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Constrained Digital Regional Pole Placer for Vehicle Active Suspension

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ABSTRACT

Thispaper presents a new nonlinear controller design method for regional pole placement of uncertain discrete-time systems. The constraint of control input saturation is considered in the design. A sufficient condition is derived for the robust stabilization and the desireddynamic performance represented by the settling time and damping ratio.The design is formulated in terms of linear matrix inequality (LMI) optimization.

The effectiveness of the proposed design isillustrated by two examples. The first one ishard disk drives, whoseuncertainty is modeled bya norm-bounded form. The resulting controller does not violate the limits, and the second example is on regional pole placement for an uncertain system with and without control saturation. Application to vehicle active suspension to achieve comfortable dynamic performance by pole placement and avoiding actuator saturation is also considered. The results are compared with passive suspension system.

Key words: constrained control, pole placement, linear matrix inequality (LMI) optimization, hard disk drives, vehicle active suspension

1. INTRODUCTION

Manyphysical systems are inherently nonlinear and subject to variation in the operating point. To overcome such difficulties, the system to be controlled isrepresented by an uncertain linear time-invariant model. The uncertainty can be cast into either polytopic or norm-bounded form.The powerful robust control techniques of liner systems can then be applied [1]. The poles of systems without uncertainty can be placed in desired locations so as to achieve good dynamic behavior in terms of settling time and damping ratio [2].However, for systems with uncertainties, the closed-loop poles can be assigned to a domain(D) or region, rather than specific locations [3]. This is termed D-stability or regional pole placement by using robust controllers against system uncertainty. Regional pole placement for continuous-time systems with polytopic uncertainty using state feedback is presented in [4], while output feedback is presented in [5]. In addition to plant uncertainty, another source of uncertainty is the controller itself, termed resilient control, which is tackled in [6]. More improvement in robust pole placement by adding a guaranteed cost constraint and fault-tolerant control is found in[7].

In many practical control problems, the actuator has limited output, called saturated (or constrained) control [8]. Combining robust pole placement with saturated control is tackled in[9]. Another example is the computer hard disk drive (HDD) servo system, which has major nonlinearities of friction and actuator saturation. When the actuator saturation is not considered in the design phase, the performance of the designed control system seriously deteriorates, as shown in [10] and the references therein.

The main contribution of this paper is the development of constrained robust controllers, which achieve pole placement in a desired region in the complex plane. Consequently, a good dynamic performance is

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attained in the presence of system uncertainty. The practical limitation of the control signal is taken into consideration in the design phase as well. A sufficient condition of the LMI frame is derived to carry out the controller design. The system uncertainty is represented in the norm-bounded form. The model is then used for the design of a discrete-time saturated robust pole placer in MATLAB simulation and applied to a hard drive system.Note that the proposed method solves the regional poleplacement problem in an entirely different and simpler approach than that given in [9]. To the best of the authors' knowledge, no research work is done for regional pole placement for discrete-time systems with state feedback control.

The ultimate goal of this paper is to present anew method to design saturating controllers for uncertain systems. In this method, the control signal is allowed to saturate while guaranteeing asymptotic stability to a bounded region which, obtained by the solution of a set of LMIs, is ellipsoidal and symmetric. The main challenge in this approach is to obtain wide range domain of initial states that ensures asymptotic stability for the system although the presence of saturations. To get around this problem, linearization of the nonlinear saturation function is presented. The purpose of the present paper is to design a saturated state feedback controller for uncertain system with pole region constraints. The proposed controller is robust against system uncertainties in both the state and input matrices. Using the LMI method, a convex optimization problem is formulated to find the controller matrix. Furthermore, the designed saturated controller is applied to three examples. The first example is an uncertain hard disk drive system and the proposed controller does not violate the design constraint. The second example shows the control limit violation and how it can be avoided using the proposed design. The third example is avehicle active suspension system in which a saturated controller is designed while achieving the best possible ride comfort via pole placement.

The paper organization is as follows: The problem formulation is presented in section 2. In section 3, the design of a saturated robust regional pole placer is developed for uncertain discrete-time systems. Section 4 is devoted to numerical simulation to demonstrate the effectiveness of the proposed design. Finally, the conclusion is presented in section 5.

Notations: Capital, small, and Greek letters denote matrices, vectors, and scalars, respectively. I denotes the identity matrix. W', W^{-1} denote the transpose and the inverse of any square matrix W, respectively. $W > 0$ ($W < 0$) denote a symmetric positive (negative)-definite matrix W. The symbol • is as an ellipsis for terms in matrix expressions that are induced by symmetry. For example,

$$\begin{bmatrix} L+(W+N+W'+N') & N \\ N' & M \end{bmatrix} = \begin{bmatrix} L+(W+N+\bullet) & N \\ \bullet & M \end{bmatrix}$$

2. PRELIMINARIES AND PROBLEM FORMULATION

The following facts [1] are used in the sequel:

Fact 1 (congruence transformation): The definiteness of a matrix W does not change under the congruence transformation $H'WH$.

Fact 2 For any real matrices W_1, W_2 and $\Delta\Box(k)$ with appropriate dimensions and $\Delta'\Delta \leq I$, $\leftrightarrow \|\Delta\| \leq 1$, it follows that

$$W_1 \, \Delta \, W_2 + W_2' \, \Delta' \, W_1' \leq \varepsilon^{-1} \, W_1 \, W_1' + \varepsilon \, W_2' \, W_2, \; \varepsilon > 0$$

where $\Delta(k)$ represents system uncertainty in the norm-bounded form. The use of this lemma is to eliminate the uncertainty.

Fact 3 (Schur complement): This fact is useful in transforming a nonlinear matrix inequality into a linear one.

For constant matrices W_1, W_2, and W_3, where $W'_1 = W_1$ and $0 < W_2 = W'_2$, it follows that

$$W_1 + W'_3 W_2^{-1} W_3 < 0 \leftrightarrow \begin{bmatrix} W_1 & W_3' \\ W_3 & -W_2 \end{bmatrix} < 0$$

Consider the discrete-time uncertain system

$$x_{k+1} = (A + \Delta A)x_k + (B + \Delta B).sat(u_k) \tag{1}$$

where x_k, u_k are the state and control vectors of dimension n, m, respectively. The pair (A, B) is assumed to be controllable. ΔA, ΔB are time-varying matrices, which represent parametric uncertainty, unmodelled dynamics, and/or nonlinearities, assumed to be norm-bounded as follows:

$$[\Delta A, \Delta B] == M\Delta_k [N, N_b] \tag{2}$$

$$\Delta_k' \Delta_k \leq I \leftrightarrow \|\Delta_k\| \leq 1 \tag{3}$$

where M and N are known constant matrices with appropriate dimensions. Δ_k is an unknown matrix with Lebesgue measurable elements, and I is the identity matrix.

The control signal is constrained due to practical limitation. The saturated controller to develop is assumed to be state feedback and symmetric and normalized as follows:

$$u = F\ x, \quad -1 < u_j < +1, \quad j = 1 \ldots m$$

or

$$sat(u_j) = \begin{cases} 1 & if\ u_j \geq 1 \\ u_j & if\ -1 < u_j < 1 \\ -1 & if\ u_j \leq -1 \end{cases}$$

$$u_k = Fx_k \tag{4}$$

Note that if the control limits are different from ±1, it can always be normalized and cast in the form (4) as shown in [8]. The saturation control system is shown in Fig. 1.

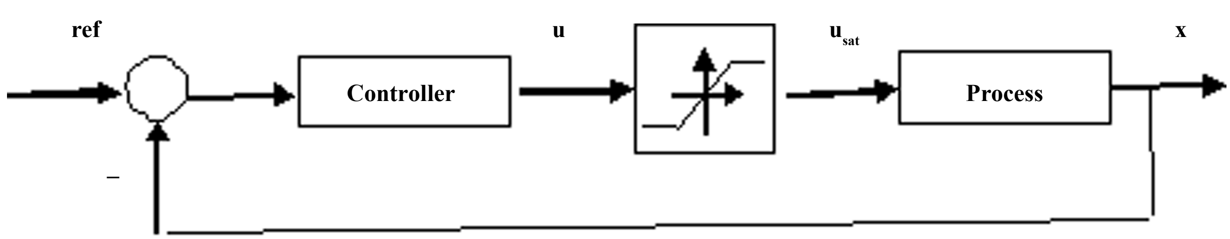


Figure 1: Feedback-saturated control systems

The closed-loop system is given by

$$x_{k+1} = \{A + \Delta A + (B + \Delta B)F\}x_k \tag{5}$$

The problem is to develop a saturated controller, which robustly stabilizes the closed-loop system (5) and ensures a good dynamic performance, described by maximum settling time and minimum damping ratio despite the system uncertainty. To achieve both constraints of max settling time and min damping ratio, the closed-loop poles mustlie in the hatched area, asshown in Fig. 2. This is termed D-stability in which the poles must lie inside the region D for all admissible uncertainties.

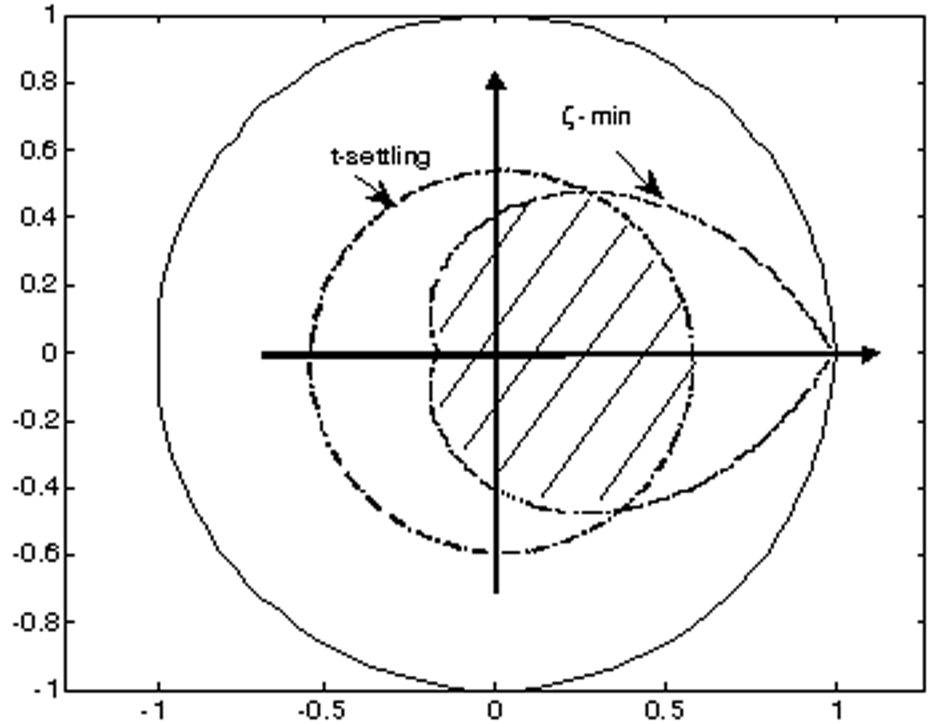


Figure 2: Desired region of closed-loop poles, hatched

3. PROBLEM SOLUTION

To design the abovementioned controller, the nonlinear saturated control function is first linearized in the equivalent form using the following two lemmas [8]:

Lemma-1: For all $u \in R^m$ and $\theta \in R^m$ such that $|\theta_j| < 1, j \in [1, m]$,

$$sat(u) \in co\{\, D_i u + D_i^- \theta,\ i \in [1, \eta]\,\}, \tag{6}$$

with *co* denoting the convex hull.

Equation (6) has the following equivalent form:

$$sat(u) = \sum_{i=1}^{\eta} \gamma_i\ [\, D_i u + D_i^- \theta\,],\ \gamma_i \ge 0$$

Here, D_i is an $m \times m$ diagonal matrix with elements either 1 or 0 and $D_i^- = (I - D_i)$, which results in $\eta = 2^m$ possible matrices. The matrices D_i and D_i^- are introduced to model the saturation function as a linear one. If D_i is selected as I, D_i^- becomes 0, and the resulting controller will be unsaturated. Recall that these controllers (6)work in a linear region and do not allow saturation to occur.

The following sets are defined:

$$D(F) = \left\{\, x \in R^n :\ -1 \le\ Fx\ \le 1 \,\right\} \text{ and } \varepsilon(P,\rho) = \left\{\, x \in R^n : x^T P x\ \le \rho\ ;\ \rho > 0 \,\right\}$$

where P is a symmetric positive-definite matrix. The sets $D(F)$ and $\varepsilon(P, \rho)$ represent, respectively, a symmetrical polyhedral and an ellipsoidal one. The following result is recalled:

Lemma-2: For a given positive scalar ρ, if there exist matrices $Y \in R^{mxn}$ and $Z \in R^{mxn}$ and a positive definite matrix $X = X' \in R^{nxn}$ andsolutions to the following LMIs:

$$[AX + B(D_i Y + D^-_i Z)] + \bullet < 0, \tag{7}$$

$$\begin{bmatrix} 1/\rho & Z_j \\ \bullet & X \end{bmatrix} > 0 \tag{8}$$

$i = 1, \ldots, \eta$; $j = 1, \ldots, m$, then when $\Delta A = 0$ and $\Delta B = 0$, the closed-loop saturated control system is asymptotically stable at the origin $\forall\ x_0 \in \varepsilon(P, \rho)$ with

$$F = YX^{-1}, \tag{9}$$

$$H = ZX^{-1}, \tag{10}$$

$$P = X^{-1}. \tag{11}$$

It is worth mentioning that LMI (7) guarantees asymptotic stability, while LMI (8) ensures that the ellipsoidal set ε (P, ρ) is contained inside the polyhedral set $D(H)$, allowing the control to be saturated. As a special case, by selecting $D_i = I$, the control works only in a linear region without reaching saturation. In this case, $D_i^- = 0$ in LMI (7), and Z_i is replacedby Y_i in LMI (8) to have $\varepsilon(P, \rho) \subset D(F)$.

Our control targetis to design asaturated controllerthat maintains asymptotic stability against system uncertainty, as well as to place the closed-loop poles in a desired region D-stability, if possible, so as to achieve a good dynamic responsein terms of settling time t_s and damping ratio ζ.

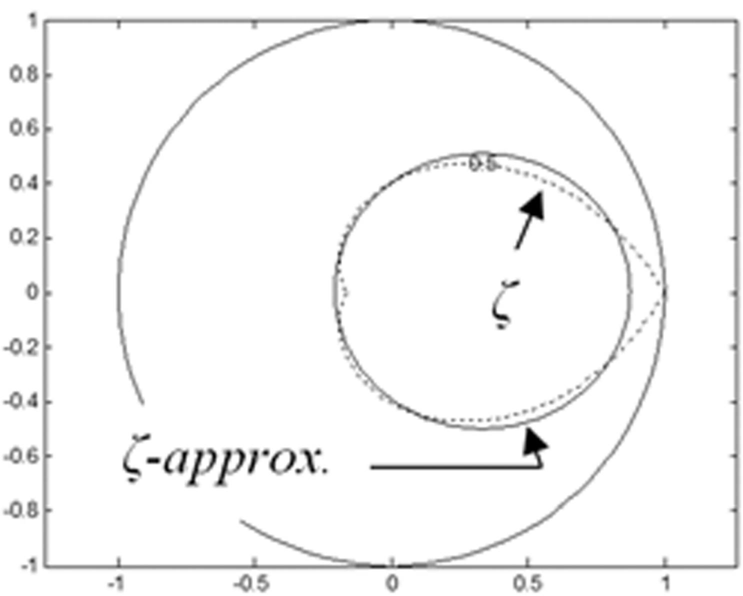
Pole placement in the region shown in Fig. 2 is difficult to solve.However, the problem can be easily solved by approximating the spiral of the constant damping ratio as a circle, as shown in Fig. 3. The problem is thus reduced to placing the poles in between the two circles, one for t_s and the other for ς_{approx}.

We will consider two design cases: without and with saturated inputs.

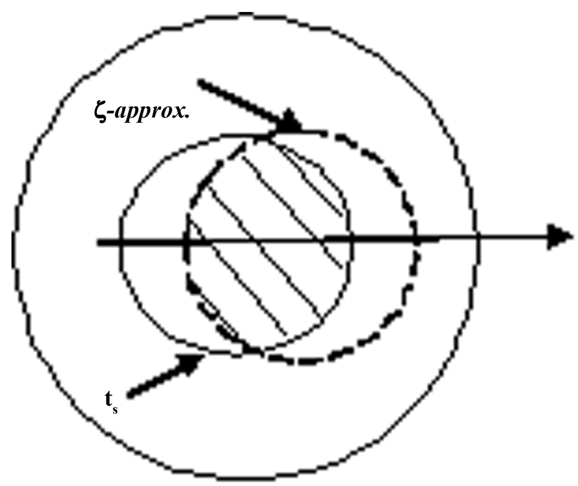
Design case 1: Unsaturated control

Consider the uncertain system with unsaturated control

$$x_{k+1} = (A + \Delta A)x_k + (B + \Delta B).\ u_k$$



(a)



(b)

Figure 3: (a) ζ circleapproximation and (b) desired region of poles, hatched

Theorem 1: If there is a feasible solution to the following LMIs, then the closed-loop poles lie inside the two circles of center q_1 and radius r_1, and center q_2 and radius r_2.

$$X = X' > 0, \varepsilon_1 > 0, \varepsilon_2 > 0, \nu_1 > 0, \nu_2 > 0,$$
$$\begin{bmatrix} -r_1^2 X & \bullet & \bullet & \bullet \\ AX + BY + q_1 X & -X + (\varepsilon_1 + \nu_1) MM' & \bullet & \bullet \\ NX & 0 & -\varepsilon_1 I & \bullet \\ N_b X & 0 & 0 & -\nu_1 I \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} -r_2^2 X & \bullet & \bullet & \bullet \\ AX + BY + q_2 X & -X + (\varepsilon_2 + \nu_2) MM' & \bullet & \bullet \\ NX & 0 & -\varepsilon_2 I & \bullet \\ N_b X & 0 & 0 & -\nu_2 I \end{bmatrix} < 0$$

Moreover, the controller is given by

$$F = YX^{-1}$$

Proof: It is well-known [3] that the eigenvalues ofmatrix$A$ lie inside a circle of center -q and radius r if and only if

$$P = P' > 0, \begin{bmatrix} -r^2 P & \bullet \\ A + qI & -P^{-1} \end{bmatrix} < 0 \tag{13}$$

Or equivalently,

$$P = P' > 0,$$
$$\begin{bmatrix} -r^2 P & \bullet \\ A + BF + qI & -P^{-1} \end{bmatrix} + \left(\begin{bmatrix} 0 \\ M \end{bmatrix} \Delta(t) \begin{bmatrix} N' \\ 0 \end{bmatrix}' + \bullet \right) + \left(\begin{bmatrix} 0 \\ M \end{bmatrix} \Delta(t) \begin{bmatrix} F'N_b' \\ 0 \end{bmatrix}' + \bullet \right) < 0$$

The last matrix inequality is satisfied if

$$P = P' > 0,$$
$$\begin{bmatrix} -r^2 P & \bullet \\ A + BF + qI & -P^{-1} \end{bmatrix} + \left(\varepsilon \begin{bmatrix} 0 \\ M \end{bmatrix} \begin{bmatrix} 0 \\ M \end{bmatrix}' + \varepsilon^{-1} \begin{bmatrix} N' \\ 0 \end{bmatrix} \begin{bmatrix} N' \\ 0 \end{bmatrix}' \right) + \left(\nu \begin{bmatrix} 0 \\ M \end{bmatrix} \begin{bmatrix} 0 \\ M \end{bmatrix}' + \nu^{-1} \begin{bmatrix} F'N_b' \\ 0 \end{bmatrix} \begin{bmatrix} F'N_b' \\ 0 \end{bmatrix}' \right) < 0$$

is satisfied, or

$$\begin{bmatrix} -r^2 P & \bullet & \bullet & \bullet \\ A + BF + qI & -P^{-1} + (\varepsilon + \nu) MM' & \bullet & \bullet \\ N & 0 & -\varepsilon I & \bullet \\ N_b F & 0 & 0 & -\nu I \end{bmatrix} < 0$$

The last matrix inequality can be linearized by pre-and postmultiplying by [P^{-1}, I, I, I], that is, applying fact 1, and letting$P^{-1} = X$, $FX = Y$.

Note that the above condition is only a sufficient condition for regional pole placement in one circle. Since we have to achieve pole placement in the area between the two circles, one for t_s and the other for ζ, we have theorem 1.

Design case 2: Saturated control

Consider the uncertain system with saturated control

$$x_{k+1} = (A+\Delta A)x_k + (B+\Delta B).\, sat(u_k)$$

Theorem 2: The regional pole placement with robust saturated state feedback control (4) for the uncertain system (1) can be achieved if there exist $X = X' > 0,\ \varepsilon_1 > 0,\ \varepsilon_2 > 0,\ \nu_1 > 0,\ \nu_2 > 0,\ Y$, and a feasible solution to the following LMIs:

$$\begin{bmatrix} -r_1^2 X & \bullet & \bullet & \bullet \\ AX + B(D_i Y + \bar{D}_i Z) + q_1 X & -X + (\varepsilon_1 + \nu_1)MM' & \bullet & \bullet \\ NX & 0 & -\varepsilon_1 I & \bullet \\ N_b(D_i Y + \bar{D}_i HX) & 0 & 0 & -\nu_1 I \end{bmatrix} < 0 \qquad (14)$$

$$\begin{bmatrix} -r_2^2 X & \bullet & \bullet & \bullet \\ AX + B(D_i Y + \bar{D}_i Z) + q_2 X & -X + (\varepsilon_2 + \nu_2)MM' & \bullet & \bullet \\ NX & 0 & -\varepsilon_2 I & \bullet \\ N_b(D_i Y + \bar{D}_i HX) & 0 & 0 & -\nu_2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} 1/\rho & Z_j \\ \bullet & X \end{bmatrix} > 0$$

for $i = 1, \ldots, \eta$; $j = 1, \ldots, m$.

Moreover, the saturated robust pole placer is given by

$$F = YX^{-1}$$

Proof: From (13), the poles of the closed-loop uncertain system (1) lie inside the circle of center $(-q, 0)$ and radius r if and only if there is a feasible solution to the following LMI:

$$P = P' > 0$$
$$\begin{bmatrix} -r^2 P & \bullet \\ A + \Delta A + (B + \Delta B)(D_i F + D_i^- H) + qI & -P^{-1} \end{bmatrix} < 0 \qquad (15)$$

Equation (15) is satisfied if the following inequality is satisfied:

$$P = P' > 0$$
$$\begin{bmatrix} -r^2 P & \bullet \\ A + B(D_i F + D_i^- H) + qI & -P^{-1} \end{bmatrix} + \varepsilon \begin{bmatrix} 0 \\ M \end{bmatrix} \begin{bmatrix} 0 \\ M \end{bmatrix}' + \varepsilon^{-1} \begin{bmatrix} N' \\ 0 \end{bmatrix} \begin{bmatrix} N & 0 \end{bmatrix} +$$
$$\nu \begin{bmatrix} 0 \\ M \end{bmatrix} \begin{bmatrix} 0 \\ M \end{bmatrix}' + \nu^{-1} \begin{bmatrix} (D_i F + \bar{D}_i H)' N_b' \\ 0 \end{bmatrix} \begin{bmatrix} N_b(D_i F + \bar{D}_i H) & 0 \end{bmatrix} < 0$$

or equivalently,

$$\begin{bmatrix} -r^2P & \bullet & \bullet & \bullet \\ A+B(D_iF+\bar{D}_i H)+qI & -P^{-1}+(\varepsilon+\nu)MM' & \bullet & \bullet \\ N & 0 & -\varepsilon I & \bullet \\ N_b(D_iF+\bar{D}_iH) & 0 & 0 & -\nu I \end{bmatrix} < 0 \qquad (16)$$

By post-and premultiplying (16) by P^{-1}, that is, by applying the congruence transformation, and substituting $P^{-1} = X$, one gets (14). This completes the proof.

4. SIMULATION RESULTS

The effectiveness of the proposed controller is illustrated by the following two examples:

Example 1: Hard disk drive (HDD)

The computer's HDDis used to store information efficiently on its tracks.Consider the basic diagram of a disk drive, as shown in Fig.4.Information is read as the HDD rotates. The control objective of the disk drive reader is to position the reader accurately on the desired track and to move from one track to another within 4 ms, if possible. For the typical parameters given in [2], the disk drivesystem can be approximated as

$$G(s) = \frac{5}{s(s+20)} \qquad (17)$$

To cope with the approximations and the neglected dynamics in the nominal model (17), we consider the parameters of the numerators and denominators as time-varying over the intervals [4.5→5.5], [16→24]. We select the two state variables as $x_1(t)$ = the position y(t) and $x_2 = dx_1/dt$.Discretizing the system at sampling time T = 1 ms withthe zero-order hold method, we get the norm-bounded uncertain system.

$$A=\begin{bmatrix} 1 & 0.0009901 \\ 0 & 0.9802 \end{bmatrix}, B=\begin{bmatrix} 2.483e-6 \\ 0.00495 \end{bmatrix}, M=\begin{bmatrix} 0 \\ -0.0742 \end{bmatrix}, N'=\begin{bmatrix} 0 \\ 0.0524 \end{bmatrix}, N_b=-0.0065$$

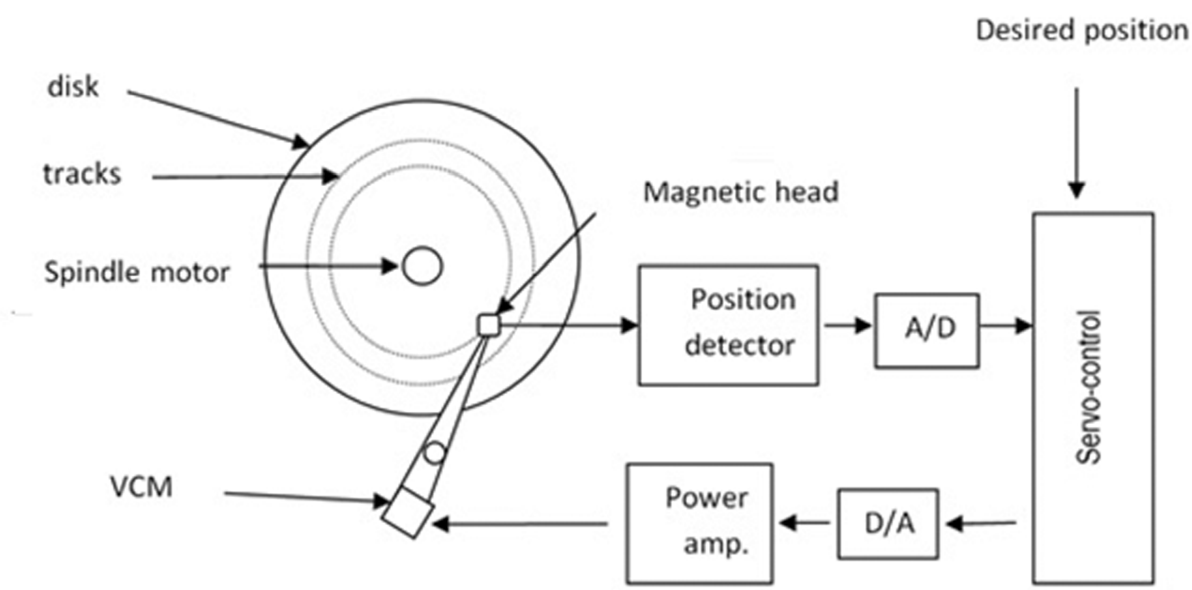


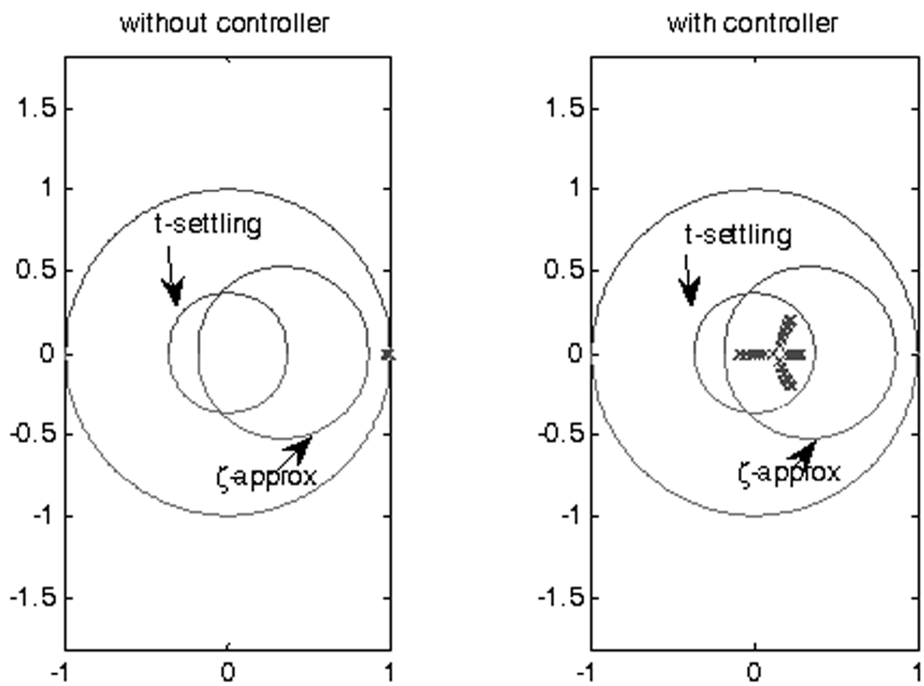
Figure 4: Closed-loop control of HDD

The control objective is to design a controller to achieve a settling time t_s of about 4 ms and a damping ratio of ζ>0.5. To achieve the first objective, we select $r_1 = 0.37$; $q_1 = 0$ for the first circle, and $r_2 = 0.53$; $q_2 = -0.34$ for the second circle.

The obtained controller using theorem 1 is

$$F= [-144070\ -260].$$

The open-loop and closed-loop poles with admissible uncertainties are shown in Fig. 5. Although the uncontrolled system is unstable, the proposed controller succeeds in allocatingthe poles in the desired region, between the two circles.



(a) Open-loop poles **(b) Closed-loop poles**

Figure 5: Poles of the disk drive uncertain system

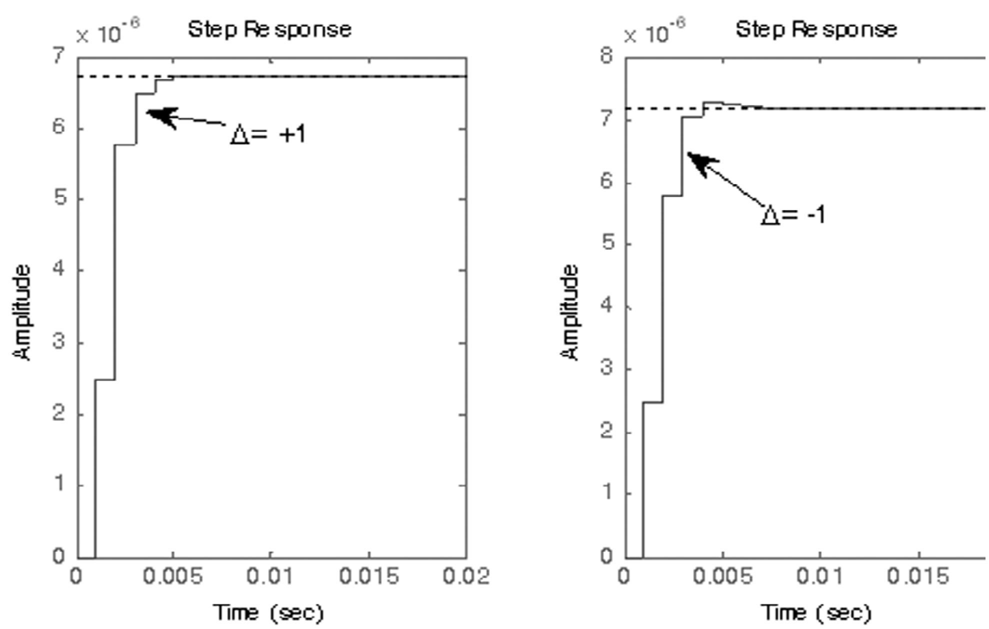


Figure 6: Step response of the disk drive head position reader $x_1$at extreme uncertainties, $\Delta(k)=\pm 1$

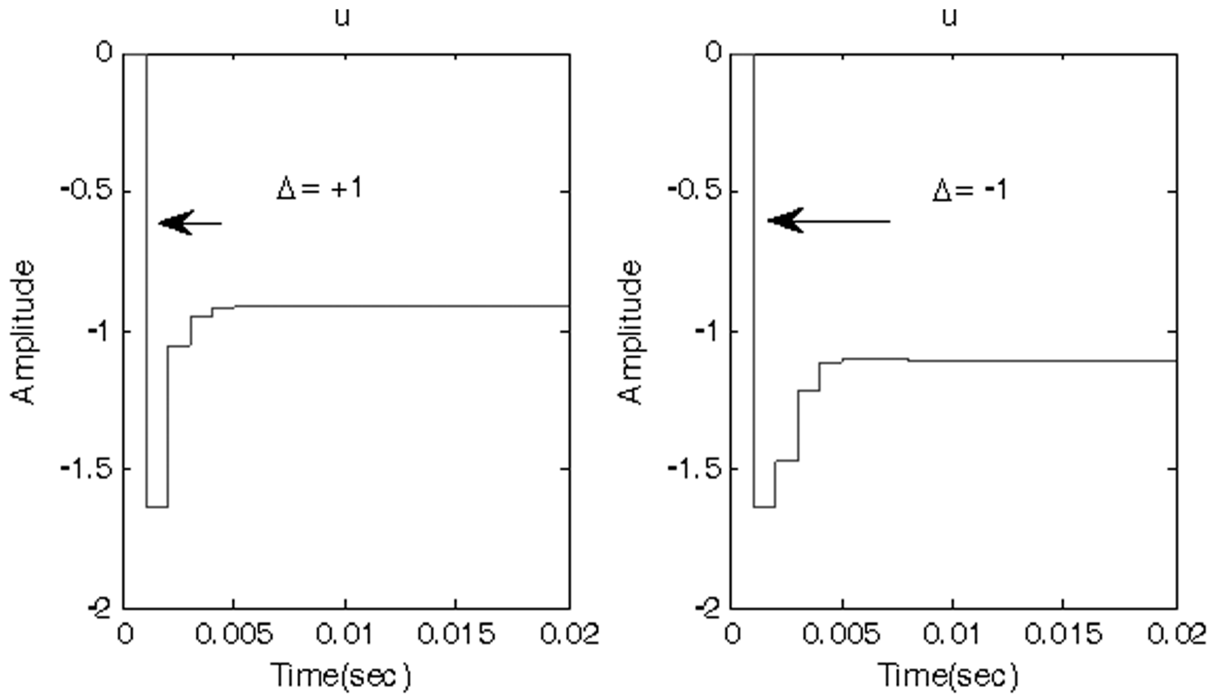


Figure 7: Control signalof the disk drive at extreme uncertainties, Δ (k) =±1

It is evident in Figs. 6 and 7 that the proposed controller achieves the desired dynamic response, ζ>0.5 and t_s<4 ms, without violating the control constraints, $|u| < 3$ volts [10].

Example 2: Consider the following example, which shows the control limit violation and how it can be avoided using theorem2.

Given the uncertain discrete-time system with sampling time T = 0.2 sec,

$$A = \begin{bmatrix} 0 & 1 \\ -0.6703 & 1.6700 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0.0154 \quad 0.0176], D = 0, M = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, N' = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, N_b = .2$$

The control objectives are the settling time <1.34 sec and the damping ratio >0.5, which require the designing of a state feedback robust pole placer without and with control limit $|u| < 1$.

The settling time constraint is achieved by a circle r_1 = 0.55, q_1 = 0, anda damping ratio r_2 = 0.53, q_2 = -0.34.

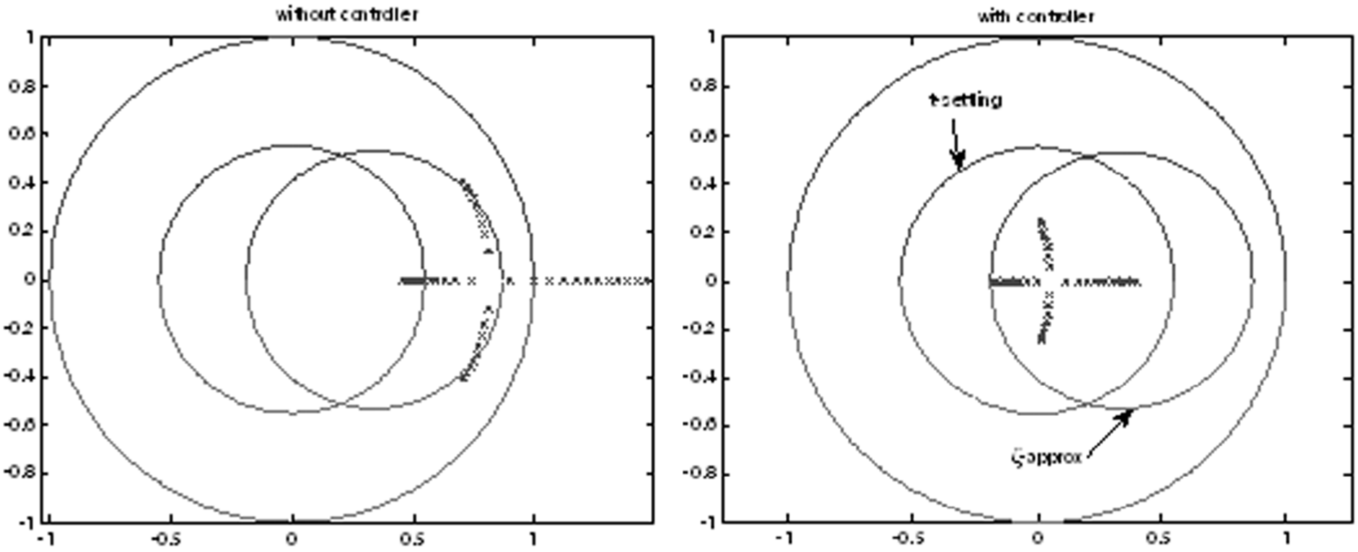
Using theorem 1, the unsaturated controller is

F = [0.6776 -1.5500]

The open-loop and closed-loop poles with the admissible uncertainties are shown in Fig.8.The system without a controller is unstable, while the poles are in the desired region, between the two circles, with the proposed controller.

For initial conditions x_0 = [2 2]', the system response and the control signal are shown in Figs. 9 and 10, respectively.

It is evident that the proposed controller achieves the desired dynamic response.Since the control signal violates the limit±1, the design has to be modified using theorem 2. The feedback gain of the saturated robust pole placer is given by



(b) Open-loop poles (b) Closed-loop poles

Figure 8: Poles of the uncertain system

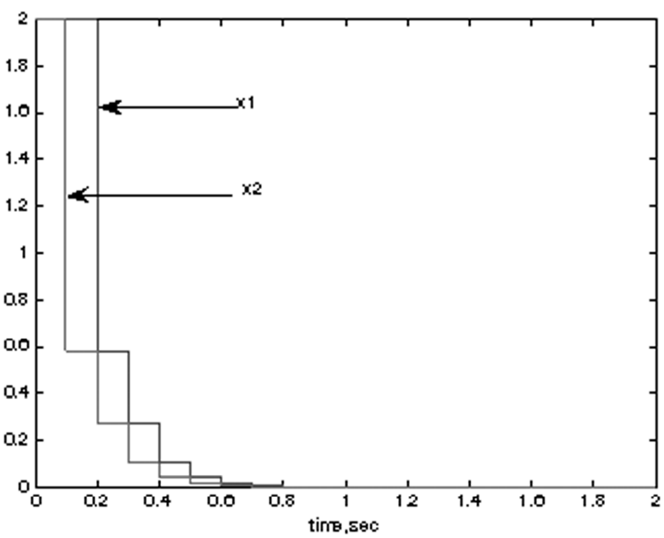


Figure 9: State response x_1, x_2 using unsaturated control

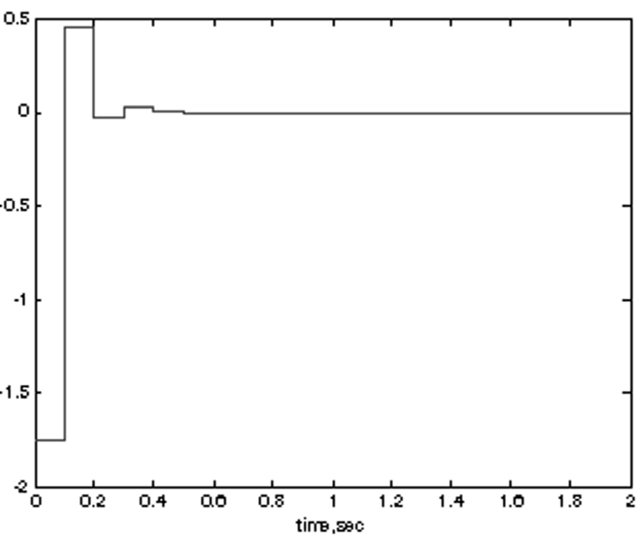


Figure 10: Unsaturated control signal

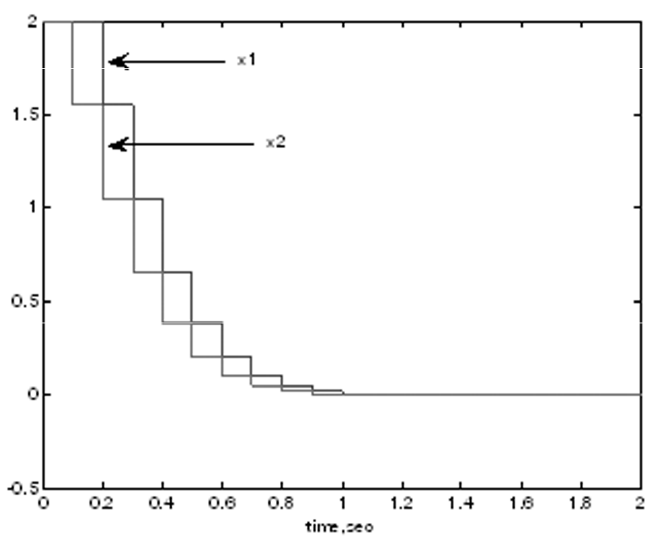


Figure 11: Stateresponse x_1, $x_2$using saturated control

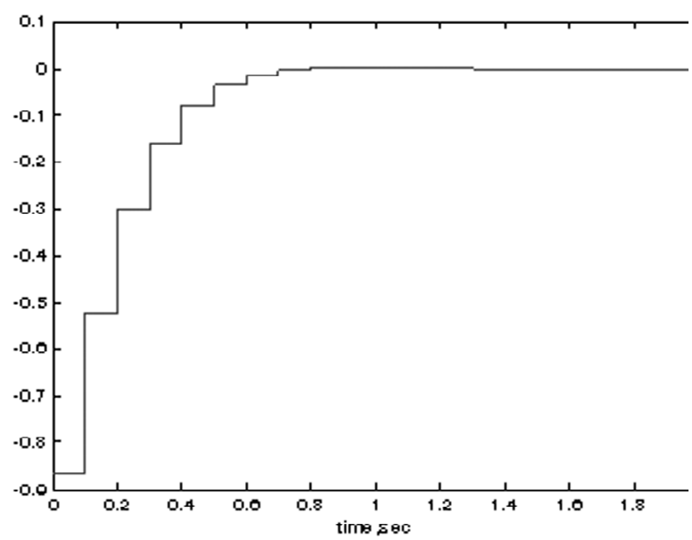


Figure 12: Saturated control signal

$$F = [0.3193\ -0.7518]$$

For the same initial conditions, the system response and the control signal are shown in Figs. 11 and 12, respectively. As shown, the control objectives are satisfied without control violation.

5. APPLICATION TO VEHICLE ACTIVE SUSPENSION CONTROL

Vehicle suspension systems are widely investigated and elaborated in the literature. For example, continuous-time control, adaptive, fault tolerant, robust, switched control techniques are studied in [11], [12], [13], and [14], respectively. Robust H_∞ approach and linear matrix inequality optimization approach is used to design an active suspension control is given [15].

Unlike the previous approaches, the proposed design introduces a digital computer or a microprocessor into the control loop as given in theorem 2. It is used to design an active suspension system for the quarter-vehicle model shown in Fig.13 [16].

The dynamics of the quarter-vehicle model can be described by [16]

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & \frac{-k_u}{m_u} & \frac{-c_s}{m_u} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \omega + \begin{bmatrix} 0 \\ \frac{u_s}{m_s} \\ 0 \\ -\frac{u_s}{m_u} \end{bmatrix} u \tag{18}$$

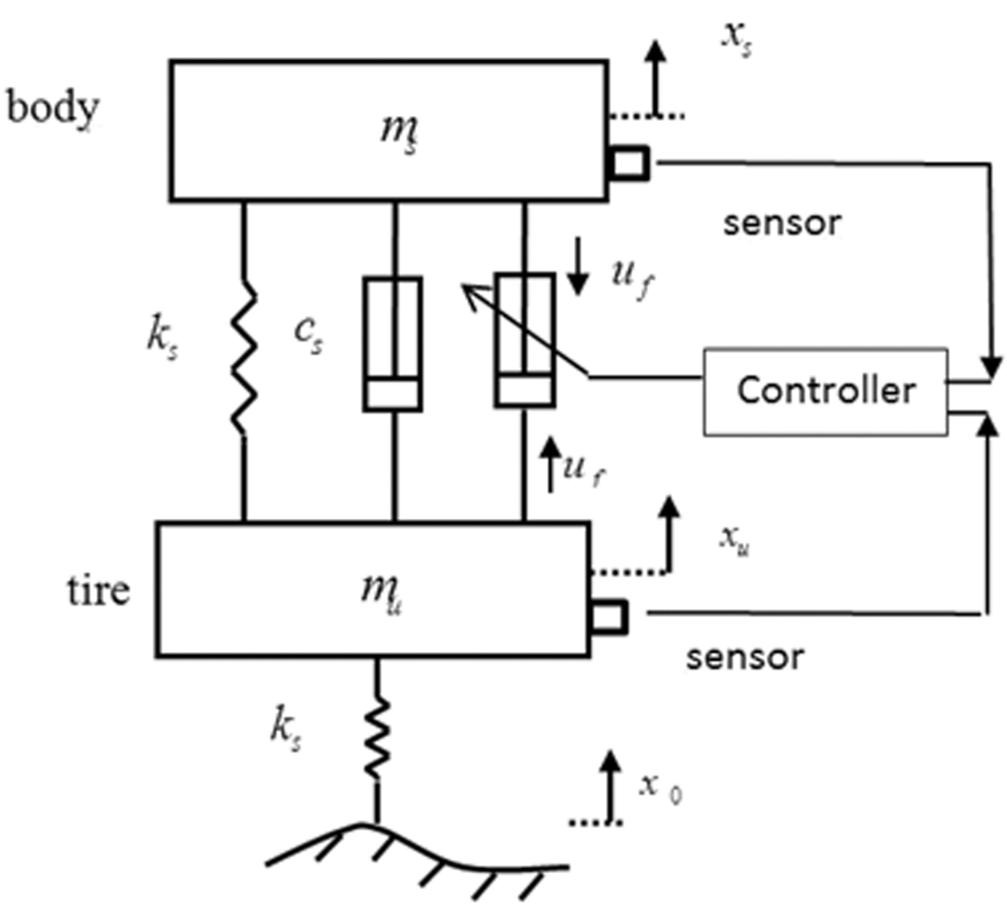


Figure 13: Quarter-vehicle model with an active suspension

where (k_s, c_s) are the parameters of the so-called passive suspension; k_u stands for the tire stiffness; m_s and m_u represent sprung and un-sprung masses, respectively. Moreover, $x_s - x_u$ is the suspension stroke, $x_u - x_0$ is the tire deflection and x_0 is the vertical ground displacement caused by road unevenness and u_f is the scalar active force generated by a hydraulic actuator. The state variables of the model (18) are defined as $x_1 = x_s - x_u$, $x_2 = \dot{x}_s$, $x_3 = x_u - x_0$, and $x_4 = \dot{x}_u$. The control input u is the normalized active force $u = u_f / u_s$ and $\omega = \dot{x}_0$ is the disturbance due to the road roughness. Note that, active forces generated by hydraulic actuators and considered as control inputs, are bounded because of actuator saturation. The system parameters and nominal values are shown in Table 1.

Table 1
Quarter-vehicle active suspension parameters

Parameter	*Value*
m_s	320 kg
m_u	40 kg
k_s	18 kN/m
k_u	200 kN/m
C_s	1 kN.s/m
u_s	1.5 KN

The normalized input is bounded as $|u(t)| \le 1$ and the active force is bounded by u_s = allowable spring stroke (±0.08 m) * spring constant.

Due to different passenger load variation, it is assumed that the mass m_s varies between 250 to 390 kg. The system is discretized with the zero order hold method at a sampling time *Ts* = 0.001 sec. Therefore, the discrete time norm-bounded model is obtained as

$$A = \begin{bmatrix} 0.9997 & 0.000986 & 0.002476 & -0.0009852 \\ -0.05546 & 0.9969 & -0.007783 & 0.003106 \\ 0.0002228 & 1.245e-5 & 0.9975 & 0.0009867 \\ 0.4433 & 0.02485 & -4.934 & 0.9727 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.089e-5 & 0.004622 & -1.857e-5 & -0.03694 \end{bmatrix}'$$

With uncertainty matrices

$$M = \begin{bmatrix} -0.0004 & -0.1367 & 0 & -0.0016 \end{bmatrix}'$$

$$N = \begin{bmatrix} -0.0729 & -0.0037 & -0.0102 & 0.0041 \end{bmatrix},$$

$$N_b = 0.0061$$

For passenger comfort, the oscillations due road bumps should be damped out within 1 sec with a minimum damping ratio $\zeta = 0.25$ [17]. To achieve the first objective, we select $r_1 = 0.9960$; $q_1 = 0$ for the first circle, while for the second, $r_2 = 0.68$; $q_2 = -0.31$.

The LMIs (14) are solved to get the saturated control gain matrix as

F = [-402.7340 -47.9208 546.4541 16.1682]

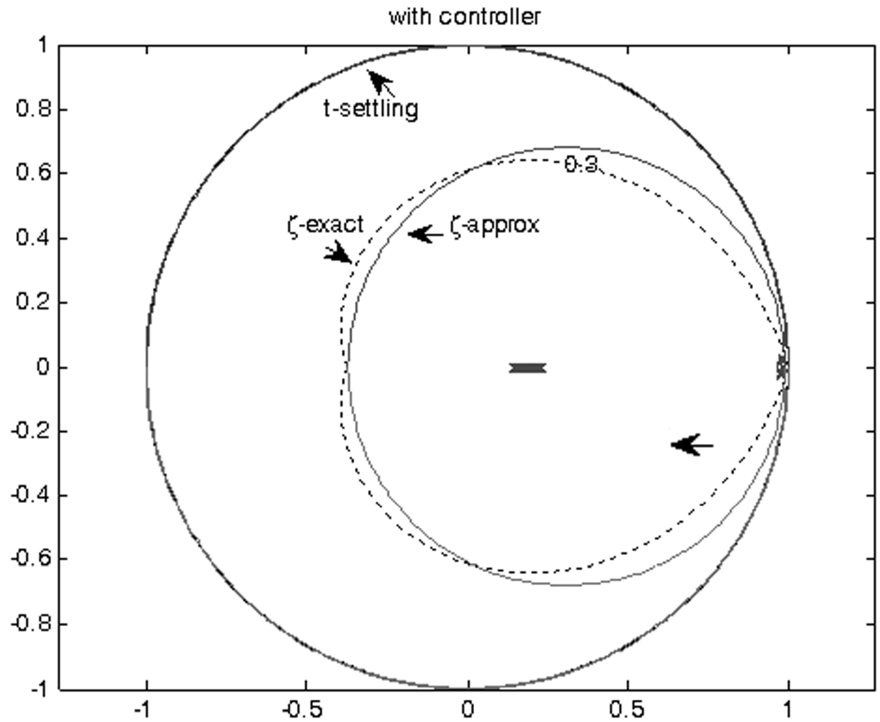


Figure 14: Closed loop poles with system uncertainties using saturated control

As shown in Fig. 14, all the closed loop poles lie inside the desired domain.

The proposed controller is tested for two cases: step, and bump road.

Case 1. Step road test

For simulation, we assume that the vehicle is subject to a 500 N unit step input due to a step road change. With passive suspension, the vehicle will oscillate for an unacceptable long time, about 3.5 s, with large overshoot. This might damage the suspension system.

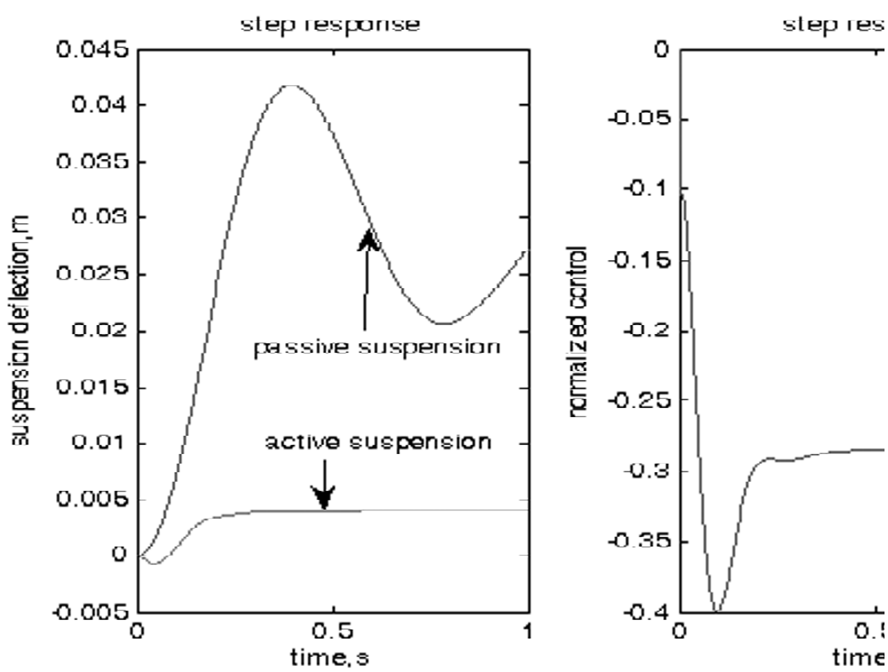


Figure 15: Step response at light load

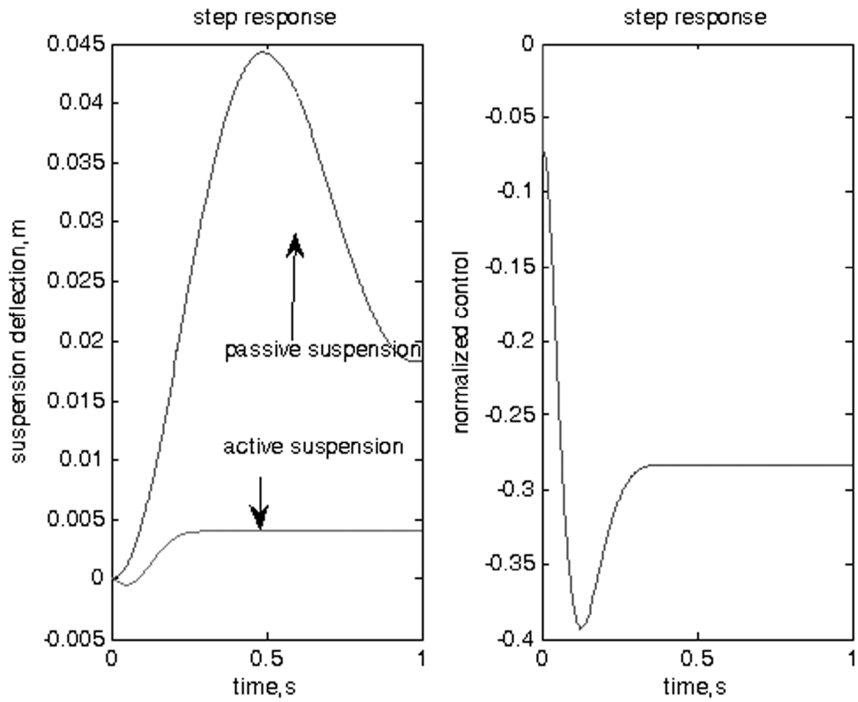


Figure 16: Step response at heavy load

Whereas the active suspension damps the oscillations in about 0.3 sec without overshoot.

Case 2. Road bump test

In order to study the system response due to road bump, the case of an isolated bump in an otherwise smooth road surface is considered [16]. The corresponding ground displacement in this case is given by

$$x_0 = \begin{cases} \frac{A}{2}(1-\cos(\frac{2\pi V}{L}t)), & 0 \le t \le \frac{L}{V} \\ 0, & t > \frac{L}{V} \end{cases} \quad (19)$$

where A and L are the height and the length of the bump and V is the vehicle forward velocity. These values are chosen as $A = 0.1$ m, $L = 5$ m and $V = 27$ km/h. The bump response, namely the suspension stroke (m) and the active force (kN) are shown in Fig. 14 and Fig. 15. In comparison with the passive suspension case, the proposed robust saturated control gives better dynamic performance in terms of less overshoot and faster damping of the suspension stroke. Moreover, the normalized control signal is bounded between ±1 as shown in Fig. 16.

Passengers comfort:

The ride comfort is usually measured by the body acceleration $\ddot{x}_s$ that is given by

$$\ddot{x}_s = \begin{bmatrix} -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \end{bmatrix} x + \frac{u_s}{m_s} u$$

This acceleration is shown in Figure 19

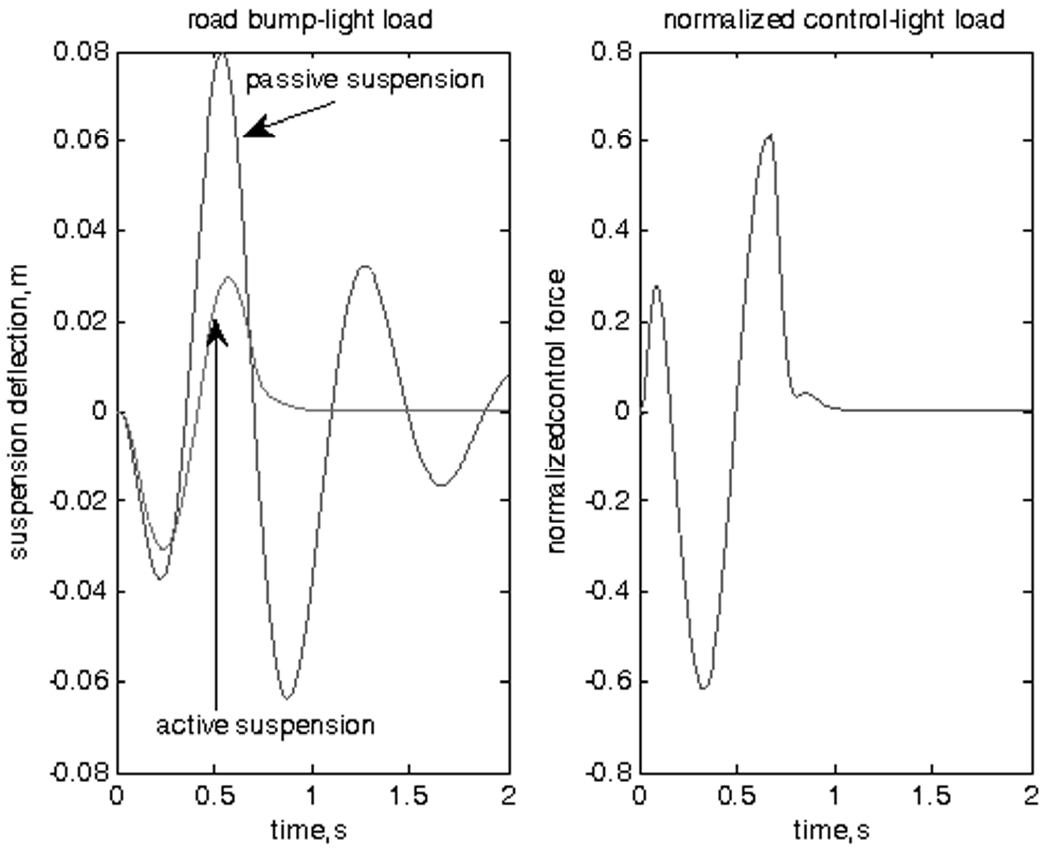


Figure 17: Road bump Response at light load

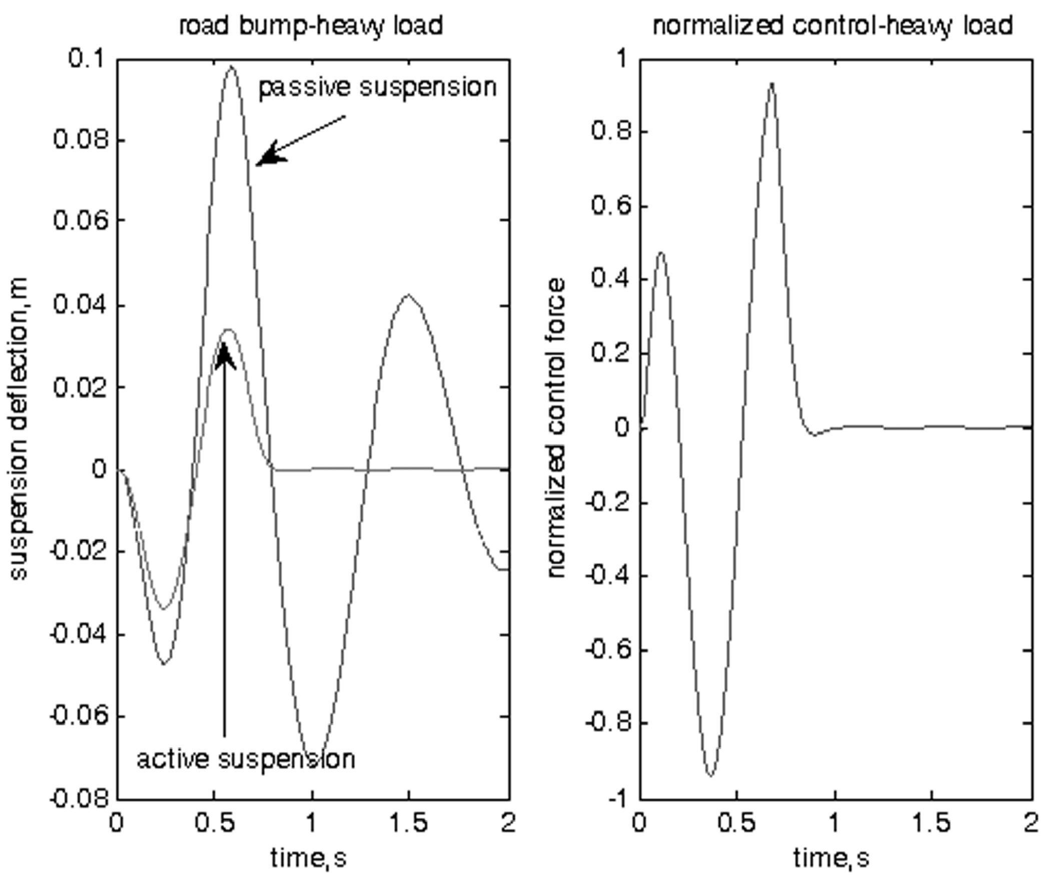


Figure 18: Road bump response at heavy load

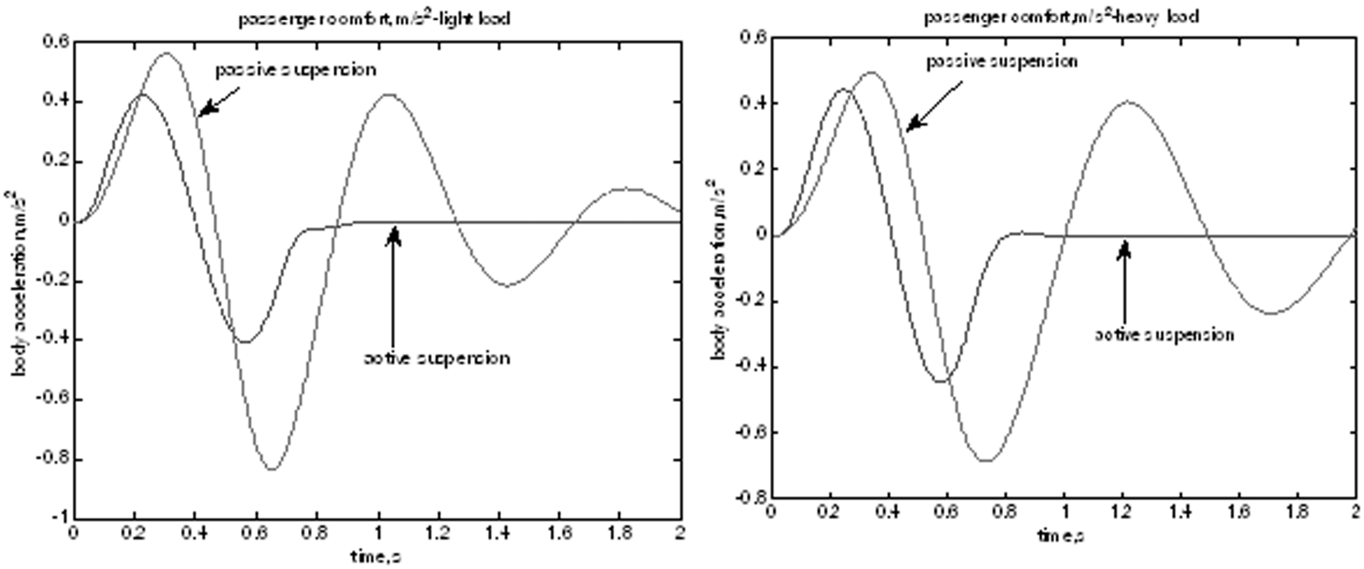


Figure 19: Passenger comfort due to road bump:light load(left), heavy load(right)

It is evident from Figs. 15-19 that the active suspension outperforms the passive one since it satisfies the control objectives (damps out the oscillationsin < 1 sec, and damping ratio >0.25) as well as it does not violate the constraints of suspension stroke (8 cm), and hydraulic actuator force limit of 1.5 KN. Active suspension also provides better passenger comfort than the passive one as it has lower body acceleration.

6. CONCLUSION

Thispaper considers placing the poles in a desired region of uncertain linear digital systems in the presence of input saturation. The new scheme is developed to design robust controllers, taking into consideration the effect of saturation nonlinearity. The proposed controller is based on LMI optimization and requires the uncertainty of model parameters to be cast in the norm-bounded form. The designed controller is applied to two numerical examples. Simulation results have also been presented to show the effectiveness of the proposed design. Moreover, the proposed control is applied to an active suspension of a quarter vehicle model. Analysis and simulation results have confirmed the potential benefit of the proposed constrained active suspension in achieving the best possible ride comfort, while keeping suspension strokes and control inputs within bounds.

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