

# Architectural Tensioned Fabric Structure in Monkey Saddle Form

Mohd Nasir B. Abdul Hadi\*, Yee Hooi Min\*\*, Kaydora Abd Ghani\*\*\* and Hayati Abdul Hamid\*\*\*\*

## ABSTRACT

Tensioned fabric structure is highly suited to be used for realizing surfaces of complex or new forms. In this study, form-finding of fabric surface bordered by Monkey Saddle has been investigated. Form-finding has to be carried out for tensioned fabric structure in order to determine the initial equilibrium shape under prescribed support condition and pre-stress pattern. The possibility of adopting the form of Monkey Saddle as surface shape for tensioned fabric structure has been studied. The combination of shape and internal forces for the purpose of stiffness and strength is an important feature of fabric surface. For this purpose, form-finding needs to be carried out. Nonlinear analysis method has been used for form-finding analysis of the fabric in the form of Monkey Saddle. Development and pattern of prestress in the resulting tensioned fabric surface is also studied. Form-finding has been found to converge for Monkey Saddle  $u = v = 0.4$  and  $u = v = 0.8$ . The way of mathematical modelling presented in this paper forms the basis for computer designer to consider the Monkey Saddle  $u = v = 0.4$  and  $u = v = 0.8$  applied in tensioned fabric structure. This study provides an alternative choice for structural designer to consider for Monkey Saddle applied in tensioned fabric structures.

**Keywords:** Tensioned Fabric Structure, Form-Finding, Nonlinear Analysis Method, Prestress.

## 1. INTRODUCTION

Tensioned Fabric Structure (TFS) is a suitable structure to be applied in large space area. Actually, TFS has been used over the past 50 years ago. Tensioned fabric structures are structures that are composed of tensioned fabric as structural members. Fabric patterns are joined together at seams and are tensioned through mechanical means or cables to rigid supporting system to typically provide a roofing structure as mentioned by [1]. [1] have mentioned that tensioned fabric structures are normally designed to be in the form of equal tensioned surface. Tensioned fabric structures employed membranes that have support geometry resulting in anticlastic membrane curvature, wherein outward loads are resisted by an increase in stress in the hogging warps about one axis of the membrane and inward forces are resisted by an increase in stress in the sagging warps about the other axis. Tensioned fabric structures are considered as form-resistant structures with doubly curved surfaces which must be pretensioned in such a way in order to resist applied environmental loading such as wind and snow. Form-finding using suitably formulated computational method has to be carried out for TFS in order to determine the initial equilibrium shape under prescribed support condition and pre-stress pattern. The principle of nonlinear analysis method is based on the large displacement finite element formulation used for analysis of structural behaviour under external loads. It can be used with suitable strategy proposed by [1] for form-finding purpose.

\* Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pau, Pulau Pinang, Malaysia, Email: [nasayz@yahoo.com](mailto:nasayz@yahoo.com)

\*\* Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pau, Pulau Pinang, Malaysia, Email: [yhoimin@yahoo.com](mailto:yhoimin@yahoo.com)

\*\*\* Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pau, Pulau Pinang, Malaysia, Email: [kaydora@ppinang.uitm.edu.my](mailto:kaydora@ppinang.uitm.edu.my)

\*\*\*\* Faculty of Civil Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia, Email: [norha454@salam.uitm.edu.my](mailto:norha454@salam.uitm.edu.my)

From [2], in 1829, Anatomy Joseph Plateau investigated the physical and geometrical properties of soap film surfaces, which are elastic in the sense that they have smallest possible area. By 1843, a mathematical theory of surfaces that minimizes their area. In the 1930s, Jesse Douglas and Tibor Radó finally showed mathematically that no matter what shape the curve in, there is always a least-area spanning the curve. Further study, including work in the field of Geometric Measure Theory, showed that there is always a least area surface spanning the curve taken from [2]. Most tension structures are designed to have a uniform prestress in their fabrics. In this condition, there is no shear stress in the fabric. This is also the condition which minimizes the fabric surface area for a given set of initial conditions. [3] presented an extension of the force density method called the natural force density method for the initial shape finding of cable and membrane structures, which led to the solution of a system of linear equations. With reference to a Helicoid soap film surface, the minimal surface associated with the prescribed boundary is obtained. [4] presented numerical methods to simulate soap film experiments as well as how they could be integrated among themselves and with structural optimization. [5] presented a study of minimum energy forms of prestressed cable nets and membranes in numerical form-finding and soap film models. Tensioned fabric structures are normally designed to be in the form of equal tensioned surface. Minimal surface such as classical minimal surfaces, Costa and Möbius strip or their variation have been studied as possible choice of surface form for TFS by [6]-[14].

TFS is highly suited to be used for realizing surfaces of complex or new forms. However, none of the new examples mentioned present any results on the Monkey Saddle  $u = v = 0.4$  and  $u = v = 0.8$  as load carrying members. Understanding of the possible Monkey Saddle  $u = v = 0.4$  and  $u = v = 0.8$  initial equilibrium shapes to be obtained will provide alternative shapes for designers to consider.

## 2. GENERATION OF MONKEY SADDLE IN TENSIONED FABRIC STRUCTURE

For this paper, the software [15] has been used for the purpose of model generation. Aspect of modelling of surface of Monkey Saddle and form as well pre-stress pattern of the resulting TFS through form-finding using nonlinear analysis method is studied. Monkey Saddle as shown in Figure 1 can be represented parametrically by the following set of equations [16]:

$$X = u, Y = v, Z = u^3 - 3uv^2 \quad (1)$$

For  $u$  and  $v$ : variables

The convergence criteria of form-finding adopted is that least square error (LSE) of total warp and fill stress deviation should be  $<0.01$  based from [1]. Figure 1 shows different views of the Monkey Saddle.

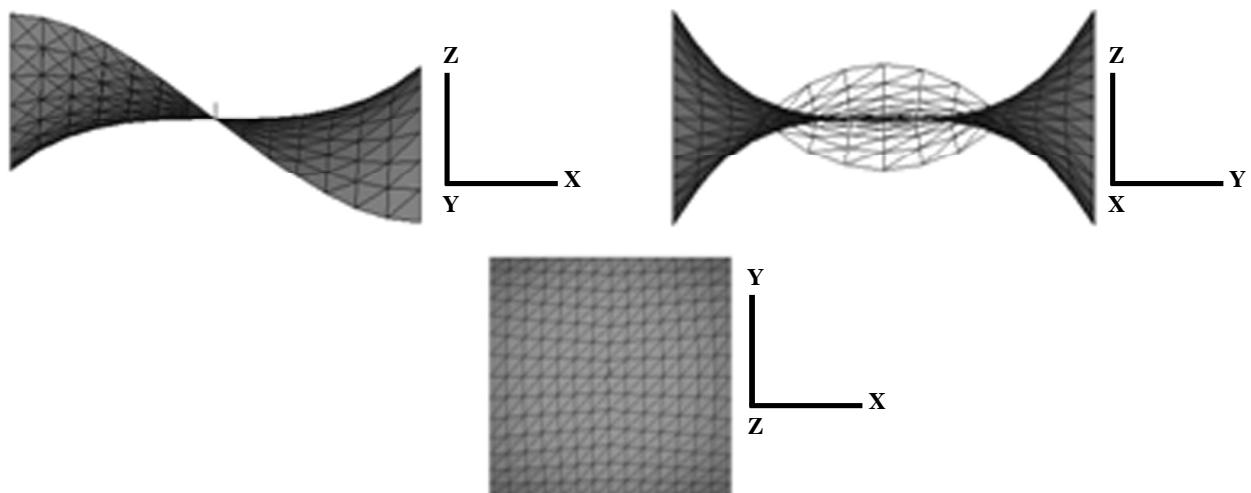


Figure 1: Different views of the Monkey Saddle.

### 3. NUMERICAL METHOD USING NONLINEAR ANALYSIS METHOD

[1] stated the principle of nonlinear analysis method is based on the large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as follows:

$$({}_0^t\mathbf{K}_L + {}_0^t\mathbf{K}_G)\mathbf{u} = {}^{t+\Delta t}\mathbf{F} - {}_0^t\mathbf{f} \quad (2)$$

Where  ${}_0^t\mathbf{K}_L$  is linear strain incremental stiffness matrix,  ${}_0^t\mathbf{K}_G$  is nonlinear strain incremental stiffness matrix,  ${}_0^t\mathbf{f}$  is vector internal forces,  ${}^{t+\Delta t}\mathbf{F}$  is load vector and  $\mathbf{u}$  is vector of increment in displacement.

A nonlinear finite element analysis program by [1] for the analysis of tensioned fabric structures has been used in this study. The procedure adopted is based on the work by [1]. 3-node plane stress element has been used as element to model the surface of TFS. All  $x$ ,  $y$  and  $z$  translation of nodes lying along the boundary edge of the Monkey Saddle have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero, Young's Modulus-Membrane, E.t, is 800000N/m, Shear modulus, Gt is 23529.41176N/m and Poisson's Ratio,  $\nu_{xy} = \nu_{yx} = 0.7$ .

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [1]. First stage (denoted as SF1) is analysis which starts with an initial assumed shape in order to obtain an updated shape for initial equilibrium surface. The initial assumed shape can be obtained from any pre-processing software and reference [1] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, E with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be  $< 0.01$ . The resultant shape at the end of iterative step n (SSn) is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of form-finding procedure adopted in this study, initial assumed shape is needed. For the generation of such initial assumed shape, knowledge of the requirement of anti-clastic nature of TFS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial assumed shape.

#### 3.1. Monkey Saddle, $u = v = 0.4$

The number of nodes and triangular elements are 225 and 392, respectively for Monkey Saddle  $u = v = 0.4$ . Figure 2 shows different views of initial assumed shape for Monkey Saddle  $u = v = 0.4$  and Figure 3 shows the different views and converged shape of the Monkey Saddle  $u = v = 0.4$ . The total warp and fill stress deviation for Monkey Saddle  $u = v = 0.4 < 0.01$ . Figure 4 shows variation of total stress deviation in warp and fill direction versus stress analysis stage for Monkey Saddle  $u = v = 0.4$ .

In this study, initial equilibrium shape of monkey saddle  $u = v = 0.4$  has been carried out. Figure 3 shows initial equilibrium shape of Monkey Saddle  $u = v = 0.4$ . The warp and fill direction for Monkey Saddle of  $u = v = 0.4$  is 0.000399 and 0.007730. Figure 4 shows convergent curve of total warp and fill stress deviation for Monkey Saddle  $u = v = 0.4 < 0.01$ .

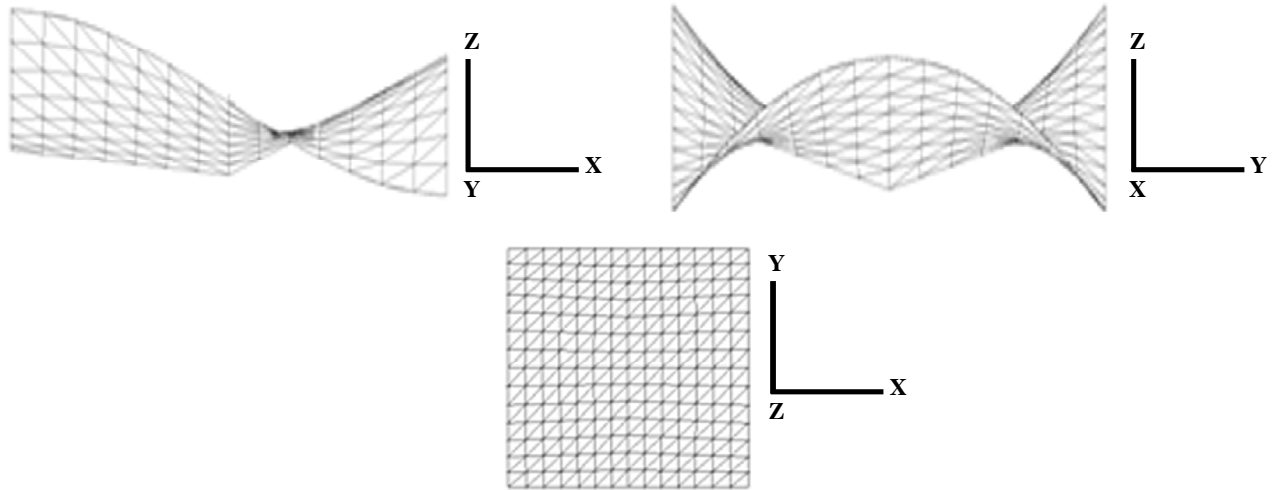


Figure 2: Different views of initial assumed shape for Monkey Saddle,  $u = v = 0.4$ .

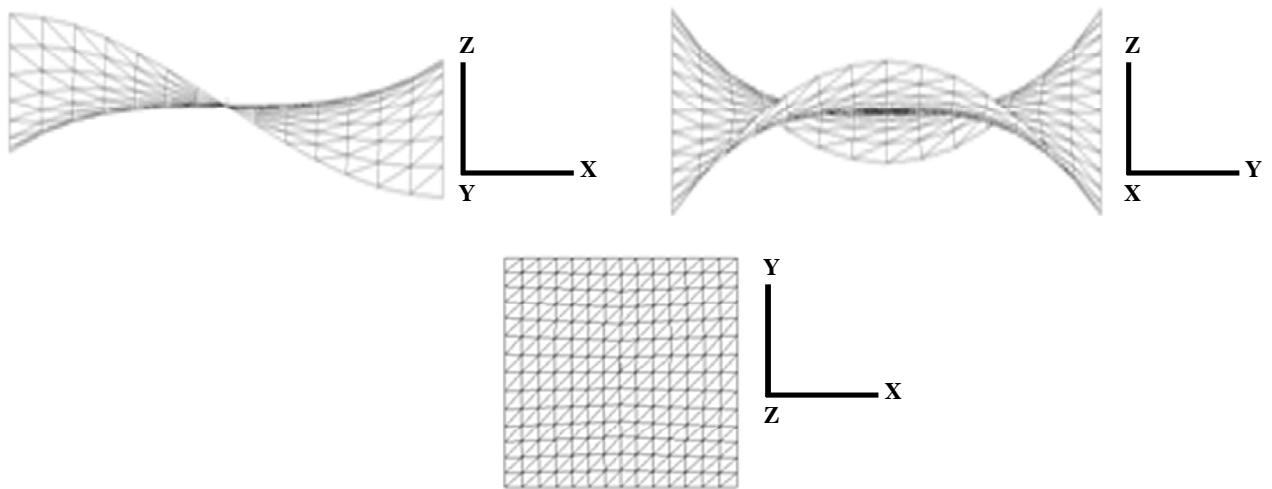


Figure 3: Different views and converged shape of Monkey Saddle,  $u = v = 0.4$ .

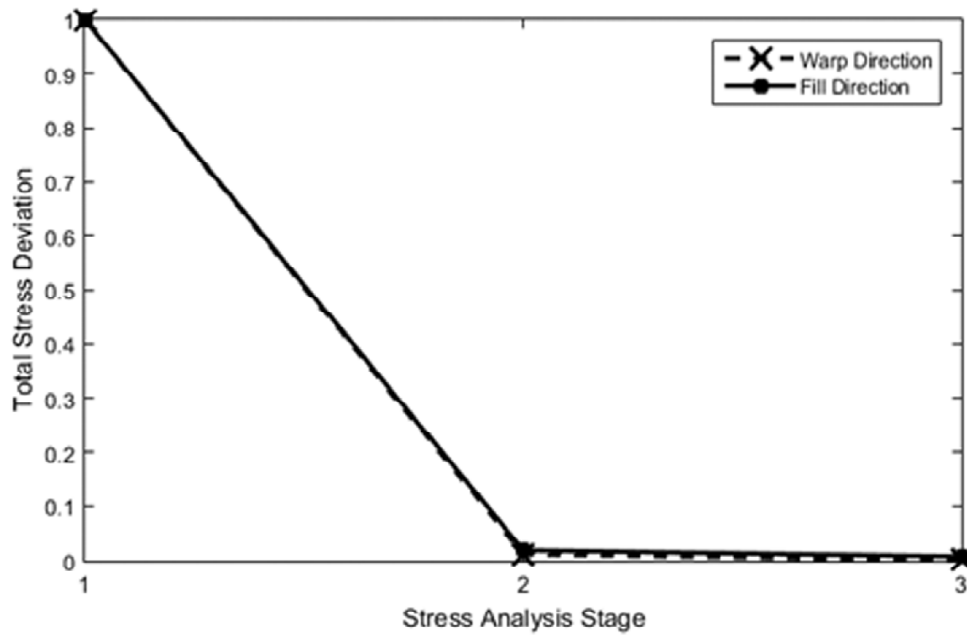


Figure 4: Variation of total stress deviation in warp and fill direction versus stress analysis stage for the Monkey Saddle,  $u = v = 0.4$

### 3.2. Monkey Saddle, $u = v = 0.8$

The number of nodes and triangular elements are 225 and 392, respectively for Monkey Saddle  $u = v = 0.8$ . Figure 5 shows different views of initial assumed shape for Monkey Saddle  $u = v = 0.8$  and Figure 6 shows

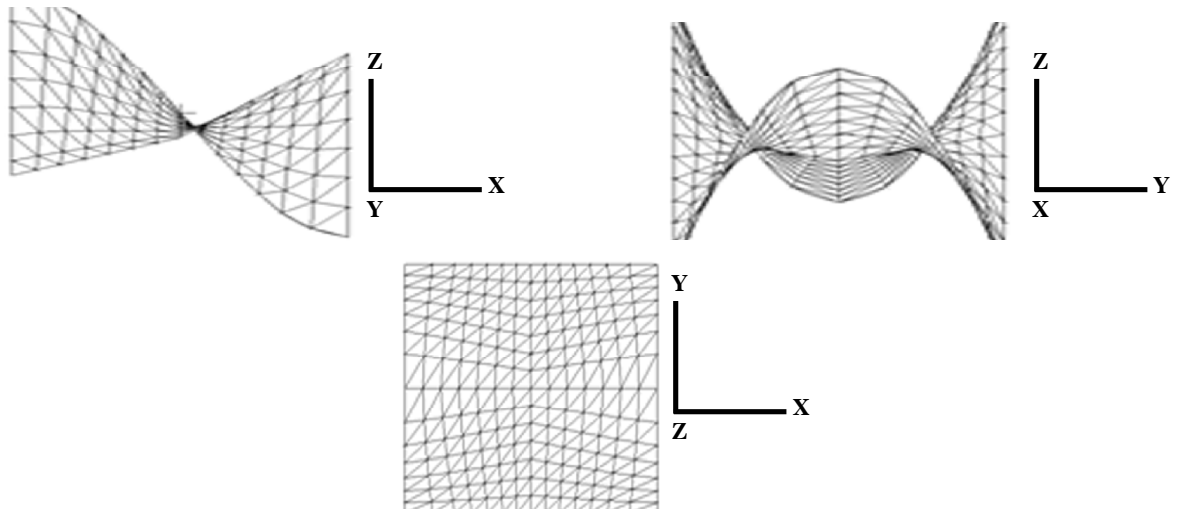


Figure 5: Different views of initial assumed shape for Monkey Saddle,  $u = v = 0.8$ .

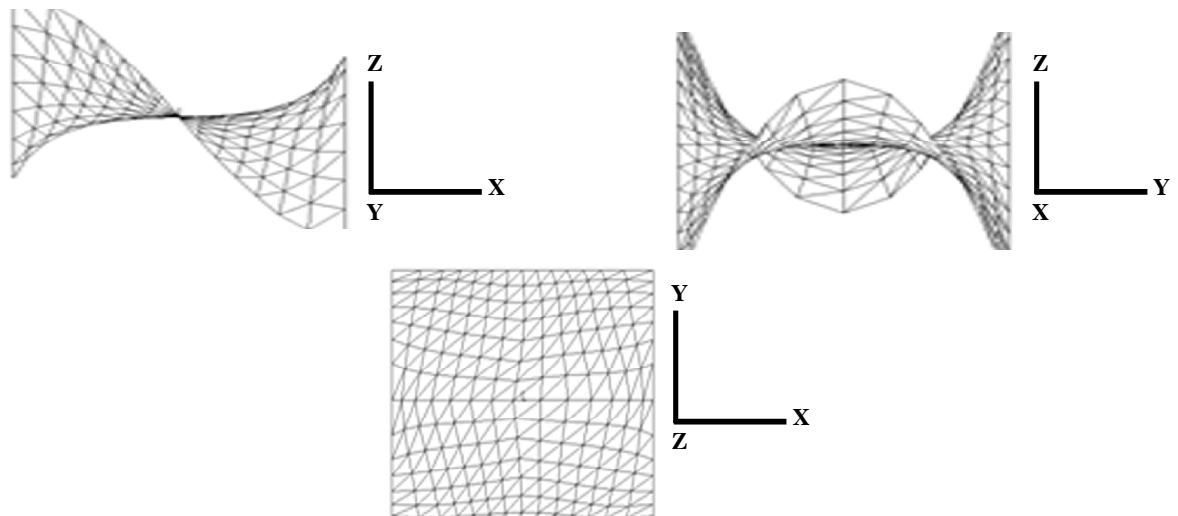


Figure 6: Different views and converged shape for Monkey Saddle,  $u = v = 0.8$ .

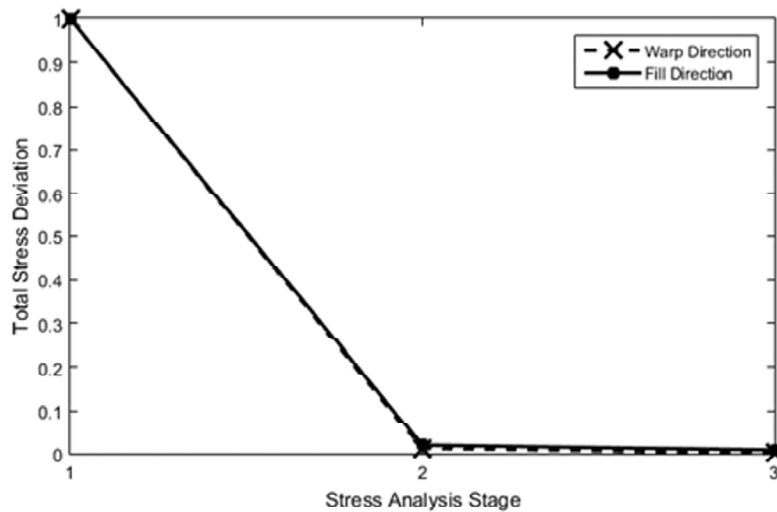


Figure 7: Variation of total stress deviation in warp and fill direction versus stress analysis stage for the Monkey Saddle,  $u = v = 0.8$

the different views and converged shape of the Monkey Saddle  $u = v = 0.8$ . The total warp and fill stress deviation for Monkey Saddle model with  $u = v = 0.8 < 0.01$ . Variation of total stress deviation in warp and fill direction versus stress analysis stage for Monkey Saddle  $u = v = 0.8$  has been show in Figure 7.

In this study, initial equilibrium shape of monkey saddle  $u = v = 0.8$  has been carried out. Figure 6 shows initial equilibrium shape of Monkey Saddle  $u = v = 0.8$ . The warp and fill direction for Monkey Saddle  $u = v = 0.8$  is 0.009461 and 0.005074. Figure 7 shows convergent curve of total warp and fill stress deviation for Monkey Saddle  $u = v = 0.8 < 0.01$ .

#### 4. CONCLUSION

Form-finding with surface in the form of Monkey Saddle with variables  $u = v = 0.4$  and  $u = v = 0.8$  has been carried out successfully using the procedure adopted which is based on nonlinear analysis method proposed by [1]. The Monkey Saddle with variables  $u = v = 0.4$  and  $u = v = 0.8$  obtained in this study will provide an alternative shapes for designers to be considered for adoption in tensioned fabric structures. The results from this study show that tensioned fabric structure in the form of Monkey Saddle is a structurally viable surface form to be considered by engineer.

#### REFERENCES

- [1] Yee, H.M. "A computational strategy for form-finding of tensioned fabric structure using nonlinear analysis method." Ph. D. dissertation, School of Civil Engineering, Universiti Sains Malaysia, Pulau Pinang, Malaysia, 2011.
- [2] Meadows, A. "Soap film geometry-minimal surfaces and isoperimetric problems in Mathematics," 2012. [Online]. Available: <http://faculty.smcm.edu/ammeadows/create/>. [Accessed: 20-June-2016].
- [3] Pauletti, R.M.O. & Pimenta, P.M. "Shape finding of membrane structures by the natural force density method," *Proceedings of the Sixth International Conference on Computation of Shell and Spatial Structures. IASS-IACM*, 2008.
- [4] Bletzinger, K.U., Wüchner, R., Daoud, F., & Camprubí, N."Computational methods for form finding and optimization of shells and membranes," *Computer Methods in Applied Mechanics and Engineering*, volume 194, issue 30-33, pp. 3438-3452, Aug. 2005.
- [5] Lewis, W.J. & Gosling, P.D. "Stable minimal surfaces in form-finding of lightweight tension structures," *International Journal of Space Structure*, volume 8, issue 3, pp. 141-166, 1993.
- [6] Yee, H.M., Choong, K.K., & Kim, J.Y., "Form-Finding Analysis of Tensioned Fabric Structures Using Nonlinear Analysis Method," *Advanced Materials Research*, volume 243-249, pp. 1429-1434, 2011.
- [7] Yee, H.M., "Form-Finding of Tensioned Fabric Structure in the Shape of Möbius Strip," *Iranica Journal of Energy & Environment*, volume 4, issue 3, pp. 251-257, 2013.
- [8] Yee, H.M., Choong, K.K., & Kim, J.Y., "Experimental Form-Finding for Möbius Strip and Ennerper Minimal Surfaces Using Soap Film Models," *International Journal of Engineering Science and Innovative Technology*, volume 2, issue 5, pp. 328-335, 2013.
- [9] Yee, H.M., Kim, J.Y., & Mohd Noor, M.S., "Tensioned Fabric Structures in Oval Form," *Applied Mechanics and Materials*, volume 405-408, pp. 1008-1011, 2013.
- [10] Mohd Noor, M.S., Yee, H.M., Choong, K.K., & Haslinda, A.H., "Tensioned Membrane Structures in the Form of Egg Shape," *Applied Mechanics and Materials*, volume 405-408, pp. 989-992, 2013.
- [11] Yee, H.M., & Samsudin, A., "Development and Investigation of the Moebius Strip in Tensioned Membrane Structures," *WSEAS Transactions on Environment and Development*, volume 10, pp. 145-149, 2014.
- [12] Yee, H.M., & Samsudin, A., "Mathematical and Computational Analysis of Moebius Strip," *International Journal of Mathematics and Computers in Simulation*, volume 8, pp. 197-201, 2014.
- [13] Yee, H.M., Choong, K.K., & Abdul Hadi, M.N. "Sustainable Development of Tensioned Fabric Green Structure in the Form of Enneper," *International Journal of Materials, Mechanics and Manufacturing*, volume 3, issue 2, pp. 125-128, 2015.