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Model Predictive Control design for a Non Linear Multivariable System with a Transmission Zero

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ABSTRACT

Most of the industrial and chemical processes are multivariable in nature. There are always complicated interactions existing between the measurement signals and control signals. Because of the interactions between several input and output variables, it is difficult to design suitable controller for multi input and multi output (MIMO) systems. In many cases, cross-coupling between inputs and outputs is low and hence conventional controllers can be employed. If multivariable systems exhibit stronger cross-coupling between process inputs and outputs, multivariable controllers should be applied in order to achieve satisfactory performance. Model predictive control (MPC) is an important multivariable control technique to control MIMO process which satisfies inequality constraints on the input and output variables. In this study, MPC is accomplished to design multivariable control for MIMO process. A benchmark quadruple tank system (QTS) is considered to illustrate the benefits of the design paradigm. The performance of this multivariable controller is studied for reference tracking and disturbance rejection cases and compared with decoupled multi loop controllers. Simulation results confirm the effectiveness of the proposed system.

Keywords: Multivariable control, PI controller, Quadruple tank process, Model predictive control.

1. INTRODUCTION

In the past decade, a great deal of activity in industry and in academia has focused on the use of process models to develop new types of multivariable controllers, generically called model predictive controller (MPC). Control design methods based on MPC concept have found wide acceptance in industrial applications and have been studied by academia. It is currently the most widely used of all advanced control methodologies in industrial applications [1]-[5] The reason for such popularity is the ability of MPC design to yield high performance control systems capable of operating without expert intervention for long periods of time. MPC offers several important advantages:

- The process model captures the dynamic and static interactions between input, output, and disturbance variables.
- Constraints on inputs and outputs are considered in a systematic manner.
- The control calculations can be coordinated with the calculation of optimum set points.
- Accurate model predictions can provide early warnings of potential problems.

The basic idea of MPC is to use a process model (either linear or nonlinear) to calculate the best changes in the manipulated variables that will achieve a specified desired result in the controlled variables. At each point in time the output variables are measured. Then the optimization procedure calculates the moves in the manipulated variables for several time steps into the future. The first of these changes is made and has a certain effect on the controlled variables. At the next time step the new values of the controlled variables are measured and incorporated into the optimization problem, which is re-solved to obtain new

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manipulated variable values. The success of MPC (or any other model-based approach) depends on the accuracy of the process model [6]-[9].

This paper presents a predictor control design methodology to solve the problem of interactions in quadruple tank process, which is a bench mark multivariable process used in control literature. To control a quadruple tank system, one essential problem is how to handle the interactions among two loops. Since MPC is more suitable for constrained MIMO control problems it is proposed to use this control strategy for QTS in minimum and non minimum operating points.

The concepts behind this study are organized as follows: Section 2 gives the description of quadruple tank process and the mathematical modelling of the system. Section 3 explains the design of model predictive control followed by results and discussions in Section 4. The conclusion is given in Section 5.

2. QUADRUPLE TANK SYSTEM

A quadruple tank apparatus which was proposed in the control literature [10] has been used in chemical engineering laboratories to illustrate the performance limitations for multivariable systems posed by ill-conditioning, right half plane transmission zeros and model uncertainties. The quadruple tank system consists of four interconnected tanks and two pumps. The schematic of the quadruple tank equipment is presented in Figure 1.

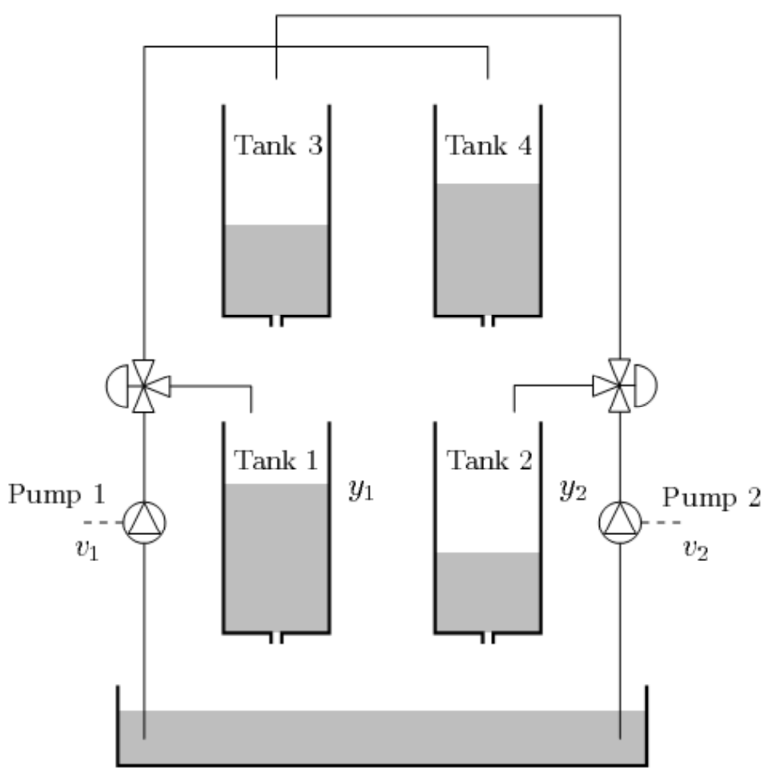


Figure 1: Schematic of Quadruple Tank System

The process inputs are u_1 and u_2 (input voltages to the pumps), and the outputs are y_1 and y_2 (voltages from level measurement devices). The target is to control the level of the lower two tanks with inlet flow rates. The output of each pump is split into two by using a three-way valve. Thus each pump output goes to two tanks, one lower and another upper, diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either in the minimum phase or in the non-minimum phase.

Let the parameter γ be determined by how the valves are set. If γ_1 is the ratio of flow to the first tank, then $(1 - \gamma_1)$ will be the flow to the fourth tank. Similarly if γ_2 is the ratio of flow to the second tank, then $(1 - \gamma_2)$ will be the flow to the third tank. The voltage applied to Pump '*i*' is V_i and the corresponding flow rate is K_iV_i. The parameters $\gamma_1\, \gamma_2 \in [0, 1]$ are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\gamma_1 K_1 V_1$ and the flow to tank 4 is $(1- \gamma_1) K_1 V_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted as '*g*'. The measured level signals are $y_1 = k_c h_1$ and $y_2 = k_c h_2$ [10].

The state equations of the four tank system are given in equations (1) to (4)

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1 \tag{1}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2 \tag{2}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \tag{3}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \tag{4}$$

where A_i is cross sectional area of Tank '*i*'

a_i is cross section of outlet hole of Tank '*i*'

h_i is water level in Tank '*i*'

The linearized state space model is given by equation (5) and (6).

$$\frac{dX}{dt} = \begin{bmatrix} -\frac{1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0 \\ 0 & -\frac{1}{r_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{r_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} X + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} U \tag{5}$$

$$Y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} X \tag{6}$$

The time constants are calculated using equation (7)

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \quad i = 1, 2, 3, 4. \tag{7}$$

The parameter values and steady state operating points of the process are assumed as per the system given in literature [10] [16]. The transfer function matrix is given in equations (8) and (9) for minimum phase and non-minimum phase operating points.

$$G_{-}(s) = \begin{bmatrix} \dfrac{2.6}{1+62s} & \dfrac{1.5}{(1+23s)(1+62s)} \\ \dfrac{1.4}{(1+30s)(1+90s)} & \dfrac{2.5}{1+90s} \end{bmatrix} \quad (8)$$

$$G_{+}(s) = \begin{bmatrix} \dfrac{1.5}{1+63s} & \dfrac{2.5}{(1+39s)(1+63s)} \\ \dfrac{2.5}{(1+56s)(1+91s)} & \dfrac{1.6}{1+91s} \end{bmatrix} \quad (9)$$

The transfer matrix G has two zeros one of them is always in the left half of s-plane, but the other can be located either in left half or right half of s-plane based on the position of three way valves. So, the system is in minimum phase, if the values of γ_1 and γ_2 satisfy the condition $0 < \gamma_1 + \gamma_2 < 1$ and in non-minimum phase, if the values of γ_1 and γ_2 satisfy the condition $1 < \gamma_1 + \gamma_2 < 2$.

3. MODEL PREDICTIVE CONTROL

The only alternative to PID controller for industrial control applications is model predictive controller as reported in control literature [11]-[13]. Because as PID, MPC also uses past, present and future values of the input and output to calculate the control action.

3.1. Model Predictive Control Strategy

The overall objectives of an MPC controller have been summarized as follows

- Prevent violations of input and output constraints.
- Drive some output variables to their optimal set points, while maintaining other outputs within specified ranges
- Prevent excessive movement of the input variables.
- Control as many process variables as possible when a sensor or actuator is not available.

The block diagram given in figure 2 represents the MPC strategy

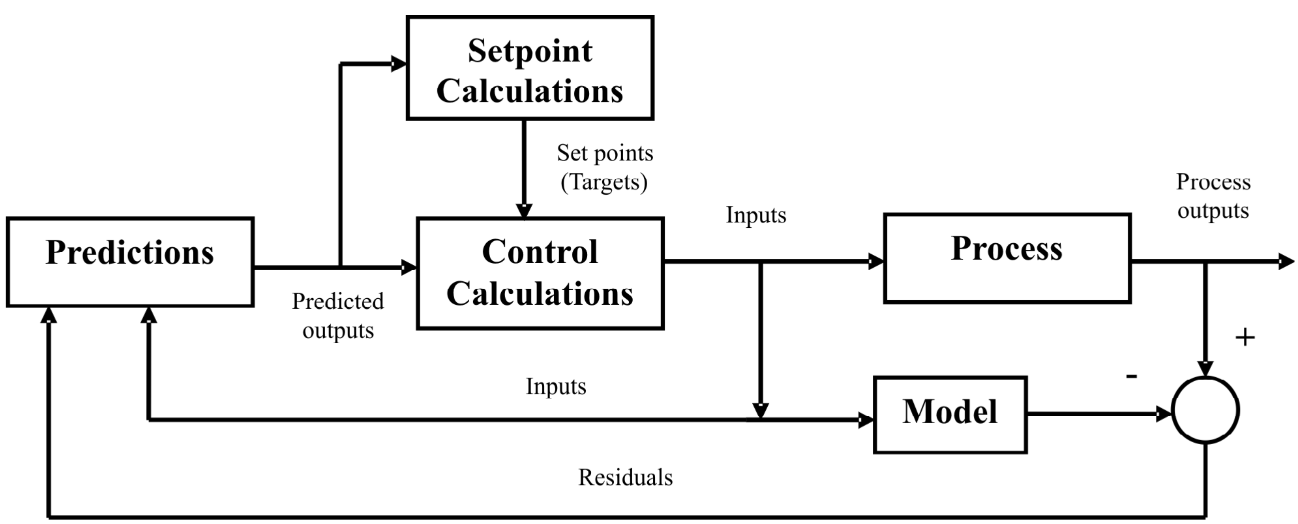


Figure 2: Block Diagram of Model Predictive Control

A process model is used to predict the current values of the output variables. The residuals, the differences between the actual and predicted outputs, serve as the feedback signal to a Prediction block. The predictions are used in two types of MPC calculations that are performed at each sampling instant: set-point calculations and control calculations. Inequality constraints on the input and output variables, such as upper and lower limits, can be included in either type of calculation. The MPC configuration is similar to both the internal model control configuration and the Smith predictor configuration because the model acts in parallel with the process and the residual serves as a feedback signal. However, the coordination of the control and set-point calculations is a unique feature of MPC. Furthermore, MPC has had a much greater impact on industrial practice than IMC or Smith predictor, because it is more suitable for constrained MIMO control problems. [14]

The set points for the control calculations, also called targets, are calculated from an economic optimization based on a steady-state model of the process, traditionally, a linear model. Typical optimization objectives include maximizing a profit function, minimizing a cost function, or maximizing a production rate. The optimum values of set points change frequently due to varying process conditions, especially changes in the inequality constraints. The constraint changes are due to variations in process conditions, equipment, and instrumentation, as well as economic data such as prices and costs. In MPC the set points are typically calculated each time the control calculations are performed.

3.2. Controller Design

The MPC calculations are based on current measurements and predictions of the future values of the outputs. The objective of the MPC control calculations is to determine a sequence of control moves (manipulated input changes) so that the predicted response moves to the set point in an optimal manner. The actual output 'y', predicted output '$\hat{y}$' and manipulated input 'u' for SISO control are shown in figure 3.

At the current sampling instant, denoted by k, the MPC strategy calculates a set of M values of the input $\{u(k + i - 1), i = 1, 2, \ldots\ldots\ldots.. M\}$. The set consists of the current input $u(k)$ and $(M - 1)$ future inputs. The input is held constant after the M control moves. The inputs are calculated so that a set of P predicted outputs $\{\hat{y}(k + i), i = 1,2,\ldots\ldots P\}$ reaches the set point in an optimal manner. The control calculations are based on optimizing an objective Function. The number of predictions P is referred to as the prediction horizon while the number of control moves M is called the control horizon.

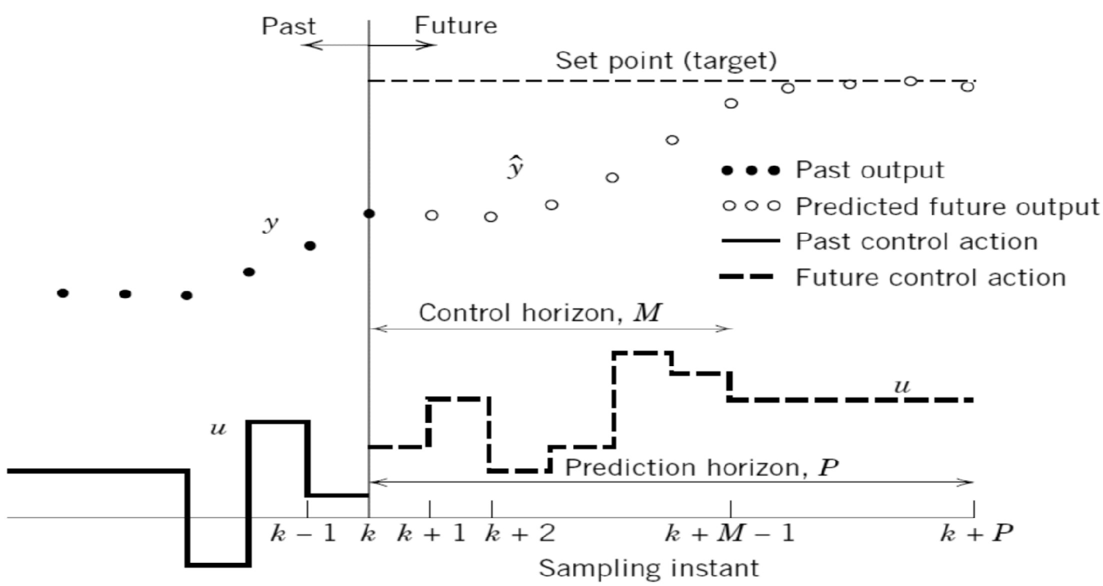


Figure 3: Control Prediction Concept for Model Predictive Control

A distinguishing feature of MPC is its receding horizon approach. Although a sequence of M control moves is calculated at each sampling instant, only the first move is actually implemented. Then a new sequence is calculated at the next sampling instant, after new measurements become available; again only the first input move is implemented. This procedure is repeated at each sampling instant.

3.3. Model Predictive Control Calculations

In each control execution of MPC algorithm, the following steps are followed. The control execution times coincide with the measurement sampling instants. New process data are acquired via the regulatory control system that is interfaced to the process. Then new output predictions are calculated using the process model and the new data. Next, the currently available outputs (CVs), inputs (MVs) and disturbance variables (DVs) for the MPC calculations are determined. Then condition number of the process gain matrix for the current control structure is determined to check for ill conditioning. The optimal operating conditions (set-point calculations) are determined and the process is moved to these set points in an optimal manner based on the control calculations. Using both types of calculations, a specified objective function is optimized while satisfying inequality constraints, such as upper and lower limits on the inputs or outputs. Finally the calculated control actions are implemented. [14]

3.4. Reference Trajectory

In model predictive control, a reference trajectory can be used to make a gradual transition to the desired set point. Let the reference trajectory over the prediction horizon *P* be denoted as

$$Y_r(k+1) \triangleq col\,[y_r(k+1), y_r(k+2), \ldots\ldots\ldots y_r(k+P)] \tag{10}$$

where Y_r is an mP dimensional vector

Equation (11) represents the reference trajectory with filtered set point as

$$Y_{i,r}(k+j) = (\alpha_i)^j Y_{i,r}(k) + [1-(\alpha_i)^j]\, Y_{i,sp}(k) \tag{11}$$

for $i = 1, 2, \ldots \ldots .\, .\, m$ and $j = 1, 2, \ldots \ldots .\, .\, P$

where $y_{i,r}$ is the ith element of y_r, y_{sp} denotes the setpoint and α_i is a filter constant and $0 < \alpha_i < 1$.

3.5. Model Predictive Control Law

The control calculations are based on minimizing the predicted deviations from the reference trajectory. Let k denote the current sampling instant. The *predicted error* vector, $\hat{E}(k+1)$ can be expressed as

$$\hat{E}(k+1) \triangleq Y_r(k+1) - \tilde{Y}(k+1) \tag{12}$$

and the predicted unforced error vector is given by

$$\hat{E}^0(k+1) \triangleq Y_r(k+1) - \tilde{Y}^0(k+1) \tag{13}$$

Where $\tilde{Y}^0(k+1)$, the corrected prediction for the unforced case is given by,

$$\tilde{Y}^0(k+1) \triangleq \hat{Y}^0(k+1) + 1[y(k) - \hat{y}(k)] \tag{14}$$

In MPC it is required to determine the control moves for M time intervals, $\Delta U(k)$ **to** minimize the quadratic objective function J which is given by Equation (15)

$$J = \hat{E}(k+1)^T Q \hat{E}(k+1) + \Delta U(K)^T R \Delta U(K) \tag{15}$$

where $\boldsymbol{Q}$ is a positive-definite weighting matrix and $\boldsymbol{R}$ is a positive semi-definite matrix. Both are usually diagonal matrices with positive diagonal elements.

The MPC control law to minimize the objective function can be calculated analytically and is given by Equation (16)

$$\Delta U(k) = (S^T QS + R)^{-1} S^T Q\hat{E}^0(k+1) \tag{16}$$

Which can be written as

$$\Delta U(k) = K_c \hat{E}^0(k+1) \tag{17}$$

Where $$K_c \triangleq (S^T QS + R)^{-1} S^T Q \tag{18}$$

K_c is a rM x mP matrix

The MPC control law is a multivariable, proportional control law based on the predicted error rather than the conventional control error. It implicitly contains integral control action because u tends to change until the unforced error o becomes zero. Thus, offset is eliminated for set-point changes or sustained disturbances. Moreover the MPC control law calculates a set of M input moves as $\Delta U(k)$, but the first control move alone is actually implemented. In the next sampling instant, new data are acquired and a new set of control moves are calculated. This is repeated for each sampling instant. This strategy is called receding horizon approach. The main advantage of this receding horizon approach is that new information in the form of the most recent measurement $y(k)$ is utilized immediately instead of being ignored for the next M sampling instants [14].

4. RESULTS AND DISCUSSIONS

The multivariable model predictor controller is designed for quadruple tank system in MATLAB/SIMULINK environment. The performance of the system in two operating conditions is tested for control challenges like multivariable interactions, setpoint tracking and measurable and unmeasurable disturbances with input and output constraints [15].

The linearized model is used for designing MPC for QTS and the controller design parameters for MPC are chosen as follows:

Sampling Period (Δt)	: 1 sec
Prediction Horizon (P)	: 10
Control Horizon (M)	: 2
Weighting matrices	: $Q = diag$ [11] and $R_{ii} = 0.1$

The simulation is carried out for minimum and non minimum phase conditions with MPC tool box using the design parameters. Simulation is carried out to analyse the following

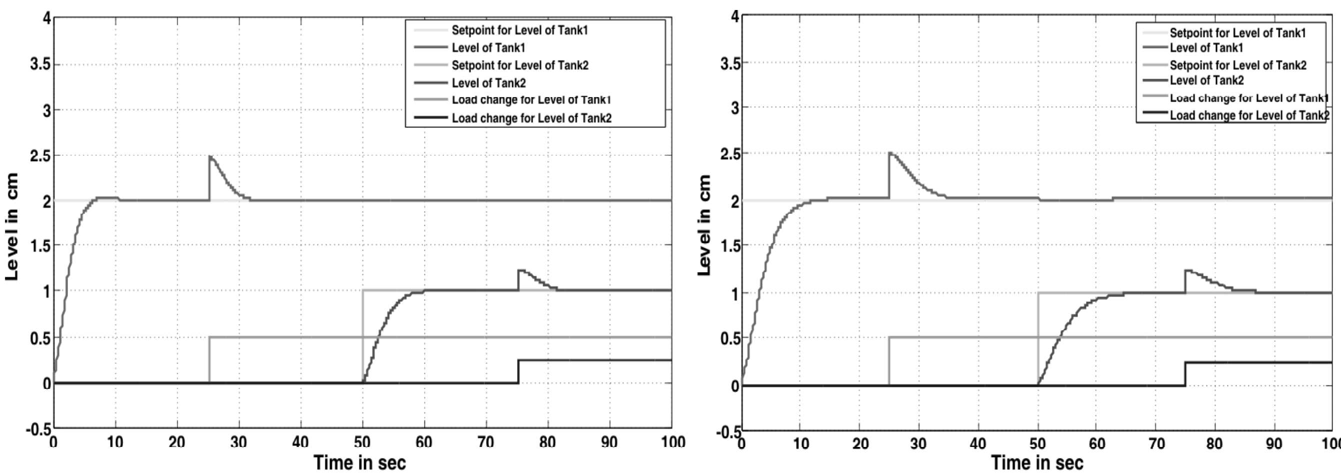
- Effectiveness in setpoint tracking and load disturbance rejection
- Robustness in handling multiloop interactions
- Robustness against disturbances in process output and controller output
- Effects on selection of design parameters (P, M, Q, R) for MPC

The simulation results are given in the following figures. Figure 4 (a) and (b) show the servo and regulatory responses of both the controlled variables in minimum and non minimum phase conditions respectively. From the responses it is clearly shown that the output variables are able to track the set points given, reject load disturbances and free from multivariable interactions.

Figure 5 (a) shows the responses of loop 1 and loop 2 in minimum phase condition with band limited white noise applied to the second output variable. Figure 5 (b) shows the responses of loop 1 and loop 2 in non minimum phase condition with band limited white noise applied to the first output variable. From these two sets of responses, it is observed that the proposed controller provides robustness against unmeasured disturbances in QTS outputs. Moreover the other loop is not getting disturbed by the application of noise.

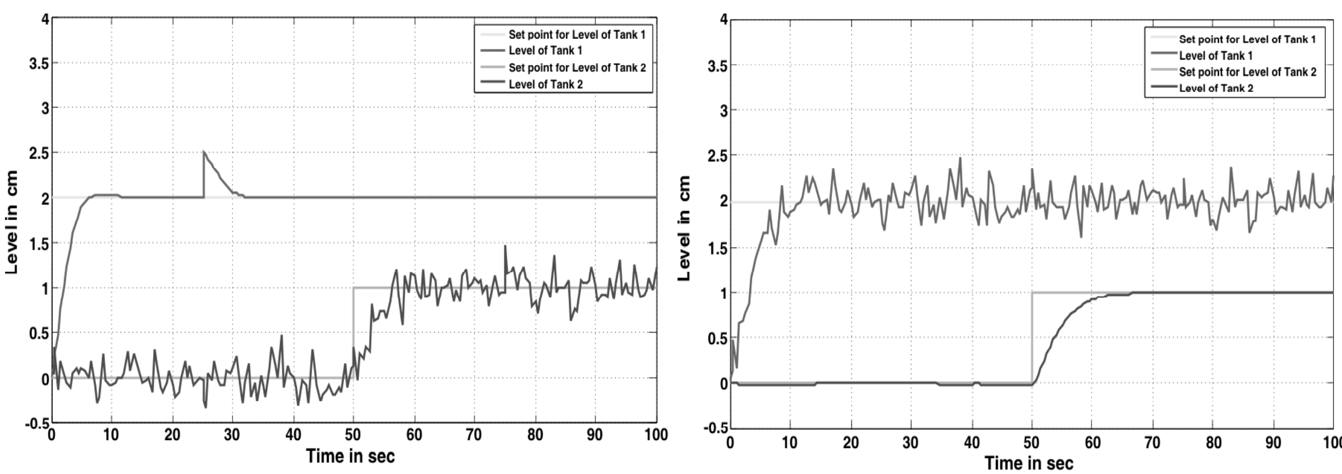
Figure 6 (a) & (b) show the quadruple tank process outputs and process inputs respectively, when the design parameters prediction horizon (P) and Control horizon (M) are low i.e. $P = 10$ and $M = 2$. Figure 7 (a) & (b) show the quadruple tank process outputs and process inputs when the design parameters prediction horizon and Control horizon are high i.e. $P = 90$ and $M = 30$. From the graphs it is found that when these design parameter values are increased, then the controller is more aggressive and the performance is degraded.

Figure 8 (a) & (b) represent the process outputs and process inputs respectively, with the output weighting matrix Q is changed as Q = diag [1 0.1]. The second loop weight is changed from 1 to 0.1. This results in performance degradation in second loop. Similarly the input weighting matrix R is changed from 0.1 to 1 for the first loop. This also reduces the first loop behaviour which is shown in Figure 9 (a) & (b).



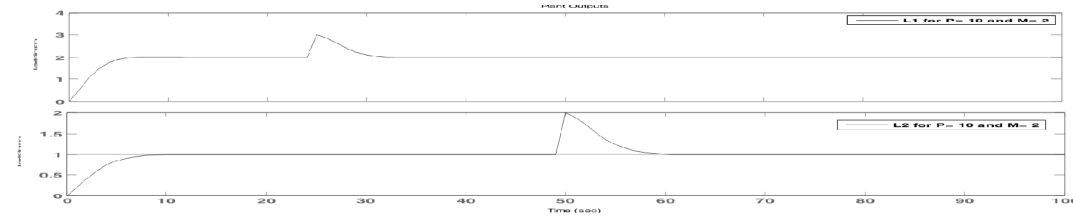
(a) **(b)**

Figure 4: QTS servo and regulatory responses with MPC in both the operating points

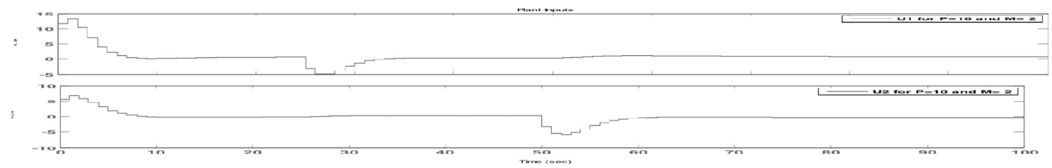


(a) (b)

Figure 5: QTS responses with MPC with noise disturbances in outputs

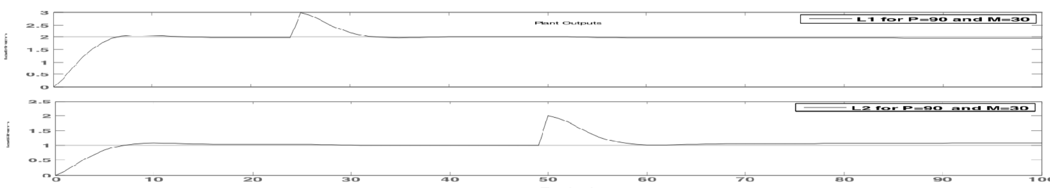


(a)

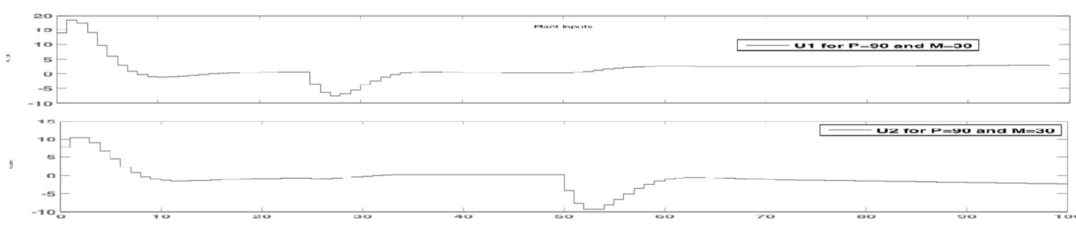


(b)

Figure 6: Process outputs and process inputs for P = 10 and M = 2

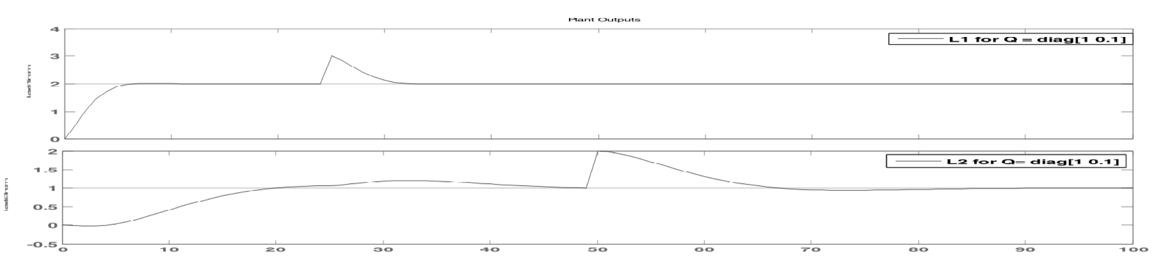


(a)

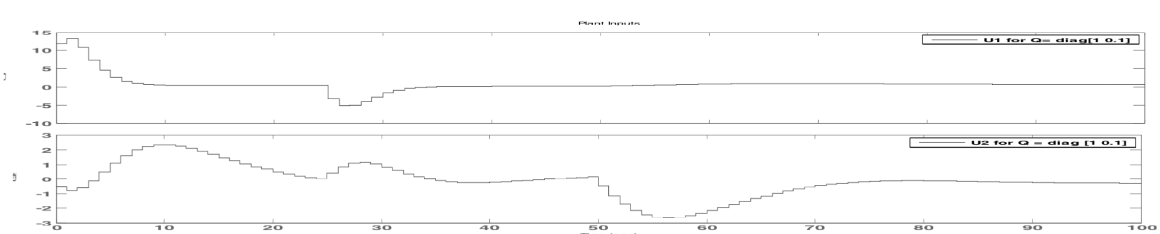


(b)

Figure 7: Process outputs and process inputs for P = 90 and M = 30

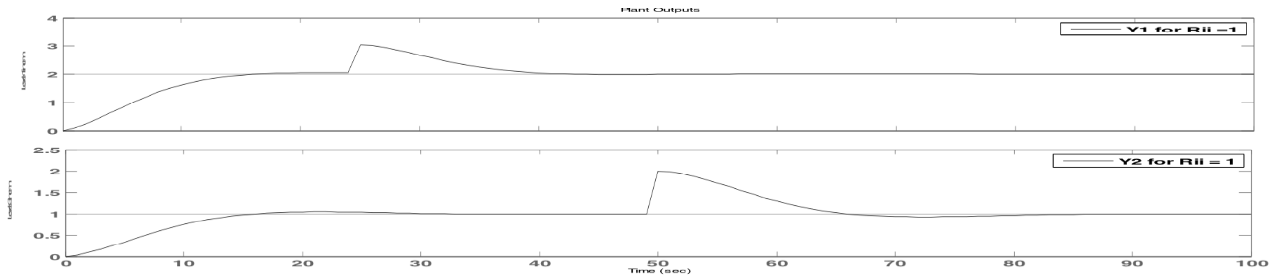


(a)

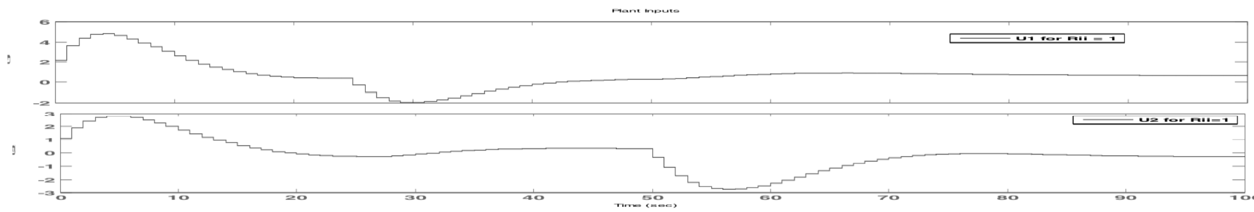


(b)

Figure 8: Process outputs and process inputs for Q = diag[1 0.1]



(a)



(b)

Figure 9: Process outputs and process inputs for R_{ii} = 1

5. CONCLUSION

In this study, the predictive controller using MPC is designed for non linear multivariable quadruple tank system in two different operating points. The levels in the lower tanks are the controlled variables which are maintained using this control system. The MPC based control system is developed and simulated in MATLAB/SIMULINK environment with suitable design parameters. It is observed through simulation results that MPC is a multivariable control which can successfully handle multivariable interactions in minimum phase and non minimum phase systems. This multivariable control system can maintain the controlled variables in the presence of measured and unmeasured disturbances present in controller outputs and process outputs and thus its robustness against non linearity is ensured.

In MPC if the prediction and control horizon values are high then the performance will be humiliated. Similarly the selection of output weighting matrix Q is selected in such a way to provide more weight for the most important variable. If the input weighting matrix R is increased, the MPC inputs become smoother and the output responses have larger deviations and longer settling times. The overall performance of this control system using MPC is best among all other multi loop controllers. MPC based control can be designed with input and output constraints for real time implementation.

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