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Triangular Tile Pasting P System and Conditional Communications

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Abstract: In formal language theory, membrane computing is a new theoretical study. It is a cell-like structure in which regions are separated by means of membranes and it is introduced by G.H. Păun. Generally using this computational system one can generate tiling patterns, two dimensional picture languages and tessellations. In this paper we introduced tissue-like triangular tile pasting P system with conditional communications with example and results. Also in this paper, the generating powers of iso-array grammars and triangular tile pasting P system have been studied in detail.

Keywords: *Iso-Triangular Tiles, Triangular Tile Pasting P System, Iso-Array Grammars, Permitting and Forbidden Conditions, Two Dimensional Pictures. (AMS subject code: 68QXX, 68RXX, 68TXX, 03DXX)*

1. INTRODUCTION

Membrane computing is a new study to both computer and non-computer researchers'. G. H. Păun introduced a new computing model called P system in [7]. It gives an abstract to computing models in the way of functioning of living cells in biological structure. P system is a distributed and parallel computing model in which evolution rules and evolving objects are considered in the regions of membranes. P system has merely a biological background and mathematical formalism. Among the basic types of P system like P system with multi rewriting rules, tissue like P system and neural like P system, tile pasting p system is a type which is introduced in [10,13] by taking square tiles in the pasting system. The pasting rules are used non-deterministically and the computation starts by applying the pasting rules to the strings maximally parallel manner in each region. By relating P systems and array grammars, Ceterachi et al. [5] began a study on linking these two areas, which were not discussed in early research. Using rules of array re-writing grammars in P system array-rewriting P system is introduced to generate two dimensional picture languages. Iso-triangular tile pasting system is defined by Kalyani et.al.in [9], which is an interesting tile pasting system. It generates iso-triangular picture languages. Connecting P system with iso-triangular tile pasting system triangular tile pasting P system is introduced in [2].

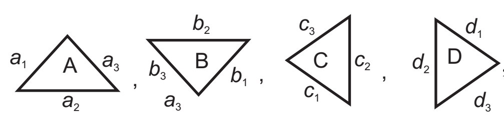
In recent years different types of P systems have been studied, among the variants of P systems, conditional communication is a new study [8], in which the communication to the membranes are controlled by the tiles or tilings not by the pasting rules. To control the communications, permitting and forbidding conditions are defined in the existing tiles or sub tiles of the picture pattern. Tissue-like P system is a type of P system to generate two dimensional pictures. In this P system membranes are considered as elementary membranes and it is differentiated from usual P system in such a way that it has no skin membranes and environment. But the computational result is collected only in the output membrane. It is not necessary that the membranes should connect or communicate to each other directly and the communications of membranes are permitted only if the membranes are designed in the system. The communication of membranes is represented graphically by using the synapses (syn). Ceterchi et.al. [6] defined a type of tissue-like P system in which membranes are placed at the nodes of a graph.

Robinson et.al. [11,12] Introduced tissue-like P system with conditional communications by connecting the two models, conditional communication on P system and Tissue-like P system. It is noticed that the evaluation rules are considered only in maximally parallel way to generate the two dimensional pictures. Among the tile types iso-triangular tiles are a new set of tiles. Iso-triangular tile pasting system and some results have been discussed in [9]. Followed by this detailed study on iso-triangular tile pasting system, triangular tile pasting P system for pattern generation is introduced. Iso-array rewriting P system with iso-array rules is studied in [3]. Tissue-like P system with active membrane is proposed with an example by using iso-triangular tiles and the generating power of that system is examined in [1]. Also we have examined the computational powers of the triangular tile pasting P system and array generating petrinets in [4]. Inspired by these studies, this paper introduces conditional communications on tissue-like triangular tile pasting P system to generate the triangular picture languages.

2. BASIC DEFINITIONS

Tiling is an art and it is a well-known theory in the applications of pattern generation. In this section we recollect the notion of triangular tiles and tile pasting system. A tile is a topological disc with closed boundary in the XOY plane, whose edges are gluable. A tiling is a family of countable tiles with no gaps or overlaps that covers the Euclidean plane.

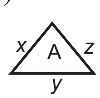
2.1. Definition

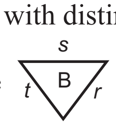
Consider the labeled iso-triangular tiles 

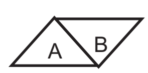
, whose horizontal (Vertical) and side edges are of length 1unit and 1/√2 unit respectively.

2.2. Definition

Generally a pasting rule is a pair (x, y) of labeled tiles with distinct edges (not necessarily distinct).

For example, the triangular tile 

and tile 

are glued by the edges (z, t), which means that the edge z of tile A is glued with the edge t of tile B, then the pattern 

is generated.

Note that the edges are of same length. The set of all edge labels is called an edge set denoted by E.

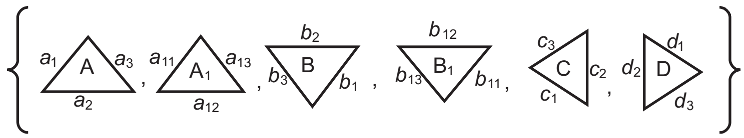
Tile pasting rules of the tiles A, B, C, D are given below :

1. Tile A can be glued with tile B by the pasting rules $\{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$ with tile C by the rule $\{(a_3, c_1)\}$ and with tile D by the rule $\{(a_1, d_3)\}$
2. Tile B can be glued with tile A by the pasting rules $\{(b_1, a_1), (b_2, a_2), (b_3, a_3)\}$ with tile C by the rule $\{(b_1, c_3)\}$ and with tile D by the rule $\{(b_3, d_1)\}$

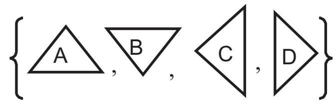
3. Tile C can be glued with tile A by the pasting rule $\{(c_1, a_3)\}$ with tile B by $\{(c_3, b_1)\}$ and with the tile D by the pasting rules $\{(c_1, d_1), (c_2, d_2), (c_3, d_3)\}$
4. Tile D can be glued with tile A by the pasting rule $\{(d_3, a_1)\}$ with tile B by $\{(d_1, b_3)\}$ and with the tile C by the pasting rules $\{(d_1, c_1), (d_2, c_2), (d_3, c_3)\}$.

Notations:

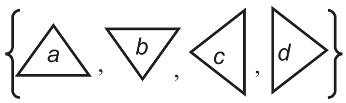
1. Iso-triangular tiles =



2. Non-terminal symbols N =



3. Terminal symbols T =



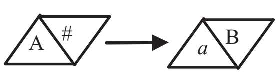
2.3. Definition

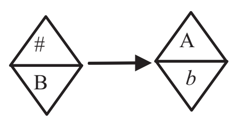
A Triangular tile pasting system (TTPS) is S = (Σ, P, t_0), where Σ is a finite set of labeled triangular tiles, P is a finite set of pasting rules and t_0 is the axiom pattern of tiles.

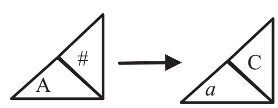
A pattern p_2 is generated from a pattern p_1 by applying the pasting rules in parallel manner to the edges of the pattern p_1, where pasting is possible. Note that the labels of pasted edges in a pattern are ignored once the tiles are pasted. The set of all patterns generated from the axiom t_0 constitutes the triangular picture language denoted by L(TTPS).

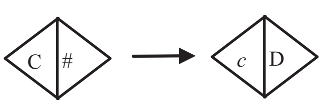
2.4. Definition

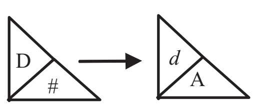
A regular iso array grammar (RIAG) is a structure G = (N, T, P, S) where N = {A, B, C, D} and T= {*a*, *b*, *c*, *d*} are finite sets of symbols (isosceles right angled triangular tiles), N ∩ T = ϕ, elements of N and T are called non-terminals and terminals respectively. S ∈ N is the start symbol of the generating pattern. P is a finite set consist the pasting rules of the following forms:

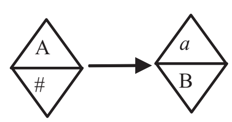
1. 

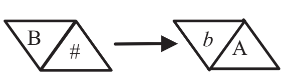
2. 

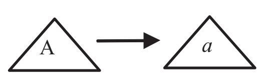
3. 

4. 

5. 

6. 

7. 

8. 

Similar rules can be given for the other tiles B, C and D. The set of all languages generated by RIAG is RIAL.

2.5. Definition

Basic puzzle iso-array grammar is a structure G = (N, T, P, S), where N = {A, B, C, D} and T= {*a*, *b*, *c*, *d*} are finite sets of symbols. The non terminal symbols are in N and the terminal symbols are in T. S ∈ N is the start symbol, P consists the rules of the following forms:

Similar rules can be given for the other tiles B, C, D. The language generated by BPIAG is L (BPIAG). The set of all languages generated by BPIAG is BPIAL.

3. TRIANGULAR TILE PASTING P SYSTEM

3.1. Definition

A triangular tile pasting P system (TTPPS) is defined as $\Pi = (\Sigma, \mu, F_1, F_2, \ldots, F_m, R_1, R_2, \ldots, R_m, i_0)$, where Σ is a finite set of iso-triangular tiles, μ is a membrane structure. In μ the membranes are labeled as 1,2,.., *m* in a one-one manner. $F_1, F_2, \ldots, F_m$ are finite sets of pictures over the iso-triangular tiles of Σ associated with *m* regions in the membranes. $R_1, R_2, \ldots, R_m$ are finite sets of triangular tile pasting rules of the type $(t_i, (x_i, y_i), 1 \le i \le n)$ associated with m regions and i_0 is the output membrane, which is called an elementary membrane. Computation starts from the axiom of pattern present in the first membrane. To each picture pattern in each region of the system, a pasting rule could be applied. The picture pattern is moved (retained) in to another region (in the same region) due to the presents of the target indication associated with the pasting rule.

A computation is successful only if the computation is stopped; the computation is stopped if there is no possibility of applying the pasting rule to the existing pattern. The result of halting triangular picture pattern is composed only by the pasting rules. The pattern is haltered in the membrane labeled by i_0. The set of all triangular picture patterns computed by a TTPPS is denoted by TTPPL (Π). The set of all such languages TTPPL (Π) is generated by the system Π is denoted by $TTPPL_m$.

Example 3.1. A class of language arrow heads is generated by the rules of RIAG with 4 distinct labeled iso-triangular tiles.

Consider the RIAG G = (N, T, P, S) where N = {A, B, C, D} and T = {*a, b, c, d*} are finite sets of non-terminal and terminal symbols respectively. S is the non-terminal iso-triangular tile A, is start symbol and P has the following rules;

The triangular picture language is shown in figure 1

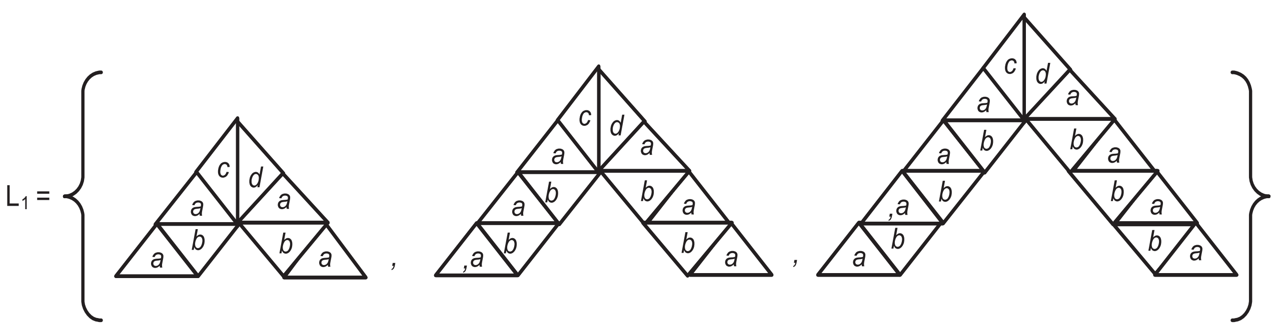
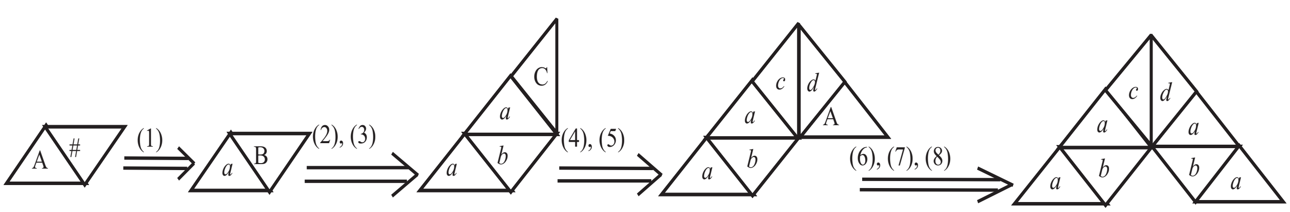
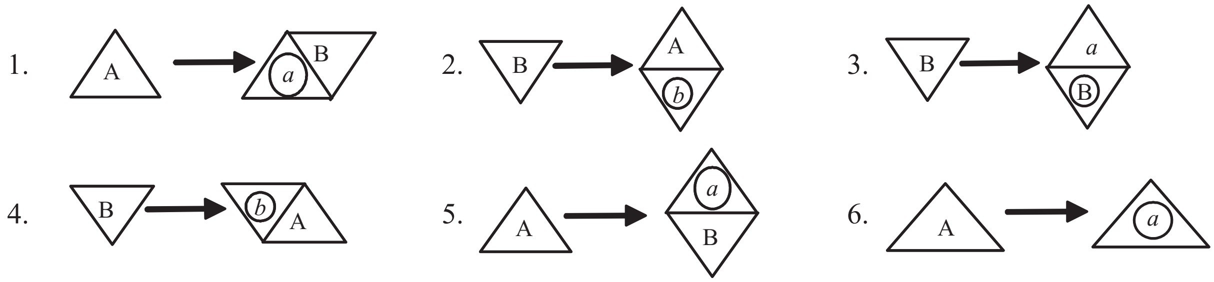


Figure 1: Language of arrow heads with the tiles "*c*" and "*d*" on the top

A member of the picture language is shown with the derivation steps and the rules are applied sequentially.



Example 3.2. Consider the BPIAG G = (N, T, P, S), where N= {A, B}, T = {*a*, *b*}, S = A is the start symbol and P consists the rules;



The picture language consists arrow heads with "*a*" tile on the top is generated by BPIAG. The picture language is shown in figure 2.

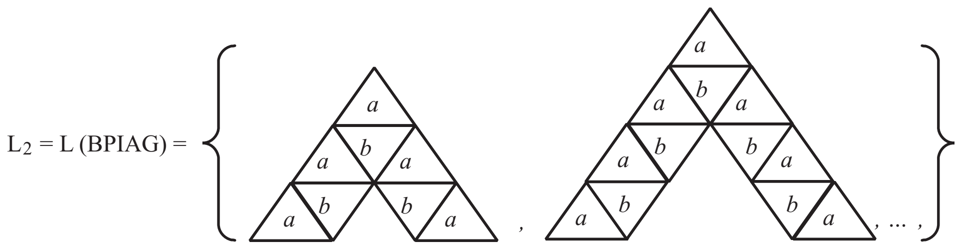


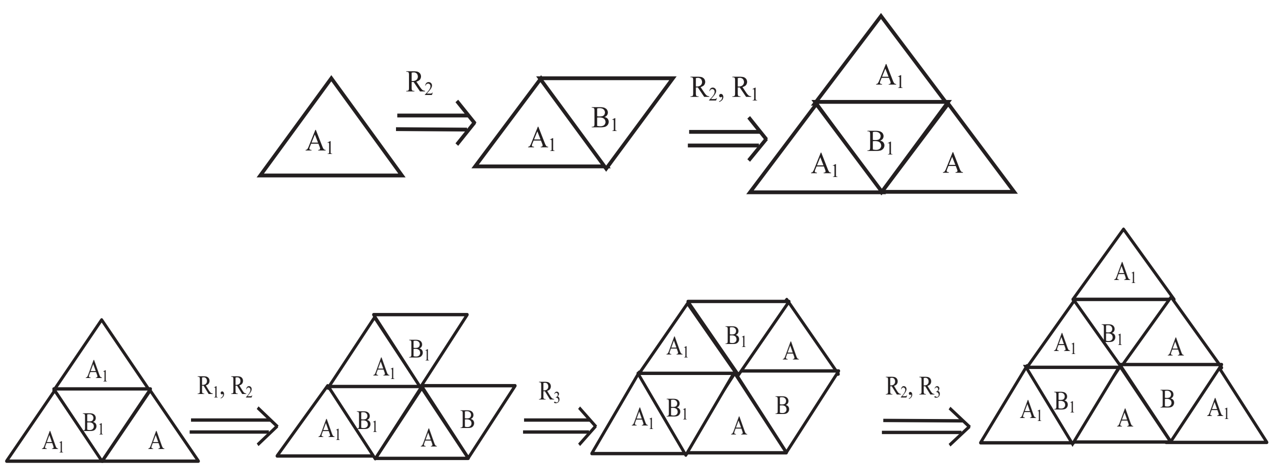
Figure 2: The language arrow heads with tile "*a*" on the top

Theorem 3.1. L (BPIAG) – L (RIAG) $\neq \phi$

Proof : From the definitions 2.4 and 2.5 all the rules of BPIAG contain the rules of RIAG. Hence it is clear that RIAG is the proper subset of BPIAG. In BPIAG the non-terminal symbol in the left hand side of the rules are replace only by the circled symbol (The circled symbol being either non-terminal or terminal). But in RIAG the non-terminal symbols of left hand side of the rules are replaced only by terminal symbols. The picture language L_2 shown in example 3.2 cannot be generated by any RIAG.

Example 3.4. The triangular tile pasting P system $\Pi_1 = (\Sigma, [_1[_2]_2[_3]_3]_1, F_1, F_2, F_3, R_1, R_2, R_3, 1)$ generates the family of two dimensional iso-triangular picture languages L_1 and L_2.

Here $\Sigma = \{A, A_1, B, B_1\}$, $F_2 = A$, $F_1 = \phi$, $F_3 = \phi$, $R_1 = \{(A, (a_3, b_3), in_2), (B, (b_{11}, a_1), here)\}$, $R_2 = \{(B, (b_2, a_{12}), in_3), (B, (b_1, a_{11}), in_3), (A_1, (a_{13}, b_{13}), in_3)\}$, $R_3 = \{(B, (b_2, a_2), in_2), (B_1, (b_{12}, a_{12}), out)\}$ and 1 is the output region. The rules of R_1, R_2 and R_3 are applied with the target indications which belongs to the set *tar* $\in$ *{here, in, out}* and then the resultant iso-triangular picture pattern is collected in the output region one finally. Two members of the Picture language L_1 is shown below with derivation steps.



On the other hand, the computation starts from the region R_2. The first member of the language "adjoin of iso-triangles" is collected in the region 1. Again the generation is started in the same region one, the 2nd member of the language is generated and it is collected in the region two. The computation is continued in this way, the members of the language are collected in the region 1 and 2 simultaneously.

In the next case of computation, with the rule (B, (b_2, a_2) the target indication out is considered in the region three. Now the computation is started in region one with tile A and the language stair case model of iso-triangles is generated. The members are collected in the output region one. The family of languages is shown in the figure 3.3, which consists of the languages adjoin of iso-triangles and step - step model of iso-triangles.

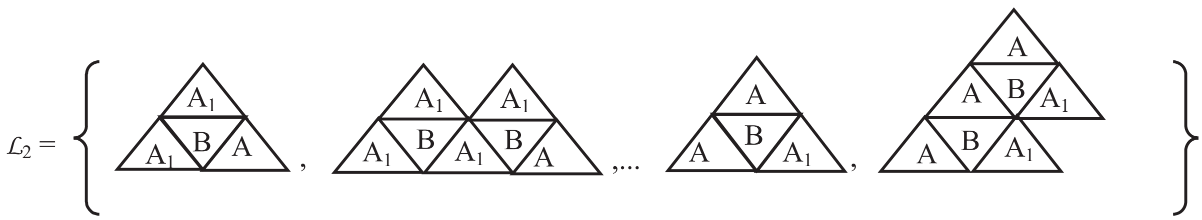
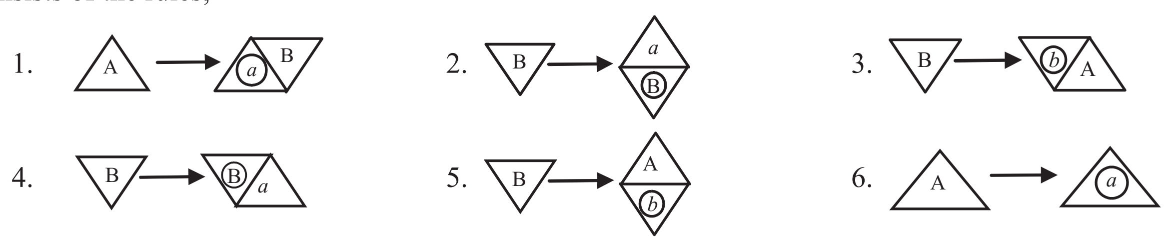


Figure 3: The family of languages of adjoin and step-step model of iso-triangles

Theorem 3.3. $TTPPL_3 - BPIAL \neq \phi$

Proof: The language L_1 is generated by the TTPPS Π_1 explained in example 3.4 with the rules $R_1 = \{(A, (a_3, b_3), in_2), (B, (b_{11}, a_1), here)\}$, $R_2 = \{(B, (b_2, a_{12}), in_3), (B, (b_1, a_{11}), in_3), (A_1, (a_{13}, b_{13}), in_3)\}$, $R_3 = \{(B, (b_2, a_2), out), (B_1, (b_{12}, a_{12}), out)\}$ (refer example 3.4) cannot be generated by any BPIAG. Since in BPIAG, the non-terminal symbols of left hand side of each rule is replaced only by the circled symbol (circled symbol may be non-terminal or terminal). By the rules of BPIAG the resultant iso-array shown in figure 3.3 cannot grow vertically. For example, consider the BPIAG $G_1 = (N, T, P, S)$, where N= {A, B}, T = {a, b}, S = A and P consists of the rules;



The rules 1, 4, 5, 6 and the rules 1, 2, 3, 6 of BPIAG generate the languages L_3 and L_4 respectively.

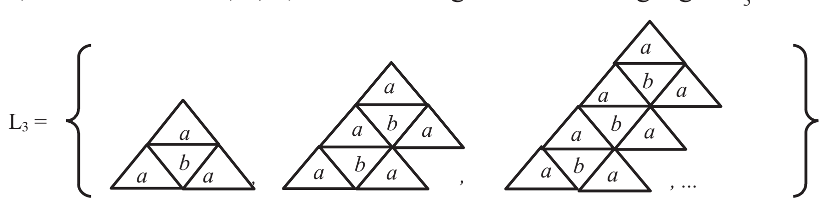


Figure 4: (*a*) Language adjoin of iso-triangles

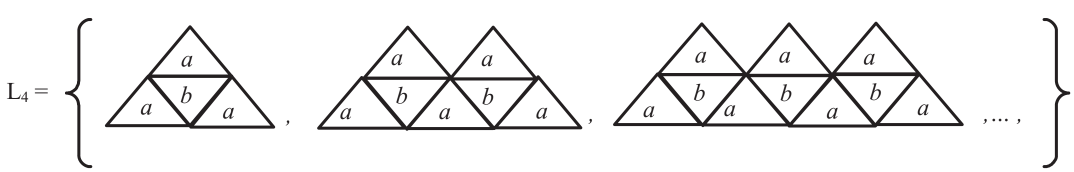


Figure 4: (*b*) Language consist overlapping iso-triangles

Theorem 3.3. $TTPPL_3 - RIAL \neq \phi$

Proof: The family of languages L_3 is generated by TTPPS $\Pi_2 = (\Sigma, [_1[_2]_2[_3]_3]_1, F_1, F_2, F_3, R_1, R_2, R_3, 3)$, where $\Sigma = \{A, B, B_1\}$, $F_1 = A$, $F_2 = \phi$, $F_3 = \phi$, $R_1 = \{(A, (a_3, b_3), here), (B, (b_2, a_2), here), (D, (a_3, c_1), in_2)\}$, $R_2 = \{(C, (c_2, d_2), here), (D, (d_3, a_1), in_3)\}$, $R_3 = \{(A, (a_2, b_{12}), here), (B_1, (b_{11}, a_1), here)\}$ and 3 is the output region. The family of language L_3 generated by the above rules are collected in the region three of the membrane 3. It is shown in the figure 5. This family of language cannot be generated by any RIAG. (refer example 3.1).

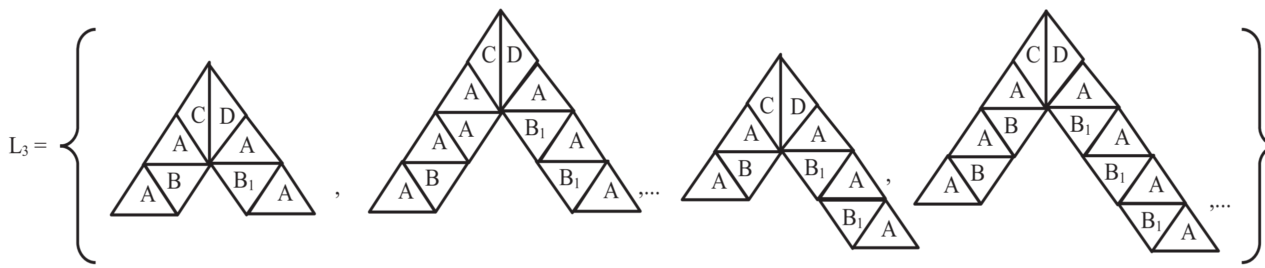


Figure 5: Language consists of arrow heads and arrow heads with tail on the right

Theorem 3.4. $TTPPL_3 \cap L(TTPS) \neq \phi$

Proof: The triangular tile pasting system $G = (\Sigma, P, t_0)$ generates the picture language L_5, where $\Sigma = \{A, B, B_1\}$, $t_0 = A$ and $P = \{(a_3, b_3), (b_2, a_2), (a_3, c_1), (c_2, d_2), (d_3, a_1), (a_2, b_{12}), (b_{11}, a_1)\}$. The language generated starts with the rule (a_3, b_3) with the tile A. Then the rules given in P are applied one by one. The resultant language is L_3. It is shown in figure 3.5. On the other hand if the computation starts with the rule (a_2, b_{12}), the language L_5 is the resultant array language which is given in the figure 3.6Thus the TTPS generate the families of languages L_3 and L_5.

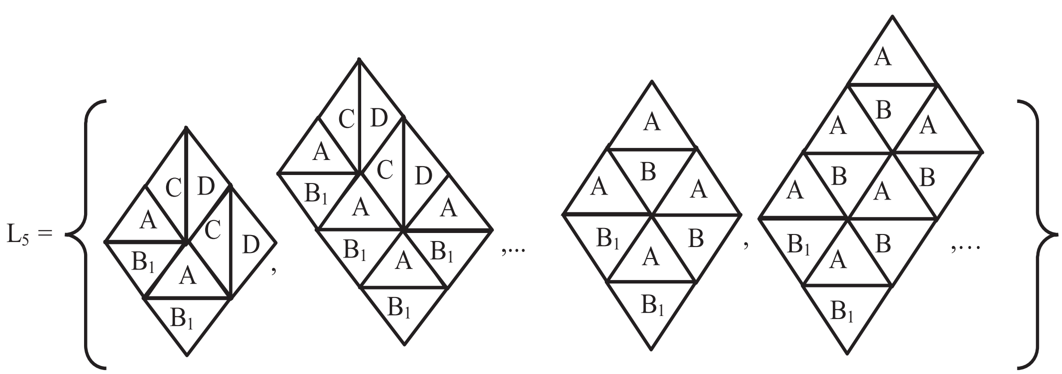


Figure 6: A language of overlaps of Rhombuses

TTPPS defined in theorem 3.3 generates the picture language L_3 shown in the figure 3.5(refer theorem 3.3). Hence the intersection of families of languages generated by TTPPS and TTPS is non-empty.

4. TISSUE-LIKE CONDITIONAL COMMUNICATION ON TRIANGULAR TILE PASTING P SYSTEM

This section introduces Tissue-like conditional communication on TTPPS with an example and discusses the generating powers of TTPS, TTPPS and *t*-CCTTPPS.

4.1. Definition

Tissue like triangular tile pasting P system with conditional communications (*t*-CCTTPPS) is defined as $\Pi = (V, \mu, (t_0)_i, \text{syn.}, (R_{(i,j)}, P_{(i,j)}, F_{(i,j)}, i_0)$. Here V is The set of all labeled iso-triangular tiles, μ is the membrane structure contains finite no of membranes, which are labeled from the set {1, 2, 3, …, *n*}. t_0 is the axiom of picture pattern present in the membrane *i* ($1 \leq i \leq m - 1$). Syn = $\{(i, j)/i, j \in i \ (1 \leq i \leq m)\}$is the set of links to the membranes.

$R_{(i,j)}$ is a finite set of evolution rules over the set of iso-triangular tiles present in the regions of the membrane system. An evolution rule is a pair $\{(\alpha, \beta) / a \in \alpha, b \in \beta\}$. $P_{(i,j)}$ is the set of permitting conditions, $F_{(i,j)}$ is the set of forbidden conditions associated with region (*i*, *j*) where $1 \leq i, j \leq m$.

The conditions can be defined as follows :

1. **Empty:** There is no restriction or condition on generating the pattern and the pattern transferred from one membrane to another membrane freely. We denote the permitting and forbidden conditions are empty.

   An empty permitting condition is denoted by (true, in_j) and an empty forbidden condition by (false, in_j).

2. **Labeled Tile Checking :** $P_{(i,j)}$ present in each region is a set of pair (α, in_j) for $\alpha \in V$ and in_j is the target indication which transfer the pattern into the membrane *j* and $F_{(i,j)}$ present in each region of membrane is a pair (β, not in_j) for $\beta \in V$. A pattern can be sent from a membrane to another membrane if the pair $(\alpha, in_j) \in P_{(i,j)}$ and if the pair $(\beta, \text{not } in_j) \in F_{(i,j)}$, β does not belongs to $P_{(i,j)}$.

3. **Sub Pattern Checking :** Each P_i is a set of pair (*u*, in_j) for $u \in V^+$ and each F_i is a set of pair (*v*, not in_j) for $v \in V$. A pattern *p* can go to another membrane only if there is a pair $(u, in_j) \in P_i$ with $u \in$ sub pattern (*p*) and for each pair $(v, \text{not } in_j) \in F_i$ with $v \notin$ sub pattern (*p*).

The working rule of this system is explained below.

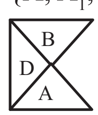
Triangular tile pasting rules present in each region is applied non -deterministically. The pattern obtained in this way is checked by the conditions in $P_{(i,j)}$ and $F_{(i,j)}$ present in the corresponding region. If the condition $p_{(i,j)}$ is satisfied in the region then pattern sent to the jth membrane. If the pattern fulfills the forbidden condition $F_{(i,j)}$ then the pattern remains in the same region. If the pattern not satisfied both the conditions then the picture pattern retained in the same region of the membrane. The computational process is continued in this manner and then the resultant triangular picture pattern is collected in the output membrane. If the set of pair in $P_{(i,j)}$ is never satisfied by the pattern, then the computation is an unsuccessful one.

The set of all such picture patterns generated by this way is denoted by L (t-CCTTPPS). The family of all such languages L (*t*-CCTTPPS) generated by the system *t*-CCTTPPS (Π) is denoted by L (*t*-CCTTPPS).

Example 4.1 Consider $\Pi = (V, \mu, t_0, \text{syn.}, (R_i, P_i, F_i), i_0 = 1)$,

where $V = \{A, A_1, B, B_1, C, D\}$

t_0 =



$R_1 = \{(a_3, b_{13}), (a_3, b_3), (b_{12}, a_2),$

$P_1 = \{(P_{1,11}, in_2), (P_{1,12}, in_2)\},$

$R_2 = \{(b_1, a_1), (a_{13}, c_1), (a_{13}, b_{13}),$

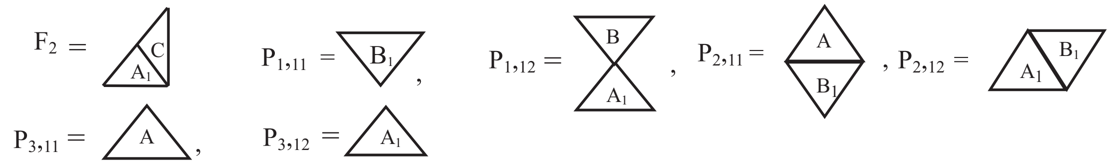
$P_2 = \{(P_{2,21}, in_3), (P_{2,22}, in_3)\},$

$R_3 = \{(b_{11}, a_{11}),$

$P_3 = \{(P_{3,1}, in_1), (P_{3,2}, in_2)\},$

$F_1 = \phi,$

$F_3 = \phi$



The first member of the picture language is shown with the derivation steps below.

The three members are shown in figure 7

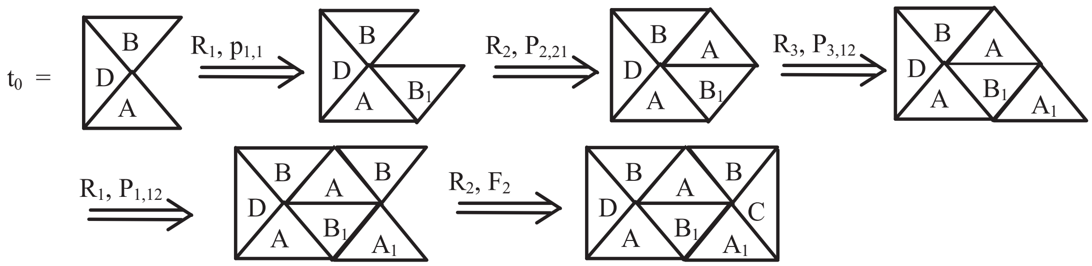


Figure 7: Language consist a class of rectangles

Each member of the language consists of $4(n + 1)$ iso-triangular tiles and n denotes the member of the languages.

Theorem 4.2. L(TTPS $\subseteq$ L(t-CCTTPPS)

It is clear from the definition from 2.3 and 4.1

Theorem 4.3. L(TTPPS) $\subseteq$ L(t-CCTTPPS)

The permitting and forbidden conditions present in the tissue-like conditional communication on triangular tile pasting P system one can generate the triangular picture language generated by TTPPS but the language generated by t-CCTTPPS cannot be generated by TTPPS always. It proves the result.

5. CONCLUSION

In this paper we have defined a new model called tissue-like triangular tile pasting P system with conditional communications. It is explained with suitable example. Iso array grammars like Regular iso array grammar and basic puzzle iso-array grammar rules have been considered with triangular tile pasting P system and the computational powers of tissue-like triangular tile pasting P system is examined. The generating powers of the iso-array grammars and the triangular tile pasting P system are also examined.

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