

Analysis of Active Vibration Control in Smart Structures

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ABSTRACT

Vibration is an important problem in engineering. There are desirable and undesirable vibrations and it is essential to control undesirable vibrations. Vibrations are controlled either by active vibration control method or passive vibration control method. In this work, the vibrations caused by the step input on a Cantilever beam is actively controlled by piezoelectric actuators when it is placed very near to the fixed end of the cantilever beam. Vibrations are very rapidly controlled when the length of the piezo electric actuator is fully along the length of the beam. The results show that PID controller is more efficient than LQR controller for the same configuration.

Keywords: vibration, piezoelectric, controller.

I. INTRODUCTION

Vibration is the most important problems in every part of engineering. There are desirable and undesirable vibrations. For controlling the undesirable vibrations two strategies can be adopted. One is passive vibration control and the other is Active vibration control. In Passive vibration control, dimensions and properties of the material is altered in order to reduce the vibration effect. But this increased the weight of the structure and the cost. Thus Active vibration control is preferred where the structure has additional materials like actuator, sensors and controller. Hence in recent years, there has been an increase in the demand of smart or intelligent structures in various engineering problems. The whole structure which comprises of the material, actuator/sensor and controller is together called smart structures. This has the high efficiency of reducing the vibration of a structure rapidly.

H. Karagulle, L Malagaca and H F Oktem [1] analyzed the active vibration control of a flexible beam containing piezoelectric patches using ANSYS. They used PID controller for controlling the vibrations of the cantilever beam and the circular plate. S. Narayanan and V. Balamurugan [2] proposed finite element modeling of piezolaminated smart structures for active vibration control with distributed sensors and actuators. Comparisons of various controllers which are used to reduce the vibrations were discussed *by them. Also they discussed about the effect of temperatures on the piezoelectric materials. Active stiffening of laminated composite beams using piezoelectric actuators was proposed by Haim Waisman and Haim Abramovich [3]. They presented that mechanical performance such as natural frequencies and mode shape can be significantly altered by inducing in-plane stresses due to piezoelectric actuators. They compared their FEA result with the published result and gave a good match.

II. LITERATURE SURVEY

S.X. Xu and T.S. Koko [4] presented the Finite element analysis and design of actively controlled piezoelectric smart structures. They developed a general purpose scheme, analyzing, designing actively controlled smart structure with piezoelectric sensor and actuators. Control of beam vibrations by means of piezoelectric

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devices: theory and experiments was given by Paolo Gaudenzi, Rolando Carhonaro and Edoardo Benzi [5]. They analysed the vibration reduction for an active cantilever beam considering a SISO control system. The sensing and actuating actions have been carried out by two identical PZT piezopatches connected at the beam in a collocated way. Young-Hun Lim, Vasundara V Varadan and Vijay K Varadan [6] proposed the closed loop finite element modeling of active structural damping in the frequency domain. They compared constant velocity feedback control and constant displacement feedback control. They concluded that resonance peak is altered without altering the structural mass or stiffness. S.H. Chen, Z.D. Wang and X.H. Liu [7] presented the active vibration control and suppression for intelligent structures. They controlled the vibration actively using negative feedback control law. Also they investigated the effect and dynamic stability of active vibration control for the intelligent structure by introducing state space equation.

In the present study, vibrations of flexible beam caused by disturbing force are actively controlled by using piezoelectric actuators. The controller used here is Linear Quadratic Controller (LQR). The LQR is designed in such a way to minimize the cost function and reduce the unwanted disturbances actively. The locations of the patches are varied and the optimal location is found out. Then the length of the patch is varied and its optimal length is found out. And again another controller called PID Controller is used and its performance is compared with that of the LQR controller.

III. METHODOLOGY

Lateral Vibrations of Beams

The beam is represented in r-y coordinates. $M(x,t)$ is the bending moment, $V(r,t)$ is the shear force and $f(r,t)$ is the external force per unit length of the beam. Euler- Bernoulli beam equation with external force is given by following equations

$$EI \frac{\partial^4 y(r,t)}{\partial r^4} + \rho A \frac{\partial^2 y(r,t)}{\partial r^2} = f(r,t) \quad \dots (1)$$

Piezoelectric Actuator

Active vibration control of flexible structures is done by using piezoelectric actuators. Piezoelectric materials possess the property by which they experience a dimensional change when an electric voltage is applied to them. The natural material is quartz crystal ($S_i O_2$). Artificial materials using ceramics and polymers such as PZT (lead zirconium titanate), PVDF (polyvinylidene fluoride) and LS (Lithium sulfate) also exhibit the piezoelectric phenomenon. When an electric voltage is applied to piezoelectric patch, it produces strain both in longitudinal and transverse directions.

Piezoelectric Strain Constant

This is the ratio of developed free strain to the applied electric field. Of particular importance are the strain constants d_{33} , d_{32} and d_{31} . The subscript d_{ij} implies that the voltage is applied or charge is collected in the i direction for the displacement or force in the j direction.

Consider a typical piezoelectric transducer, which has been poled in the three directions and is then subjected to an electric field along that direction.

For one-dimensional motion, the strain of the piezoelectric element in the z direction is given by,

$$\varepsilon^3 = \frac{d_{33}}{t_p} U(r,t) \quad \dots (2)$$

While the transducer is in the actuator mode, it will deflect in the x and y directions with the resultant strains

$$\varepsilon^2 = \frac{d_{32}}{t_p} U(r,t) \quad \dots (3)$$

$$\varepsilon^1 = \frac{d_{31}}{t_p} U(r,t) \quad \dots (4)$$

Considering the actuator mode strain along the x direction is given by the equation (3). Stress induced in the piezoelectric patch along the length.

$$\sigma = E_p \varepsilon = \frac{(E_p d_{31})}{t_p} U(r,t) \quad \dots (5)$$

Bending moment due to the stress along the piezoelectric patch is given by,

$$M_p = \frac{(E_p d_{31})}{t_p} U(r,t) w_p y dy \quad \dots (6)$$

$$M_p = E_p d_{31} w_p (t_b + t_p) U(r,t) / 2 \quad \dots (7)$$

$$M_p = C_a U(r,t) \quad \dots (8)$$

where

$$C_a = E_p d_{31} w_p (t_b + t_p) / 2$$

Assumed Mode Approach

When the piezoelectric actuator is patched on the beam, then the equation of beam is given by,

$$EI \frac{\partial^4 y(r,t)}{\partial r^4} + \rho A \frac{\partial^2 y(r,t)}{\partial r^2} = f(r,t) + \frac{\partial^2 M_p}{\partial r^2} \quad \dots (9)$$

$$EI \frac{\partial^4 y(r,t)}{\partial r^4} + \rho A \frac{\partial^2 y(r,t)}{\partial r^2} = f(r,t) + C_a \frac{\partial^2 U(r,t)}{\partial r^2} \quad \dots (10)$$

The main idea of the assumed modes approach is to expand the function $y(r, t)$ as an infinite series in the following form

$$y(r,t) = \sum_{i=1}^{\infty} \beta_i(r) q_i(t) \quad \dots (11)$$

where $\beta_i(r)$ are the Eigen functions or mode shapes satisfying ordinary differential equations and boundary conditions of the cantilever beam. Substituting $y(r,t)$ into the equation (10) then multiplying the Bernoulli-Euler equation (1) by $\beta_i(r)$ and integrating over $[0, L]$ we have,

$$\begin{aligned} EI \int_0^L \sum_{i=1}^{\infty} \beta_i''''(r) q_i(t) \beta_j(r) dr + \rho A \int_0^L \sum_{i=1}^{\infty} \beta_i(r) q_i''(t) \beta_j(r) dr \\ = \int_0^L f(r,t) \beta_j(r) dr + C_a \int_0^L \frac{\partial^2 U(r,t)}{\partial r^2} \beta_j(r) dr \end{aligned} \quad \dots (12)$$

Orthogonal properties are given by the following equations

$$\int_0^L \beta_j(r)\beta_j(r) \rho A dr = \rho AL^3 \delta_{ij} \quad \dots (13)$$

$$\int_0^L \beta_i''(r)\beta_j''(r)\rho A dr = \rho AL^3 \omega_i^2 \delta_{ij} \quad \dots (14)$$

Where δ_{ij} is the kronecker delta function, that is, $\delta_{ij} = 0$ for all i, j and is equals on if $i = j$. From these orthogonal conditions and by using the following equation, beam equation derived.

$$\beta_i''''(r) = \lambda^4 \beta_i(r) \quad \dots (15)$$

Bernoulli-Euler equation is modified to the following form,

$$\begin{aligned} EI \int_0^L \sum_{i=1}^{\infty} \lambda^4 \beta_i(r) q_i(t) \beta_j(r) dr + \rho A \int_0^L \sum_{i=1}^{\infty} \beta_i(r) q_i''(t) \beta_j(r) dr \\ = \int_0^L f(r, t) \beta_j(r) dr + Ca \int_0^L \frac{\partial^2 U(r, t)}{\partial r^2} \beta_j(r) dr \end{aligned} \quad \dots (16)$$

$$\begin{aligned} \rho A \int_0^L \sum_{i=1}^{\infty} \omega_i^2 \beta_i(r) q_i(t) \beta_j(r) dr + \rho A \int_0^L \sum_{i=1}^{\infty} \beta_i(r) q_i''(t) \beta_j(r) dr \\ = \int_0^L f(r, t) \beta_j(r) dr + Ca \int_0^L \frac{\partial^2 U(r, t)}{\partial r^2} \beta_j(r) dr \end{aligned} \quad \dots (17)$$

$$\begin{aligned} Ca \int_0^L \frac{\partial^2 U(r, t)}{\partial r^2} \beta_j(r) dr \\ = Ca \int_0^L [\delta'(r_1) - \delta'(r_2)] \beta_j(r) U(t) dr \\ = Ca [\beta_i'(r_1) - \beta_i'(r_2)] U(t) \end{aligned} \quad \dots (18)$$

$$\begin{aligned} \rho AL^3 (q_i''(t) + \omega^2 q_i(t)) \\ = \beta_i(r_w) F(t) + C_a [\beta_i'(r_1) - \beta_i'(r_2)] U(t) \end{aligned} \quad \dots (19)$$

Equation (19) is the equation of beam is a function of time variable. When the beam vibrations are controlled the controller term will not be there in the equilibrium equation. When the modal damping of the beam considered then the equation (19) transformed to the following form.

$$\begin{aligned} \rho AL^3 (q_i''(t) + 2\zeta \omega q_i'(t) + \omega^2 q_i(t)) \\ = \beta_i(r_w) F(t) + C_a [\beta_i'(r_1) - \beta_i'(r_2)] U(t) \end{aligned} \quad \dots (20)$$

Mode Shape

This gives the significance of the relative movement of each and every particle from its mean position at respective natural frequency of the beam. Beam will be having infinite number of natural frequencies. But only first four natural frequencies of the beams are considered and actively controlled. Considering equation (20) it is an ordinary differential equation of the order four. Solution of the differential equation is given by the complementary function of the equation. There are four roots for the above equation $\pm \lambda_1, \pm \lambda_1$. Therefore the solution is given by,

$$\beta_i(r) = C_1 e^{\lambda_i r} + C_2 e^{-\lambda_i r} + C_3 e^{i\lambda_i r} + C_4 e^{-i\lambda_i r} \quad \dots (21)$$

where C_1, C_2, C_3 and C_4 are the constant which are calculated from substituting the boundary conditions of the beam.

$$\beta_i(r) = A_1 \sin(\lambda_i r) + A_2 \cos(\lambda_i r) + A_3 \sinh(\lambda_i r) + A_4 \cosh(\lambda_i r) \quad \dots (22)$$

Where A, B, C and D are the constants which are calculated by using the boundary conditions of the beam. Boundary conditions of the beam at the free and forced end are to be considered to find the mode shape equation coefficients.

At fixed end: $y(0,t) = 0$,

$$EI \frac{\partial y(r,t)}{\partial r} = 0 \quad \dots (23)$$

At fixed end:

$$EI \frac{\partial^2 y(r,t)}{\partial r^2} = 0,$$

$$EI \frac{\partial^3 y(r,t)}{\partial r^3} = 0 \quad \dots (24)$$

After substituting the boundary condition into the solution of differential equation then the mode shapes are given by the following equation (25)

$$\beta_i(r) = L[\cosh(\lambda_i r) - \cos(\lambda_i r) - \frac{(\cos(\lambda_i L) + \cosh(\lambda_i L))}{(\sin(\lambda_i L) + \sinh(\lambda_i L))} * \sinh(\lambda_i r) + \sin(\lambda_i r)] \quad \dots (25)$$

The roots of the equation represent the value of λ_i . This is used to find the mode shapes at different points along the length of the beam at the respective natural frequencies of the beam.

$$1 + \cosh(\lambda_i L) \cos(\lambda_i L) = 0 \quad \dots (26)$$

Natural Frequency of the Beam

The natural frequency of the beam depends on the Young's Modulus of the beam, moment of inertia of the beam, mass density of the beam, cross sectional area of the beam and λ_i which is a factor depending on mode shape of the beam. By substituting all material properties of the beam, the frequencies of the beam are calculated. Using modal analysis in ANSYS 9.0 the frequency values are compared. The natural frequency of the beam is given by,

$$\omega_i = \sqrt{(EI / \rho A)} * \lambda_i^2 \quad \dots (27)$$

Sensor

Position sensors are used for active vibration control. Due to the external disturbances whatever the changes occurred in the beam, can be sensed using position sensors. Accelerometers can also be used for measuring end point acceleration. As the acceleration at the end point is a function of time integrating it, displacement of the beam can be measured.

State Variable Approach

Rather than directly transforming the differential equations which describe a dynamic system in to the Laplace domain, an alternative approach is to recast the time domain equations into a standard form, in terms of the internal state variables of the system. It is then possible to manipulate this state variable representation, using well established matrix methods, to derive a number of useful properties about the

system, such as its stability, its controllability and observability and the effect of various forms of feedback control.

State Space Representation

When the modal damping is not considered, then the equation of the beam for forced vibration is given as follows.

$$\rho AL^3(q_i''(t) + 2\omega^2q_i(t)) = \beta_i(L)F(t) + C_a[\beta_1'(r_1) - \beta_1'(r_2)]U(t) \quad \dots (28)$$

When the modal damping is also considered then the equation of beam for forced vibration condition is given by the equation (28)

$$\begin{aligned} &\rho AL^3(q_i''(t) + 2\zeta\omega q_i'(t) + \omega^2 q_i(t)) \\ &= \beta_i(L)F(t) + C_a[\beta_1'(r_1) - \beta_1'(r_2)]U(t) \end{aligned}$$

Let state is represented by matrix form,

$$x(t) = [q_1(t), q_1'(t), \dots, q_N(t), q_N'(t)] \quad \dots (29)$$

When the controller is not presented in the system, then the controller term will be zero. Thenspace equations are given as follows.

$$x'(t) = Ax(t) + B_f F(t) \quad \dots (30)$$

$$y(t,r) = C(r)x(t) \quad \dots (31)$$

$$y'(t, r) = D(r)x(t) \quad \dots (32)$$

When the controller is present in the system then the state space equations are given as follows.

$$x'(t) = Ax(t) + BU(t) + B_f F(t) \quad \dots (33)$$

$$y(t,r) = C(r) x(t) \quad \dots (34)$$

$$y'(t,r) = D(r) x(t) \quad \dots (35)$$

Where $y(t)$ and $y'(t)$ are displacement at the free end of the beam and velocity at the free end of respectively.

$$A = \begin{pmatrix} 0 & 1 \\ -\omega^2 & -2\zeta_n \omega_i \\ 0 & 0 \\ -\omega_n^2 & -2\zeta_n \omega_n \end{pmatrix}$$

$$B = \frac{C_a}{\rho AL^3} [0 \beta_1'(r_1) - \beta_1'(r_2) \cdot B_n'(r_1) - b_n'(r_2)]^T$$

$$B_f = \frac{1}{\rho AL^3} [0 \beta_1'(r) \dots 0 \beta_n(r)]^T$$

$$C = [\beta_1(r) 0 \dots \beta_n(r) 0]$$

$$D = [0 \beta_1(r) \dots 0 \beta_n(r)] \quad \dots (36)$$

Due to the external force the state is going to change. These changes in the state due to external disturbances are to be measured by the sensors and controlled by using actuators which are externally controlled by control system. Control voltage given to actuating piezoelectric patch and corresponding gain matrix are represented in state space form by the following equation. (37)

$$U(t) = -Kx(t) \quad \dots (37)$$

Substituting equation (37) in equation (33) we get,

$$x'(t) = Ax(t) - BK_x(t) + B_f F(t) \quad \dots (38)$$

$$x'(t) = (A - BK)x(t) + B_f F(t) \quad \dots (39)$$

$$x'(t) = A_c x(t) + B_f F(t) \quad \dots (40)$$

Equation (40) will represent the controlled state for the external disturbances. Equations(31) and (34) will represent uncontrolled and controlled responses respectively for the same input. Equations (32) and (35) will represent uncontrolled and controlled velocity at the free end of the beam for the same input.

Optimum Controller Design

Optimization refers to the science of maximizing or minimizing the objectives. Optimization requires a measure of performance. When mathematically formulated, this measure of performance is called the objective function. Optimization of control system is called optimal control.

Linear Quadratic Regulator

Optimal controller called linear quadratic regulator is considered, which is having a quadratic cost function of states and controls. The formulation of the linear quadratic regulator for a linear quadratic regulator for a linear system is as follows.

$$x' = Ax + Bu \quad \dots (41)$$

$$y = Cx \quad \dots (42)$$

A control function $u(t)$ has to be found that will minimize the cost function, J given by the equation (43)

$$J = \int_0^{\infty} \{x(t)^T Q x(t) + u(t)^T R u(t)\} dt \quad \dots (43)$$

The function inside the integral in equation (43) is a quadratic form and then matrices Q and R are usually symmetric. It is assumed that R is positive definite and Q is Positive semi definite. If R is very relative to Q , which implies that the control energy is penalized heavily, the control effort will diminished at the expense of larger values for the state. When Q is very relative to R , which implies that the state is penalized, the control effort rises to reduce the state, resulting in a damped system. Q and R respective weights on different states and they control channels respectively. Several procedures are available to solve the LQR problem. One approach to find a controller that minimizes the LQR cost function is based on finding the positive-definite solution of the following, Algebraic Riccati Equation (ARE).

$$A^T P - PA + Q - PBR^{-1} B^T P = 0 \quad \dots (44)$$

$$U = -KX, \quad K = R^{-1} B^T P \quad \dots (45)$$

Spatial LQR Control of a Laminate

The main concept behind this is to utilize the spatial property of the model of the laminate derived using the assumed mode approach. The system described by the equilibrium equation has a finite dimensional state vector since we consider the first 'N' modes of the beam. A spatial LQR controller attempts to minimize the vibration of the entire structure by minimizing a cost function that relates to the spatial behavior of the composite system.

The vibration of the beam Y_{tip} is measured by using a position sensor on the beam. This information is feedback to the controller, which applies control voltage $u(t)$ to the actuating piezoelectric patch.

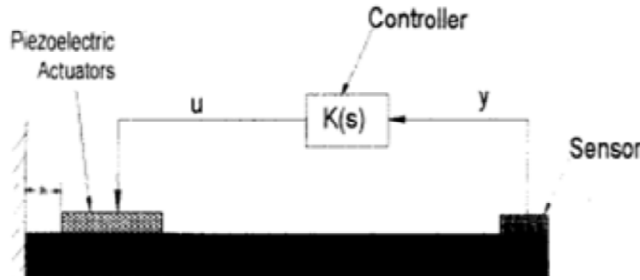


Figure 1: Cantilever beam with Actuator, Sensor and Control System

Cost Function Related to Total Energy of the Beam

LQR problem corresponds to the following cost function equation which is related to minimizing the total deflection of the beam. Therefore, by minimizing the elastic deflection cost function, it is possible to bring the beam to the rest position quickly and as smoothly as possible.

$$J = \int_0^\infty \left\{ \int_0^L [y(t,r)]^2 dr + R[u(t)]^2 \right\} dt \quad \dots (46)$$

If only vibrations on a certain region of the beam are of importance, then the quadratic cost function could be modified as follows.

$$J = \int_0^\infty \left\{ \int_0^L [y(t,r)]^T f(r) y(t,r) dr + [u(t)]^T R u(t) \right\} dt \quad \dots (47)$$

Where $\phi(r)$ is a weighing function emphasizing the region where the vibrations are to be minimized. In this case, the cost function will be,

$$J = \int_0^\infty \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} dt \quad \dots (48)$$

where

$$Q = C(L)^T f(r) C(L) \quad \dots (49)$$

Based on the $\phi(r)$ value, the vibrations can be reduced at a particular point or throughout the length of the beam. When $\phi(r)=1$, then the vibrations are controlled throughout the length of the beam, and the value of is given by

$$Q = \text{diag} (L^3, 0, \dots, L^3, 0) \quad \dots (50)$$

This type of controller design is superior to standard techniques, when the vibration reduction in a global sense is required. It attempts to design a controller which minimizes the elastic deflection along the length of the beam.

**Table 1
Parameters of the Beam and Patch**

<i>Beam material</i>	<i>Aluminum</i>
Young's Modulus (N/m ²)	6.75e10
Beam Length (m)	500e-3
Beam width (w _b)(m)	30e-3
Beam thickness (t _b)(m)	1.5e-3
Mass Density (kg/m ³)	2752.3
Moment of Inertia I = w _b t _b ³ / 12 (m ⁴)	8.4375E-012
Area of cross section of Beam A = w _b t _b ³ (m ²)	4.5E-005

Piezoelectric material	PZT
Young's Modulus (N/m ²)	139 x 10 ⁹
Patch Length (m)	50e-3
Patch width (w_p) (m)	30e-3
Patch thickness (t_p) (m)	0.5e-3
Charge constant (d31) (m/V)	11 x 10 ⁻¹¹
Voltage Constant (g31)(Vm/N)	0.010
Maximum electric Field (V/mm)	1

Initially we use **Lead Zirconate Titanate (PZT)** as a piezoelectric patch. The given values are to be substituted into the state space equation. Then it is transformed into transfer function in order to plot the response curves. The transfer function is calculated by generating code in MATLAB.

First the response at the free end of the beam due to step input at free end without the controller is plotted. Then the patch is placed in the beam. The location of the patch is varied continuously and the corresponding response is plotted. For all the variation of locations the Controller weighting matrix [R] is taken as $[6 \times 10^{-10}]$.

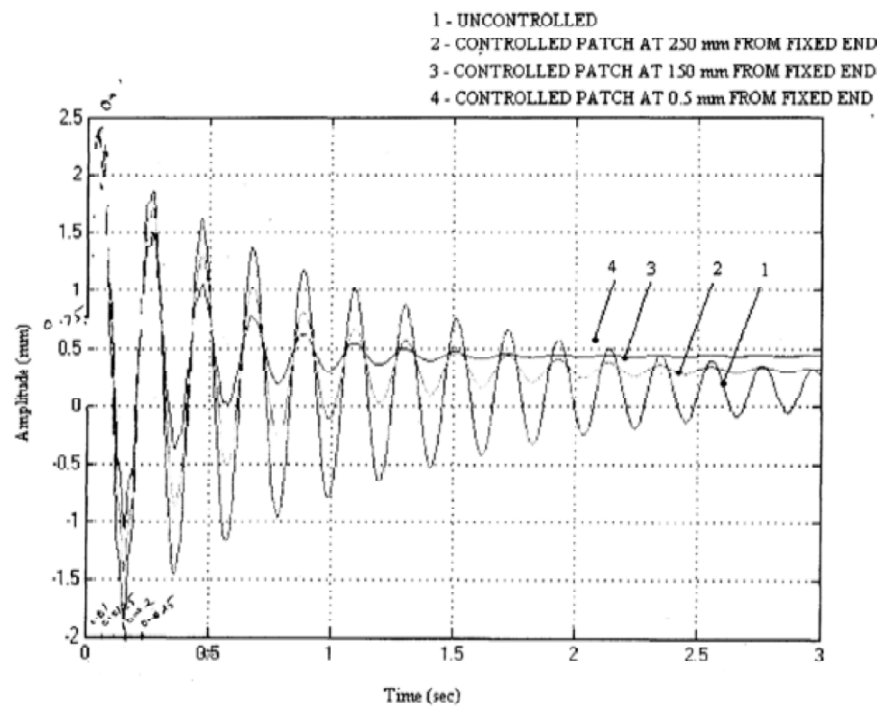


Figure 2: Step Response at the free end of the beam when the actuator (PZT) is placed at different locations along the length of the beam

Inference from the Graph

Without the controller, considering modal damping for the step input response plot is drawn. This response will decay after certain interval of time due to the material damping. But by considering controller with the minimization of the elastic deflections of the beam cost function these vibrations will be controlled in short interval of time. Also the effect of varying the location of the PZT patch is carried out and its effect can be clearly viewed from the response plot. At the particular time of 0.5 sec the amplitude will be Controlled Vibration Decay.

Table 2
For 0.5 sec the amplitude for various distance from fixed end

<i>Distance from fixed end (in meters)</i>	<i>Amplitude (mm)</i>
250E-3	0.993
150E-3	0.827
100E-3	0.779
75E-3	0.757
25E-3	0.714
1E-3	0.698

Hence from the above table we can clearly conclude that when the actuator is placed nearer to the fixed end the vibrations can be controlled very rapidly and the amplitude is reduced to a significant extent. The length of the piezoelectric patch is varied and the optimal length can be concluded from the obtained response plot. Here the piezoelectric patch used is PZT only. The Controller Weighting Matrix [R] is taken as $[6 \times 10^{-8}]$ for all cases.

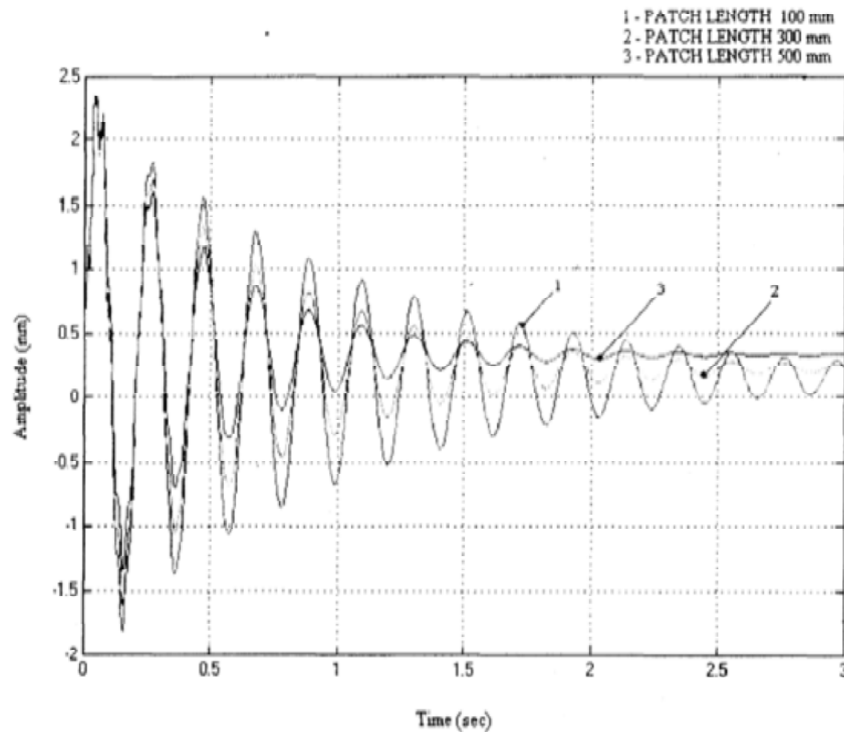


Figure 3: Step Response at the free end of the beam when the actuator (PZT) size is varied along the length of the beam

Inference from the Graph

Here the length of the piezoelectric actuator is varied along the length of the beam and its corresponding response is plotted and it can be clearly viewed that when the actuator length is more the vibrations are damped more frequently. For a particular time the amplitude is measured and it is tabulated.

Table 3
For 1 sec the amplitude for various patch length

<i>Length of the patch (in meters)</i>	<i>Amplitude (in meters)</i>
100E-3	0.422
300E-3	0.256
500E-3	0.0746

From the tabulated value, it can be inferred that the amplitude is very less in case of more patch length. Therefore full length patch can be used for rapid damping of vibrations. In this section, the length of the patch is fixed to be 50 mm and the location of the patch is placed from 10 mm from the fixed end. But here the piezo electric patch material is varied and its performance is measured from the response plot.

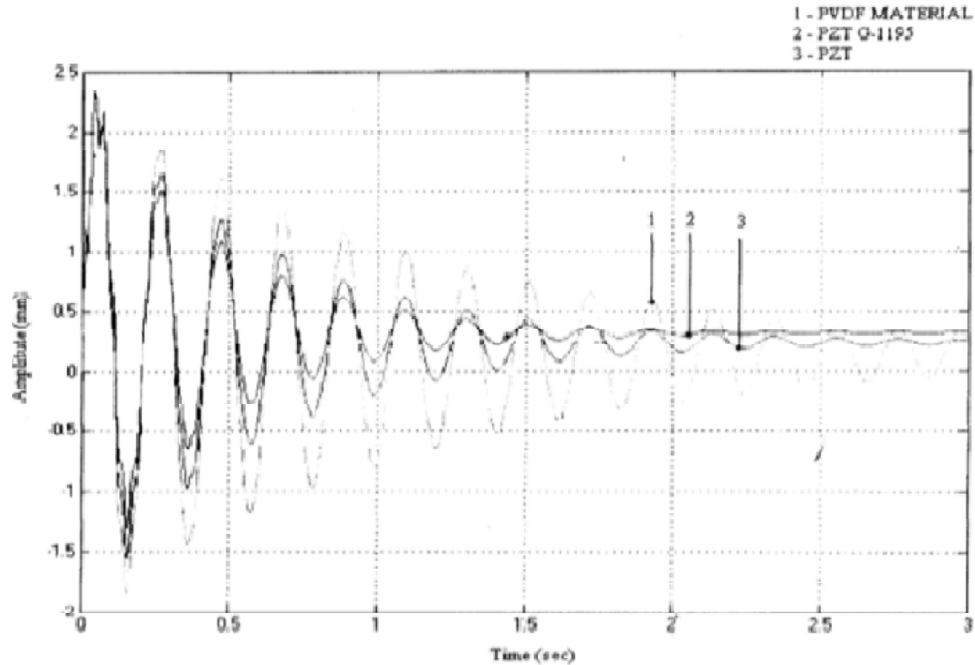


Figure 4: Step Response at the free end of the beam when the different actuator are along the length of the beam

Hence, it can be concluded from the response plot that PZT G-1195 is more efficient than other patches taken into consideration. In all the previous simulations the controller used is LQR and all the above are simulated using MATLAB using SIMULINK option. The Block diagram is shown below.

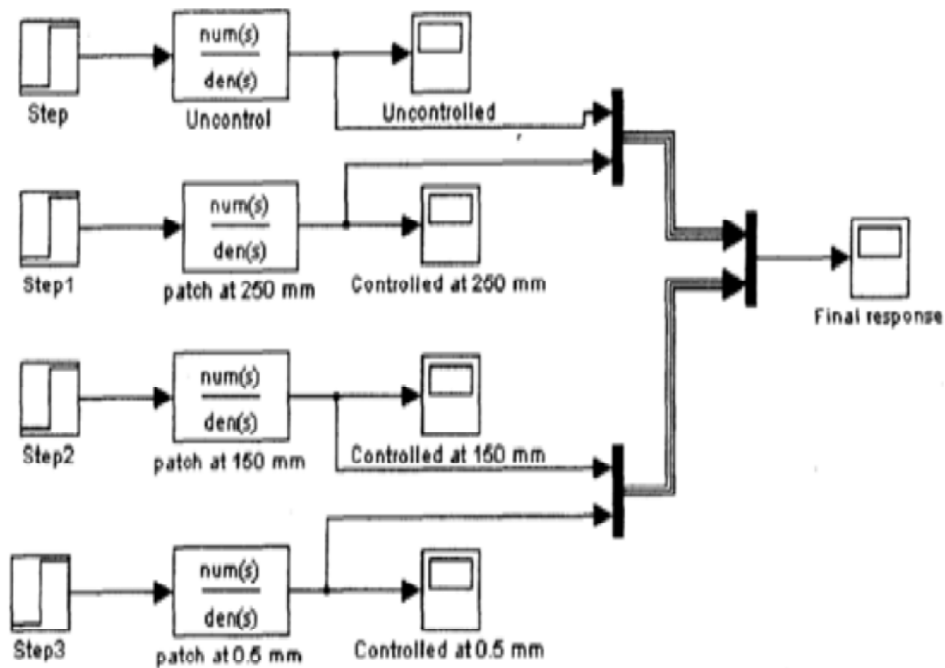


Figure 5: Simulink Block used for Simulation

In this section, instead of LQR controller PD controller is used and its performance is studied. For this controller the simulation is performed using ANSYS. The working of the PID controller is explained below. The strain is calculated at the selected sensor location and it is multiplied by K_s and then it is subtracted from zero. The zero value is the reference input value to control the vibration. The difference between the input reference and the sensor signal is called the error signal. The error value is multiplied by K_c and K_v and it is feed back into the actuator to damp the vibrations. Now for the same material parameters and using the same PZT patch the response is plotted in ANSYS.

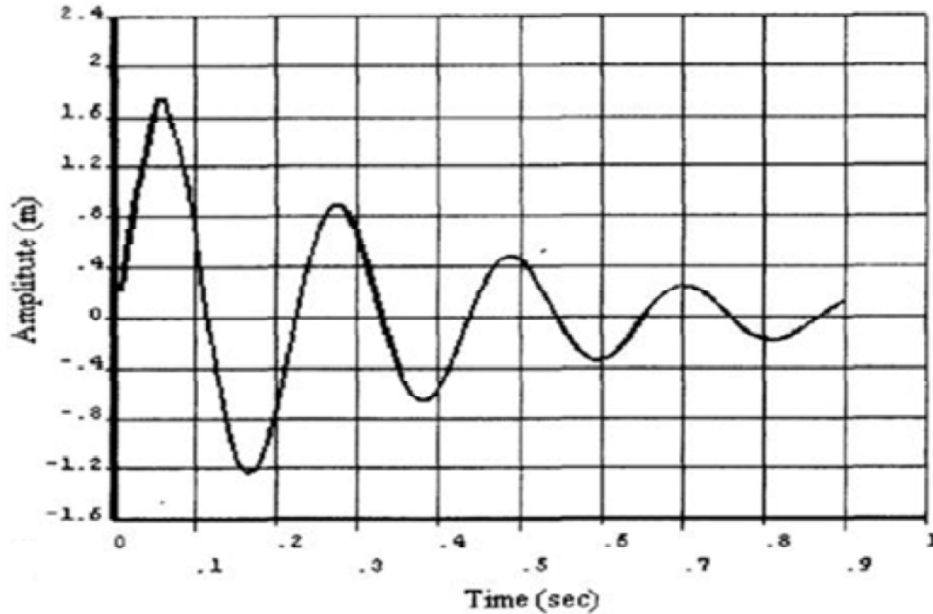


Figure 6: Response plot for PID Controller

The graph is plotted in POST 26 and the response can be viewed. From the graph it can be concluded that the amplitude is less than in LQR controller. Similarly for the same case the amplitude is less. Hence it can be concluded that PID controller is better than LQR controller.

IV. CONCLUSIONS

In this paper, the vibration caused by the step input on a Cantilever beam is controlled by using different piezoelectric actuators and based on the obtained result the conclusions are vibrations are actively controlled at the free end of cantilever beam using piezoelectric actuators. Optimal Linear Quadratic Regulator is designed for active vibration control based on the cost function related to minimization of elastic deflections of the beam. Vibrations are very rapidly controlled when the piezoelectric actuator is placed very near to the fixed end of the cantilever beam. Vibrations are very rapidly controlled when the length of the piezo electric actuator is fully along the length of the beam. Different piezo electric materials have been used and PZT G-1195 is found to be efficient than other actuators taken into consideration. When two controllers are used for the same configurations, LQR and PID controller, it is found from obtained result that PID controller is more efficient than LQR controller.

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