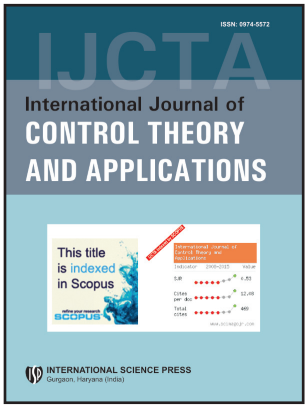

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Lattice Reduction Aided Pre-Processor for MIMO Detection

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Abstract: Multiple Input Multiple Output system has been adopted in various communication standards because of its link robustness and high throughput. Due to high computation complexity, one challenging tasks in MIMO systems is data detection at the receiver.

Lattice Reduction (LR) is an effective technique that can obtain better conditioned channel matrix by factorizing the channel matrix into the product of well-conditioned matrix and a uni-modular matrix thus increases the performance of sub optimal MIMO detector. Various LR algorithms are Lenstra, Lenstra, and Lovász (LLL) algorithm, Seysen's Algorithm (SA) and Brun's Algorithm. Among these algorithms SA finds the reduced basis with minimum number of iterations compared to the commonly used LLL algorithm. Also, SA based reduction of channel matrix, outperforms the existing LLL algorithm based reduction for sub optimal linear detectors in terms of bit error rate (BER) and computational complexity. The performance of Seysen's Algorithm (SA) for Lattice reduction (LR) is analyzed and the possibilities of architectural modification of SA for reduced complexity in MIMO data detection are exploited**.** MIMO detectors which improves the performance of these detectors.

***Keywords*:** Multiple Input Multiple Output (MIMO) system, Lattice Reduction (LR), Lenstra, Lenstra, and Lovász (LLL) algorithm, Seysen's Algorithm (SA).

1. INTRODUCTION

The higher data rate, better spectral efficiency and maximum degree of freedom of Multiple Input Multiple output (MIMO) technology has made it most robust technology for next generation wireless standards. In principle multiple transmit antennas increases the transmission rate, where multiple receiving antennas improves reliability and communication range. Even though MIMO system improves performance, designing a low complexity high performance detector in hardware is difficult.

Unlike traditional communication systems design, in which algorithmic and hardware developments can be separated, currently, engineers must have knowledge of both algorithms and architectures to realize such intricate MIMO baseband receivers. For algorithmic development, the performance of an algorithm can only be evaluated rigorously by actual implementation, especially from the complexity viewpoint

The important challenge in MIMO detection is to achieve the required performance with low computational complexity and high throughput. ML detectors achieve best BER performance and full diversity, whereas hardware implementation of Maximum Likelihood (ML) detectors is major concern. Linear detectors (LD) on other hand meet the computational complexity requirement but compromises on diversity and BER performance.

A significant improvement in performance of linear detectors is achieved if the channel matrix is preprocessed and made more orthogonal. Lattice Reduction is a powerful preprocessing technique. The channel matrix is considered as a basis of a lattice and LR attempts to find a more orthogonal basis for the same lattice [1]. Linear detection on this modified lattice improves the performance with affordable computational complexity.

Among the various LR algorithms available in the literature, Lenstra, Lenstra, and Lovász algorithm (LLL) algorithm has been used exclusively for LR-assisted data detection. The LLL algorithm allows linear detectors to exploit all of the available diversity [2], as well poses many silicon implementation challenges due to its high computational complexity and nondeterministic execution time.

Seysen's algorithm (SA) as shown in[2-4] is promising alternative for LLL algorithm in LR assisted MIMO detection, where lattice basis and its dual are reduced simultaneously to attain minimum Seysen's orthogonality measure in an efficient manner. More orthogonal lattice basis are obtained as Seysen's orthogonality defect tends to zero. It is shown in that the SA can achieve very good results in the sense of efficiently finding the shortest lattice basis vector for lattices of moderate size.

Despite these, SA has lower number of iterations and better error-rate performance over LLL algorithm. In this paper preprocessing stage of linear MIMO equalizer using SA and its VLSI implementation are presented.

2. MIMO DETECTION

2.1. MIMO System Model

Consider a MIMO system with N_T transmit and N_R receive antennas, where $N_T \geq N_R$. The N_T dimensional transmit vector is $x \in X_T^N$ and $X \in CZ$ corresponds to the underlying scalar complex constellation from a quadrature amplitude modulation (QAM) alphabet. The associated complex baseband input output relation is stated as follows.

$$r = \mathbf{Hx} + n \tag{1}$$

where, 'r' is the N_R dimensional receive vector, 'H' stands for the $N_R \times N_T$ complex valued channel matrix, and 'n' is N_R dimensional noise vector. The task of the MIMO detector is to compute an estimate based on the received vector r and knowledge of the channel matrix H.

A. Lattice Reduction

The goal of Lattice Reduction is to generate a more orthogonal lattice basis B such that B = HT where T is a $N_R \times N_T$ uni-modular matrix, i.e., $|\det(T) = 1|$. SA performs the above operation on the MIMO channel matrix. The constellation of the received symbol vector is relaxed to $x \in CZ_T^M$ [5,6], and the input output relation get modified as

$$r = Bs + n \tag{2}$$

with, $s = T^{-1}x$, the reduced lattice basis B facilitates the search for the lattice point that is closest to r.

2.2. Lattice Reduction Algorithms

There are three sub optimal LR algorithms that can be used for MIMO detection: (1) Lenstra, Lenstra, and Lovász algorithm (LLL algorithm); (2) Seysen's algorithm (SA); and (3) Brun's algorithm. Among these, the LLL algorithm is the most popular LR method. However, LLL poses many implementation challenges due to its high computational complexity and non-deterministic execution time[7,8]. Another important LR algorithm is SA, which has been recently applied to the MIMO detection problem. SA performs the size reduction globally compared to the local pairwise reduction approach of LLL. Brun's algorithm can be applied to poorly conditioned channel matrices and have much lower complexity than LLL and SA, but comes at the expense of poor performance. This paper exploits the possibility of using SA for MIMO detection preprocessing.

2.3. Seysen's Algorithm

Orthogonality measure of SA named Seysen's metric is given as

$$S(H) = \sum_{i=1}^{MT} H_i H_i^{\#} \tag{3}$$

SA iteratively decreases S(H) to obtain a reduced lattice basis. In it was shown that the computational complexity and memory requirements are reduced if SA is performed on the Gram matrix $G = H^H H$ equivalent of H and its dual $G^{\#} = G^{-1}$.

Each iteration of SA performs the following four steps

1. **Computation of Update Values** λ: Computes the complex update values λ for all index pairs i and j with $1 \le i, j \le N_T$ and $i \ne j$ as

$$\lambda_{i,j} = \mathbf{1/2(\{G^{\#}_{j,i}\, G_{j,j} - G_{j,i} G^{\#}_{i,i}\}/G^{\#}_{i,i} G_{j,j})} \tag{4}$$

   If all the update values are zero the algorithm terminates.

2. **Computation of Improvement values Δ:** For every update value λ, the corresponding improvement value Δ is computed.

$$\Delta_{i,j} = \mathbf{-2(G_{i,i} G^{\#}_{i,i}\, |\lambda_{i,j}|^2 - G_{j,j} R\{\lambda_{i,j} G^{\#}_{j,i}\} + G^{\#}_{i,i} R\{\lambda_{i,j} G^{\#}_{i,j}\})} \tag{5}$$

   If all the improvement values are zero the algorithm terminates.

3. **Update Index pair selection:** According to the index selection scheme, an index pair {s, t} resulting in largest reduction of Seysen's Metric is chosen

$$\boldsymbol{s, t = \arg_{i,j} \max \Delta_{i,j}} \tag{6}$$

4. **Basis Updation:** The basis is updated with λ on the selected index pair

$$\begin{aligned} &\mathbf{G'_{s,j} = G_{s,j} + \lambda^{\#}_{s,tj} G_{t,j}} \quad \boldsymbol{j < s} \\ &\mathbf{G'_{j,s} = G^{*}_{s,j}} \quad \boldsymbol{j > s} \\ &\mathbf{G'_{s,s} = G_{s,s} + 2R\{\lambda^{*}_{s,t} G_{t,s}\} + |\lambda^{\#}_{s,t}|^2\, G_{t,t}} \end{aligned} \tag{7}$$

   And on dual of gram matrix $\mathbf{G^{\#}}$

$$\begin{aligned} &\mathbf{G'^{\#}_{s,j} = G^{\#}_{t,j} - \lambda_{s,t} G^{\#}_{s,j}} \quad \boldsymbol{j < t} \\ &\mathbf{G'^{\#}_{t,j} = G'^{\#}_{t,j}} \quad \boldsymbol{j > t} \\ &\mathbf{G'^{\#}_{t,t} = G_{t,t}{}^{\#} - 2R\{\lambda_{s,t} G^{\#}_{s,t}\}\, |\lambda_{s,t}|^2\, G^{\#}_{s,s}} \end{aligned} \tag{8}$$

$$t'_s = t_s + \lambda^{\#}_{s,t}\, t_t$$
$$t_t^{-1} = t_t^{-1} - \lambda_{s,t}\, t_s^{\#-1} \quad (9)$$

If all λ or Δ values becomes zero the algorithm terminates.

3. ARCHITECTURE

The MIMO receiver considered in this paper is shown with the preprocessing[9] unit in Figure 1. The received data symbol is synchronized to the receiver domain, and is then demodulated and passed to pre computation unit, which consist of three sub blocks (1) Channel estimation (2) Matrix preprocessing and (3) Equalizer[10,11]. Preprocessing unit compute all necessary operation based on channel state information for detection phase.

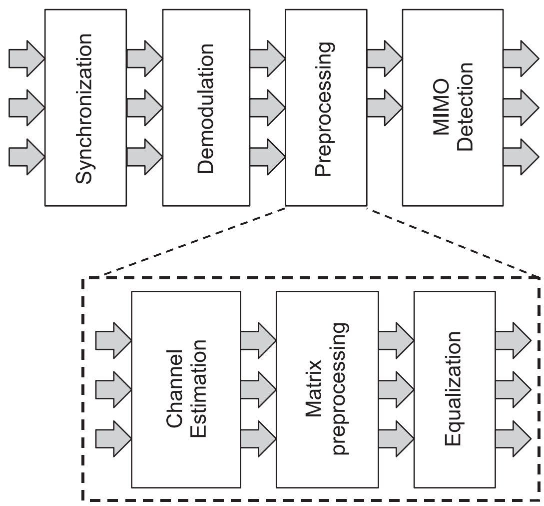

Figure 1: Typical MIMO receiver

The complete preprocessing unit using Seysen's algorithm is illustrated in Figure 2.

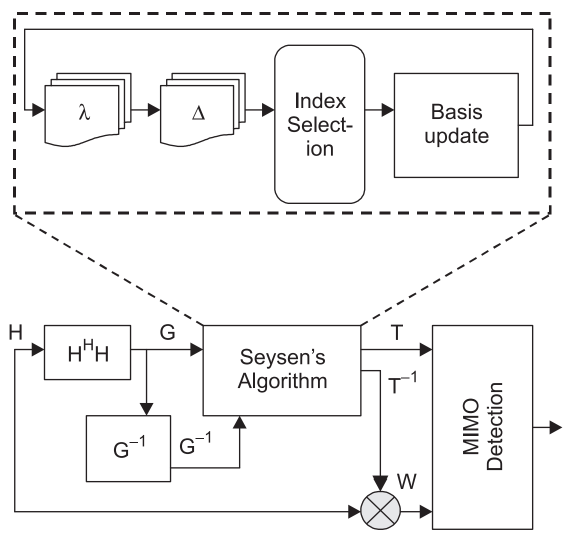

Figure 2: SA based LR preprocessing unit

Lattice reduction using Seysen's algorithm is carried over gram matrix 'G' and it's inverse. Initial step in preprocessing is to calculate gram matrix and its dual from channel matrix by multiplying Hermitian transpose of channel matrix with channel matrix itself.

SA computation blocks consists of four sub blocks (1) Lambda calculation block (2) Delta calculation block (3) Index selection block and (4) Basis update block.

1. *Lambda Calculation block:* All possible λ update values are calculated from elements of gram matrix based on (4). The dynamic range of the update coefficient $\lambda_{i,j} = a + jb$ is limited to $a, b = (-1, 0, 1)$ to reduce the computational complexity. The division in (4) can be removed by setting

$$R\{\lambda_{i,j}\} = 0 \text{ if } |R\{G^{\#}_{j,i}G_{j,j} - G_{j,i}G^{\#}_{i,i}\}| = 0$$
$$R\{\lambda_{i,j}\} = -1 \text{ if } |R\{G^{\#}_{j,i}G_{j,j} - G_{j,i}G^{\#}_{i,i}\}| < G_{j,j}G^{\#}_{i,i}$$
$$R\{\lambda_{i,j}\} = +1 \text{ if } |R\{G^{\#}_{j,i}G_{j,j} - G_{j,i}G^{\#}_{i,i}\}| \geq G_{j,j}G^{\#}_{i,i}$$

This modification is applicable to imaginary part of lambda as well. Since $a, b = (-1, 0, 1)$ all multiplication in $\lambda_{i,j}$ stage can be replaced with conditional addition or subtraction. Elimination of complex multiplication and division reduces the hardware complexity. Depending on the number of clock cycles available for the unit, multiple instances of the λ calculation unit are instantiated in parallel to compute multiple candidates in one cycle. Figure 3 shows a modified λ calculation unit.

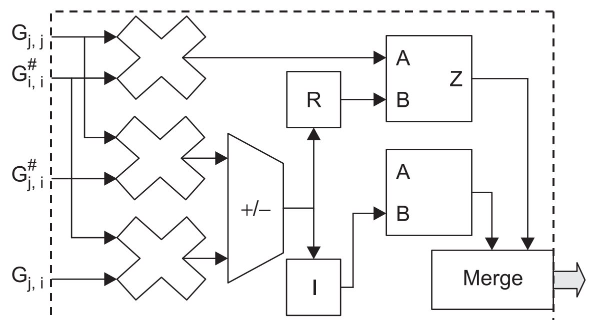

Figure 3: Division free λ calculation unit

2. *Delta Calculation block:* The computed Lambda update values are fed into the following delta calculation block, where all possible improvements $\Delta_{i,j}$ of Seysen's metric for the actual basis are calculated. As the dynamic range of $\lambda_{i,j}$ is restricted the computation steps in $\Delta_{i,j}$ also gets reduced. The multiplication in (5) can be replaced by conditional addition/subtraction.

The main computational block of the $\Delta_{i,j}$ calculation unit is shown in Figure 4. Here complex multipliers are replaced with conditional adders or subtraction unit.

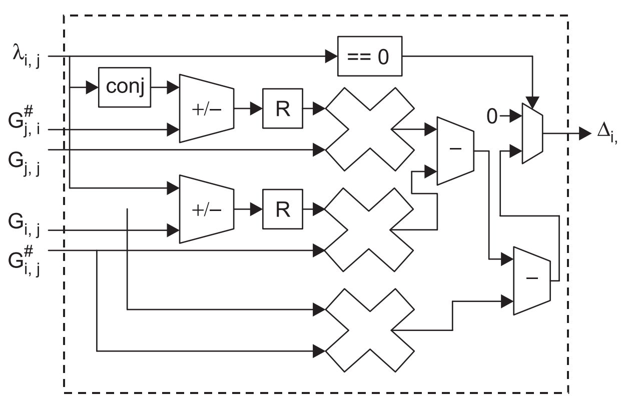

Figure 4: Δ calculation unit

3. *Index selection block:* After the computation of all $\Delta_{i,j}$ a greedy selection is performed using (6) Thereby the locally optimal index pair with the largest reduction of Seysen's metric is chosen. The selected indices and the corresponding update value are then forwarded to the matrix update unit.

4. *Matrix update block:* Updation of the original basis is performed based on the selected indices (s, t) and $\lambda_{i,j}$ using (7), (8), (9). Since λ values are limited to $(-1, 0, +1)$, there is No. complex multiplications in the matrix update block.

Once the maximum improvement on Seysen's metric is achieved or if the specified iteration limit defined in the SA is reached, the LR for the specific channel matrix is terminated and the transformation matrices T and $T^{\#}$ are forwarded to the detector block. Iteration is computed by feeding back the transformation matrices and the updated matrices G and $G^{\#}$ to the lambda updation block.

4. SIMULATION RESULTS

The number system, in turn the number of bits chosen has a large impact on the BER. All the hardware blocks in the architecture use 32 bit IEEE floating point format. Simulation results of various blocks of SA are given. The simulation is carried out using Mentor graphics Model Sim 10.1.

Figure 5 shows the gram matrix calculation from random channel matrix value by multiplying Hermitian transposed channel matrix with channel matrix itself.

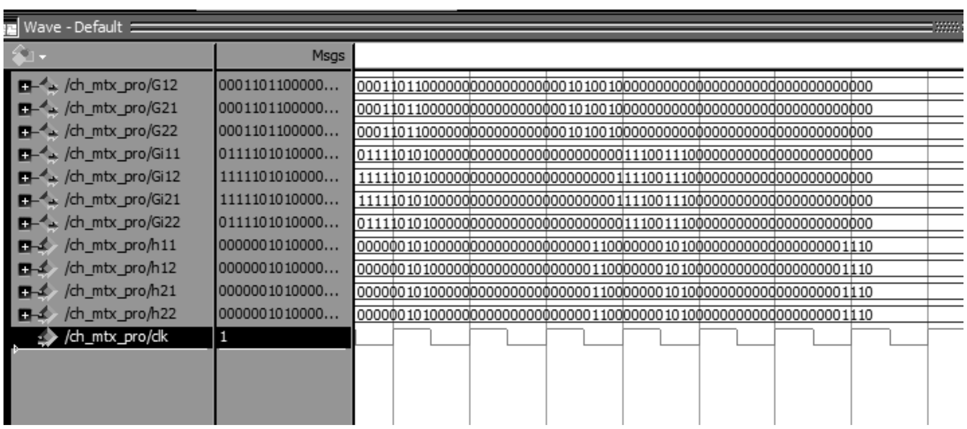

Figure 5: Gram matrix computation result

Lambda calculations are computed over resulting gram matrix and it's dual. Typical lambda results are shown in Figure 6.

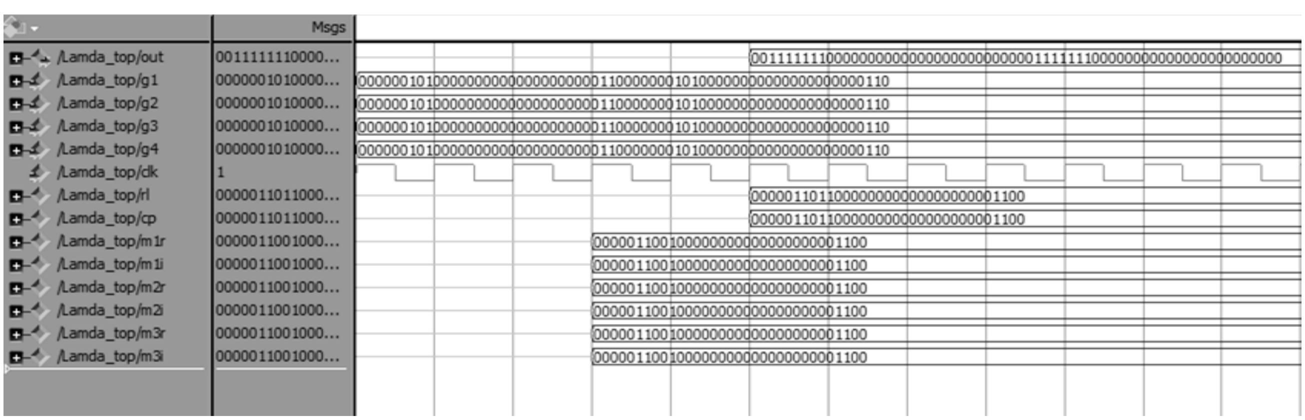

Figure 6: λ simulation based on gram matrix

All possible delta values are calculated for each lambda value until further optimum Seysen's metric is not possible. A sample delta calculation scheme is illustrated in Figure 7. The simulation is carried out until particular iteration limit is reached according to Seysen's algorithm.

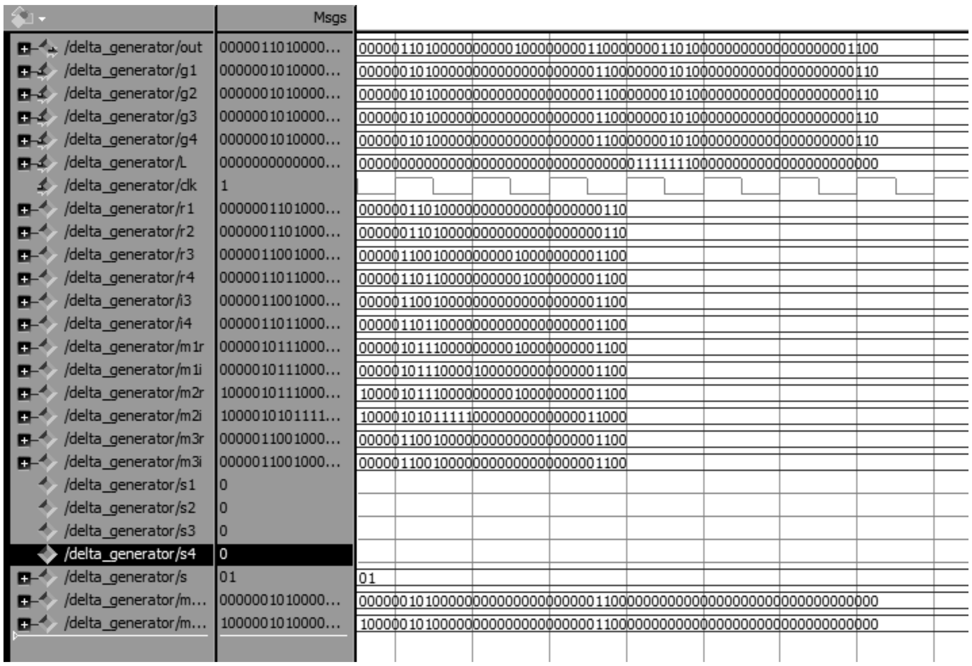

Figure 7: Δ simulation result

The functionality of each block is verified by simulation.

5. ASIC IMPLEMENTATION

The proposed VLSI architecture after verification for functionality is synthesized using Cadence Encounter tool. The synthesis results are tabulated in Table 1.

Table 1
Synthesis report

Implementation	*ASIC*
Technology	180 nm CMOS
Architecture	Parallel processed & Pipelined
Gate count	110 kGE
Clock	250 MHZ
Power	4.7 mW

As the placement and routing has not yet completed it is not possible to compare the results with existing preprocessing cores.

6. CONCLUSION

In this paper complexity reduced LR based preprocessing core for MIMO detection is proposed. The dynamic range of various parameters of SA is limited and the complexity is reduced. The various blocks of the algorithm are parallel processed and internal units of the blocks are pipelined to improve the performance.

The BER achieved due to the algorithm modification has to be analyzed by combining the preprocessing core with linear detector. Also the placement and routing of the preprocessor using Cadence Encounter tool is in progress.

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