms like LMS and RLS are used so that filter weights are optimized to minimize a cost function associated with adaptive filter.

LMS algorithm is based on method of minimizing a cost function which is mean square error. It is different from steepest descent search method in a way that it takes instantaneous values of the input and desired signals. It produces satisfactory results and provides a perfect solution to adaptive filtering problem. Computational efficiency is improved by modifying LMS algorithm and passing on partial information to LMS process. RLS algorithm is used to recursively minimize the weighted sum of squared error estimates.

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2. ADAPTIVE FILTER ALGORITHMS

2.1 LMS Algorithm

In Least Mean Square algorithm, the output is calculated using some initial value of tap-weights. This output is compared with desired signal and error is calculated. The algorithm works in such a manner that it tries to minimize the mean square error.

If $x(n)$ is the input signal and $d(n)$ is the desired signal, then error signal $e(n)$ is calculated as the difference of $x(n)$ and $d(n)$. Based on the error signal, tap-weights are optimized iteratively so as to minimize the mean square error. It uses the auto-correlation matrix R , between input signal and cross-correlation matrix p , between input signal and desired signal. E represents the expectation operator and using equation 1, the mean square error is minimized.

$$E[e^2(n)] = E[d^2(n)] + E[p^t W] + E[W^t p] + E[W^t R W] \quad [1] \quad \dots\dots(1)$$

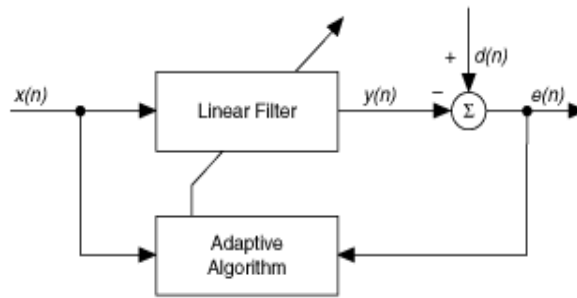


Figure. 1 Block diagram of Adaptive Filter

The block diagram of adaptive filter shown in figure 1 shows how the adaptive algorithm updates the weights and minimizes the error signal. The weight update process takes place continuously using gradient and step-size parameter using equation 2.

$$W(n+1) = W(n) + u * x(n) * e(n) \quad \dots\dots(2)$$

The convergence and speed of convergence of LMS algorithm depends upon the initial value of step-size. Its value is less than unity and typically bounded by the condition

$$0 < u < 2 / \lambda$$

This conventional LMS algorithm takes the least number of iterations to converge on to optimum filter tap-weights. But the computational efficiency is a concern.

2.2 Sign-Error LMS Algorithm

This is one of the variants of conventional LMS algorithm in which the weight update equation remains the same. In order to increase the efficiency of computation and faster implementation using hardware, only sign of error signal is used. The modified LMS becomes

$$W(n+1) = W(n) + u * x(n) * \text{sign}[e(n)] \quad \dots\dots(3)$$

Where sign returns the sign of $e(n)$. Further, step-size u is chosen as power of 2 to facilitate multiplication. With this, hardware implementation becomes easy. The computational efficiency is increased at the cost of degraded performance. It is evident from equation 4 that convergence is slow initially due to large value of step-size.

$$W(n+1) = W(n) + (u/|e(n)|) * x(n) * e(n) \quad \dots\dots(4)$$

2.3 Sign- data LMS Algorithm

As in the case of previous variant of LMS algorithm, here sign of input samples is used. This partial information that is passed onto the weight update process, results in better results as compared to sign-error LMS algorithm.

$$W(n+1) = W(n) + (u/|x(n)|)*x(n)*e(n) \quad \dots(5)$$

As it is clear from equation 5 that the convergence is independent of error. It has better performance as compared to sign-error.

2.4 Sign- Sign LMS Algorithm

As clear from the name, it uses only the signs of both, input signal and error signal. The absence of actual information makes it weaker as compared to its counterparts.

$$W(n+1) = W(n) + u*\text{sign}[x(n)]*\text{sign}[e(n)] \quad \dots(6)$$

3. RLS ALGORITHM

Recursive Least Square algorithm is based on principle of minimizing the weighted sum of squared error signal related to input signal. RLS gives excellent performance when operating in time varying environments. The enhanced performance is achieved at the cost of increased computational cost and some stability problems. In this algorithm, the filter tap-weights are updated using equations

$$\begin{aligned} W(n+1) &= w^T(n) + k(n)*e(n) \\ K(n) &= u(n)/(\lambda + X^T(n)u(n)) \\ u(n+1) &= w^{-1}(n)X(n) \end{aligned}$$

4. RESULTS

Conventional LMS algorithm gives the best performance when compared with its variants. It converges onto optimum set of weights taking minimum number of iterations. To improve the computational efficiency, sign-sign LMS performs better but it does not assure convergence.

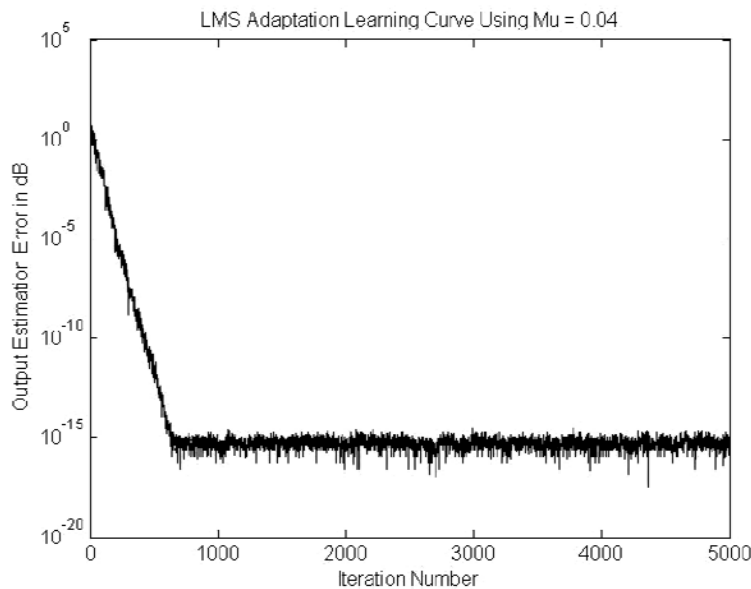


Figure.2 Learning curve of LMS algorithm

Figure 2 shows the learning curves of conventional LMS, Sign-data LMS, Sign-error LMS and Sign-Sign LMS algorithms. It has been plotted between output estimated error and number of iterations. It shows that conventional LMS algorithm converges very fast as compared to other variants and RLS algorithm. Due to its high computational complexity, its variants are also popular.

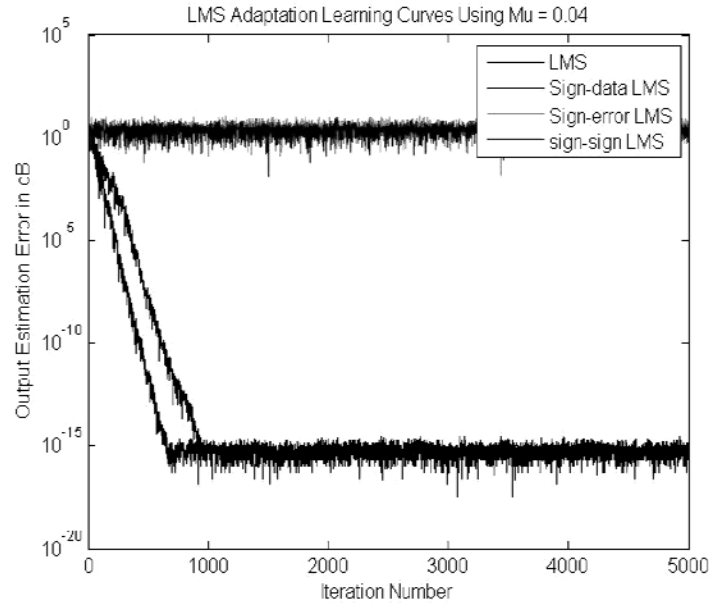


Figure.3 Learning curves of LMS algorithm and its variants

In RLS, the cost function is minimized by method of least squares in a recursive manner. The input and output plots are shown in figure 4.

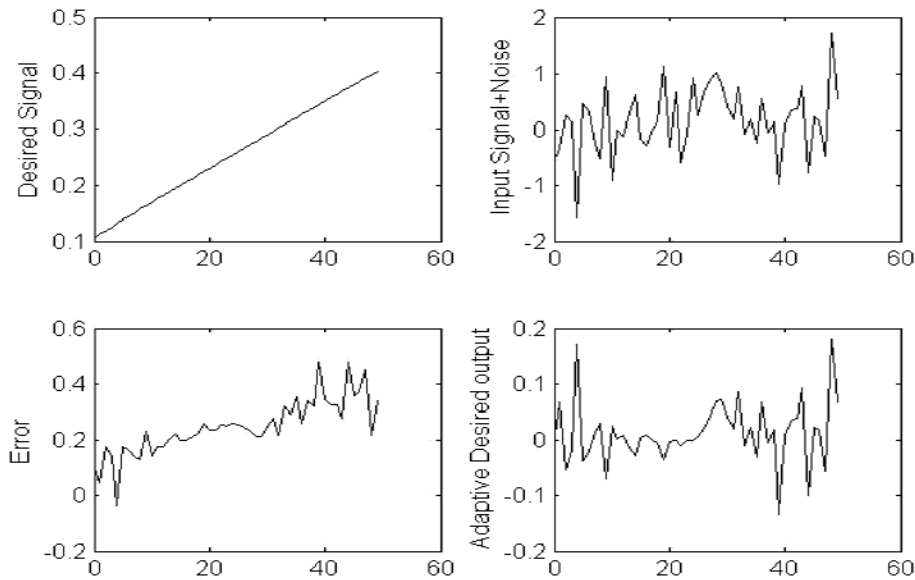


Figure. 4 RLS algorithm input and output

Table 1 shows the comparison of LMS and its variants along with RLS algorithm. Mean square error is minimum case of LMS algorithm and convergence speed is the fastest of all.

Table 1
Performance Comparison

<i>Parameter</i>	<i>LMS algorithm</i>				<i>RLS</i>
	<i>Conventional LMS</i>	<i>Sign-Data</i>	<i>Sign-Error</i>	<i>Sign-Sign</i>	
MSE	2.29 *10 ⁻¹⁶	9.155 *10 ⁻¹⁶	3.6226	0.6369	0.0428
Complexity	High	Low	Low	Very low	Very high
Stability	Less stable	Less stable	Less stable	Less stable	Highly stable

5. CONCLUSION

RLS algorithm provides better stability under changing environment and improves further with iterations. LMS algorithm has lesser computational complexity with respect to RLS, but it is less efficient computationally when it is compared with its variants.

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