

# Robustness of PI Controller for Different Fractional Values of an Integrator for a Conical Tank Process

N. Gireesh\* and G. Sreenivasulu\*\*

**Abstract:** Fractional calculus is more popular in these days. These concepts are applied to control systems. Fractional order PID controller is an enhancement of conventional PID controller. Fractional order PI controller is used in this paper to control the output variable of the process. In this paper the conical tank was used as a process. Height of the tank acts as output variable. In this paper the controller parameters (proportional gain and integral gain) are calculated by using conventional controller tuning methods like Zeigler Nichols, Cohen-Coon, Chein-Hrones-Reswick and Kappa-Tau methods and the response of the process is studied for various fractional value of an integrator ( $\lambda$ ) of a controller. The performance of the fractional order controller is compared for different values of  $\lambda$  and also studied the robustness of the controller for a change in 25% and 40% time constant of the process.

**Keywords:** Performance Evaluation, Conical tank, Fractional order controller, Tuning methods, Robustness

## I. INTRODUCTION

In process control applications, control of process parameters is a challenging job. The important reason is process parameters are uncertain and time-varying unmeasured variables, delay in inputs and measured outputs, constraints on manipulated variables and load or set point variations. Maintaining the height of the liquid in the tank is a problem in Process control industries. If height is too low or too high problems may arise because of spillage of material or improper chemical reaction or penalty for sequential operations. In this paper conical tank is used as a plant and mathematical model of the conical tank is nonlinear. Radius of the tank is different at different locations. Control theory concepts have been used to maintain the response at desired value. Majority control theory deals with design of controllers for linear process. PID controllers are widely used in Process control industries because they are well known and simple to the field operator. PID controller proved to be the best perfect controller for simple and linear processes. Practically all the systems are not linear. Desired characteristic of the process can be improved by introducing the fractional values in the PID controller.

Last few years research on fractional calculus has been increased. By extending the ordinary differential equations fractional order differential equations can be obtained. It has numerous applications in the field of control system because the advancements in computation power allow simulation and implementation of systems with adequate precision. Podlubny proposed a generalization of the PID controller, namely the  $PI^{\lambda}D^{\mu}$  controller [1]. The FOPID controller has by five parameters and the parameters are the proportional gain, the integral gain, the derivative gain, the derivative order and the integrating order. Different methods have been proposed for tuning of fractional order controller. Pole distribution (Petras, 1999), frequency domain approach (Vinagre, Podlubny, Dorcak & Feliu, 2000), state-space design (Dorcak, Petras, Kostial & Terpak, 2001), two-stage or hybrid approach (Chengbin & Hori, 2004), Ziegler-Nichols rules (Valerio, D. & Sa Da Costa, J., 2006)[3], Fractional Ms constrained integral gain optimization (YangQuan Chen,

\* Department of E. I. E., Sree Vidyanikethan Engineering College, Tirupati, India

\*\* Department of E. C. E., S. V. U. College of Engineering, Tirupati, India

Tripti Bahskaran & Dingyu Xue, 2008)[4], Optimization based tuning (Fabrizio Padula & Antonio Visioli, 2011)[5].

In this paper the response of the fractional order controller for the different values of  $\lambda$  was studied and proportional gain and integral gain values of the fractional order controller were calculated using Zeigler-Nichols, Cohen Coon, Chein-Hrones-Reswick and Kappa-Tau methods. Time domain specifications and performance indices Integral square error (ISE), Integral absolute error (IAE) and Integral time absolute error (ITAE) are compared for different tuning methods with different values of  $\lambda$ .

This paper is organized as following. Fractional calculus briefly discussed in section 2. Experimental setup and Mathematical modeling the process discussed in section 3 and section 4. System identification is discussed in section 5. Integer order and fractional order controller were discussed in section 6. In section 7 tuning methods are discussed. Computer simulations and results are given in section 8 and section 9 ends with conclusion.

## II. FRACTIONAL CALCULUS

In mathematics calculus is one of the main topic to discuss on limits, derivatives and integration. Normally it deals with integer order. More than 300 years back Leibniz (1665) was addressed on fractional order. Many mathematicians contribute in the field of fractional calculus, including Euler's, J.L.Lagrange, P. S. Laplace, J. B. J. Fourier, N. H. Abel, G.F.B Reimann, Oustaloup, I. Podlubny, etc [2, 18]. Fractional calculus [6] is a generalization of integration and differentiation to non-integer order operator  ${}_a D_t^\alpha f(t)$ , where  $a$  and  $t$  denote the limits of the operation and  $\alpha$  denotes the fractional order.

$${}_a D_t^\alpha f(t) = \frac{d^\alpha f(t)}{[d(t-a)]^\alpha} \quad (1)$$

$${}_a D_t^\alpha f(t) = \begin{cases} \frac{d^\alpha f(t)}{dt^\alpha} & ; \quad \Re(\alpha) > 0 \\ 1 & ; \quad \Re(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & ; \quad \Re(\alpha) > 0 \end{cases} \quad (2)$$

Riemann-Liouville (RL) and the Grunwald–Letnikov (GL) definitions are commonly used for the general fractional differentiation and integration.

GL definition is

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\lfloor \frac{(t-a)}{h} \rfloor} (-1)^j \binom{\alpha}{j} f(t - jh) \quad (3)$$

RL definition is

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (4)$$

where  $n$ ,  $h$  and  $\lfloor \frac{(t-a)}{h} \rfloor$  are integers.

## III. EXPERIMENTAL SETUP

Experimental setup of a conical tank system consists of a conical tank, a water reservoir, motor, rotameter, a level transmitter, an electro pneumatic converter (I/P converter), a pneumatic control valve, an interfacing data acquisition module and a Personal Computer (PC). The level transmitter output is interfaced with computer through NI USB 6008 data acquisition module. This module supports 8 analog input and 2 analog output channels with the voltage range of  $\pm 10$ volts. The sampling rate of the analog input 10K samples per sec with 12-bit resolution. Figure 1 shows the real time experimental setup and figure 2 shows the block diagram of the experimental setup of the process. Table I shows the technical specifications of the setup.



Figure 1: Experimental setup of conical tank system

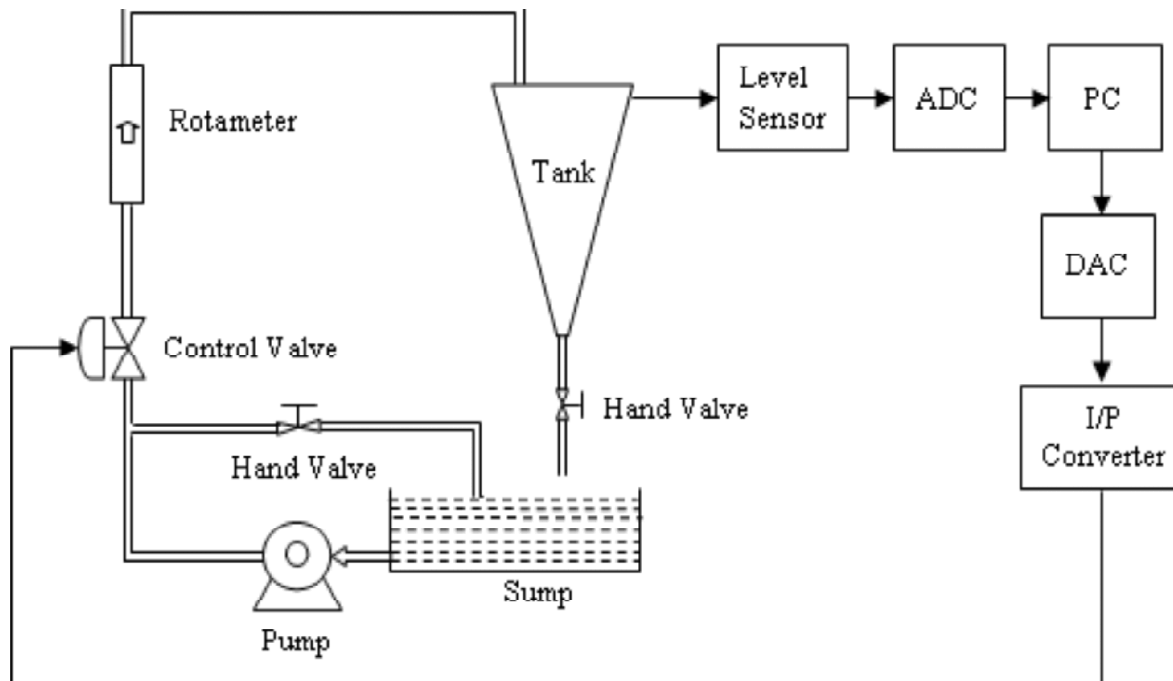


Figure 2: Block diagram of the experimental setup

**Table I**  
**Technical Specifications of the Setup**

<i>Part Name</i>	<i>Details</i>
Conical tank	Stainless Steel Height – 35cm Top diameter -13cm Bottom diameter – 0.8cm
Level Transmitter	Capacitive Type Range – 550mm Output – 0-5VDC
Pump	Single Phase AC motor Centrifugal regenerative 0.5HP
Control Valve	Equal percentage valve Air to open Size-1/4"
Rotameter	Range – (0-1200)LPH
Electro pneumatic converter	Input – (4-20)mA Output – (3-15)psi Supply – 20psi

#### IV. MATHEMATICAL MODELING OF PROCESS

The schematic diagram of the conical tank system is shown in Figure 3. The mathematical model [7, 9, 10] of the conical tank can be determined by considering level as the control variable and inflow to the tank as the manipulated variable.

The operating parameters are

$F_{in}$  – Inflow rate of the tank

$F_{out}$  – Outflow rate of the tank

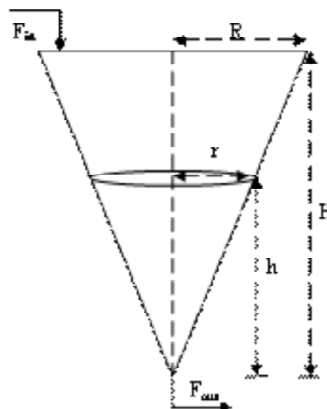
$H$  – Total height of the conical tank.

$R$  – Top radius of the conical tank.

$h$  – Liquid height in the tank.

$r$  – Radius of the liquid in the tank.

$K$  – Valve coefficient



**Figure 3: Schematic diagram of conical tank level system**

The conical tank area of cross section is

$$A = \pi r^2 \quad (5)$$

$$\tan \theta = \frac{r}{h} = \frac{R}{H} \quad (6)$$

$$r = R * \frac{h}{H} \quad (7)$$

$$A = \pi \frac{h^2 * R^2}{H^2} \quad (8)$$

According to mass balance equation, accumulation rate = rate of inflow – rate of outflow.

$$A \frac{dh}{dt} = F_{in} - F_{out} \quad (9)$$

$$F_{out} = K\sqrt{h} \quad (10)$$

using (4) & (6) in (5),

$$\frac{dh}{dt} = \frac{F_{in} - K\sqrt{h}}{A} \quad (11)$$

Linearize the equation around  $h = h_s$  using Taylor's series

$$f(h, F) = \frac{F_{in} - K\sqrt{h}}{A} \quad (12)$$

By Taylor's expansion,

$$f(h, F) = \frac{F_{in} - K\sqrt{h}}{A} + \frac{1}{A} (F - F_{ins}) - \frac{K(h - h_s)}{2A\sqrt{h_s}} \quad (13)$$

The first term on RHS is zero, because the linearization is about a steady state point

$$f(h, F) = \frac{1}{A} F' - \frac{Kh_s'}{2A\sqrt{h_s}} \quad (14)$$

$$\frac{dh_s'}{dt} = \frac{1}{A} F' - \frac{Kh_s'}{2A\sqrt{h_s}} \quad (15)$$

This is similar to the first order equation. The transfer function of the system is

$$\frac{h(s)}{F_{in}(s)} = \frac{k}{\tau s + 1} \quad (16)$$

Where,

$$\tau = \frac{2A\sqrt{h}}{K} ; \quad k = \frac{2\sqrt{h}}{K}$$

## V. SYSTEM IDENTIFICATION

From the mathematical modeling, the process is first order system. For dynamic analysis first order plus dead time model is used. To obtain this model controller is disconnected to the experimental setup and operated in open loop. Initially the hand valves are at mid position. Applied fixed input flow rate of the water to the tank. The height of the water in the tank reached at a steady state value. When there is a step change in the input flow rate the output reached a new steady state value. The process gain is calculated using Ziegler-Nichols open loop method. Identification of inflection point is difficult in Ziegler-Nichols open loop method. Sundaresan and Krishnaswamy [8] proposed dead time and time constant for to avoid the difficulty of identification of inflection point. The proposed times  $t_1$  and  $t_2$  are estimated from step response corresponds to 35.3% and 85.3% response times.

Process gain,  $K = \frac{\text{New steady state} - \text{Initial steady state}}{\text{change in input}}$

Time constant,  $\tau = 0.67(t_2 - t_1)$

Dead time,  $L=1.3t_1 - 0.29t_2$

The transfer function of a real time experiment setup is  $G(s) = \frac{12 e^{-2.05s}}{53.6s+1}$  at an operating condition of 675-750 LPH.

## VI. CONTROLLERS

### A. Integer order controller

The transfer function of the integer order PID controller is

$$G_c(s) = K_c \left( 1 + \frac{1}{\tau_i s} + \tau_d s \right) \quad (20)$$

Where,  $K_c$  = proportional gain

$\tau_i$  = integral time

$\tau_d$  = derivative time

The response is faster as, proportional gain increases. At steady state offset error will be obtained. Steady state error can be minimized by increasing the gain, sometimes it goes to unstable for large variation in the proportional gain. The offset error can be eliminated and the steady state error reaches to zero by adding the integral control. Derivative control reduces the overshoot and improves the rise time.

### B. Fractional order controller

The fractional order PID controller transfer function is

$$G_c(s) = K_c + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (21)$$

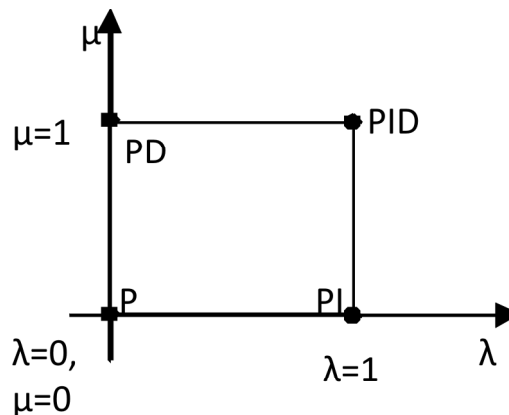


Figure 4: General Form of fractional order PID controller

If  $\lambda = \mu = 1$ , the controller is an integer order PID controller. The general form of fractional order PID controller is shown in Figure 4. Normally the values of  $\lambda$  and  $\mu$  lie between 0 and 1 or 1 and 2.

In this paper the PI controller is used by assuming  $K_d$  is zero.

## VII. CONTROLLER TUNING METHODS

Ziegler Nichols, Cohen-Coon, Chien-Hrones-Reswick and Kappa-Tau methods [11] are used in this paper to tune the controller parameters  $K_c$  and  $\tau_i$ . Two classical methods proposed by Ziegler Nichols one is open

loop method and another is closed loop method [12]. These are more popular methods. The parameters are calculated based on gain, time constant and delay of the FOPDT model in the first method. In the second method the parameters are calculated based on ultimate gain and ultimate period. Poor performance for process with dominant delay and large overshoot are the main disadvantages. Cohen and Coon [13] design method is the second popular method after Ziegler Nichols method. This method is similar to the Ziegler Nichols reaction curve method in that it makes of the FOPDT model to develop the tuning parameters. The controller settings are based on the three parameters  $K$ ,  $L$ , and  $\delta$  response of the open loop step. The main design requirement is the rejection of load disturbances. Chien-Hrones-Reswick (CHR) method [14] of tuning was developed from the Ziegler Nichols open loop method for better performance of response speed and overshoot. The quickest aperiodic response is labeled with 0% overshoot and the quickest oscillatory process is labeled with 20% overshoot. Kappa-Tau method [15] is developed based on dominant pole design with criterion on the rejection of load disturbance and constraints on the maximum sensitivity ( $M_s$ ). 1.2 to 2 is the range of typical values of  $M_s$ . Larger values of  $M_s$  give systems that are less robust but faster. This method gives good tuning for processes with long dead time.

For fractional order controller, the value of the fractional order ( $\bar{\epsilon}$ ) was varied for different values. If the order is one then the controller becomes the integer order. The order was varied between 0 and 1 and between 1 and 2. The other parameters  $K_c$  and  $\delta_i$  are taken from the integer order controller.

The PI controller tuning rules [16] of the above mentioned methods for the FOPDT model  $G(S) = \frac{K e^{-LS}}{\tau S + 1}$  is given in the Table II.

**Table II**  
**Different Tuning Methods for Integer Order PI Controller**

Tuning Methods	$K_c$	$\tau_i$
Ziegler Nichols (Open Loop)	$\frac{0.9\tau}{KL}$	3.33L
Cohen-coon	$\frac{1}{K} \left(\frac{\tau}{L}\right) \left[1 + \frac{L}{3\tau}\right]$	$L \left[ \frac{(30 + \frac{3L}{\tau})}{(9 + \frac{20L}{\tau})} \right]$
CHR (0% overshoot)	0.35 $\tau$ /KL	1.2 $\tau$
	0.6 $\tau$ /KL	4L
CHR (20% overshoot)	0.6 $\tau$ /KL	$\tau$
	0.7 $\tau$ /KL	2.3L
Kappa-Tau ( $M_s=1.4$ ) $x = L/(L + \tau)$	$[(0.29\tau /LK)\exp(-2.7x + 3.7x^2)]$	$[8.9L\exp(-6.6x + 3x^2)]$
Kappa-Tau ( $M_s=2$ ) $x = L/(L + \tau)$	$[(0.7\tau /LK)\exp(-4.1x + 5.7x^2)]$	$[8.9L\exp(-6.6x + 3x^2)]$

## VIII. SIMULATION AND RESULTS

Proportional gain and integral time of the PI controller are calculated [17] for the system  $G(S) = \frac{12 e^{-2.05S}}{53.6S+1}$  using different tuning methods are given in Table III.

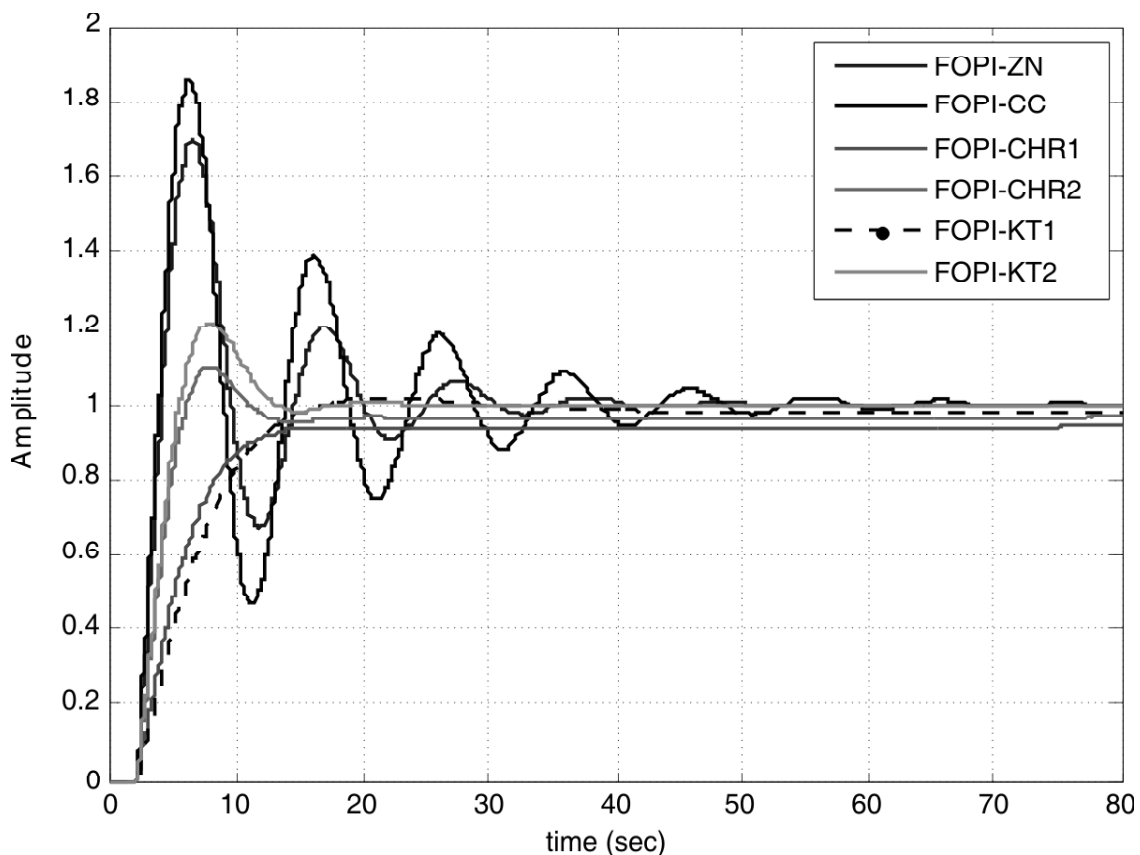
**Table III**  
**Proportional Gain And Integral Time Values Of Pi Controller**

Tuning Method	$K_c$	$\tau_i$
Ziegler Nichols (Open Loop)	1.960	6.8265
Cohen-coon	2.204	6.324
CHR (0% overshoot)	0.762	64.32
CHR (20% overshoot)	1.307	53.6
Kappa Tau (Ms=1.4)	0.574	14.35
Kappa Tau (Ms=2)	1.320	14.35

The step response of fractional order controller for a conical tank process is shown in Figure 5, Figure 6 and Figure 7 for  $\lambda = 0.7$ ,  $\lambda = 1$  and  $\lambda = 1.3$  respectively. Time domain specifications and performance indices are shown in Table IV for  $\lambda = 0.7$ , 1 and 1.3.

It is observed that by increasing the fractional order ( $\tilde{\epsilon}$ ) all the parameters are increased in Ziegler Nichols method, except settling time all other parameters are increased in Cohen Coon method, in CHR (0% overshoot) and CHR (20% overshoot) methods performance indices are more for fractional order; overshoot decreases in CHR (20% overshoot) method and in Kappa-Tau methods rise time is decreased.

Step response of the process with 25% and 40% change in time constant of the plant using fractional order PI controller for  $\lambda = 0.7$ , 1 and 1.3 shown in Figure 8 to 13. Time domain specifications and performance indices are tabulated in Table V and VI. It is observed that the rise time value increased for increasing in integral order for all methods. ITAE, Peak overshoot, settling time values are less in Kappa-Tau(Ms=2.0) method for  $\lambda = 0.7$ . CHR(20% overshoot) has less overshoot, settling time and performance indices for  $\lambda = 1$  and  $\lambda = 1.3$ . Steady state error for CHR methods for an integral order is less than one.



**Figure 5: Step response of the process with  $\lambda = 0.7$**



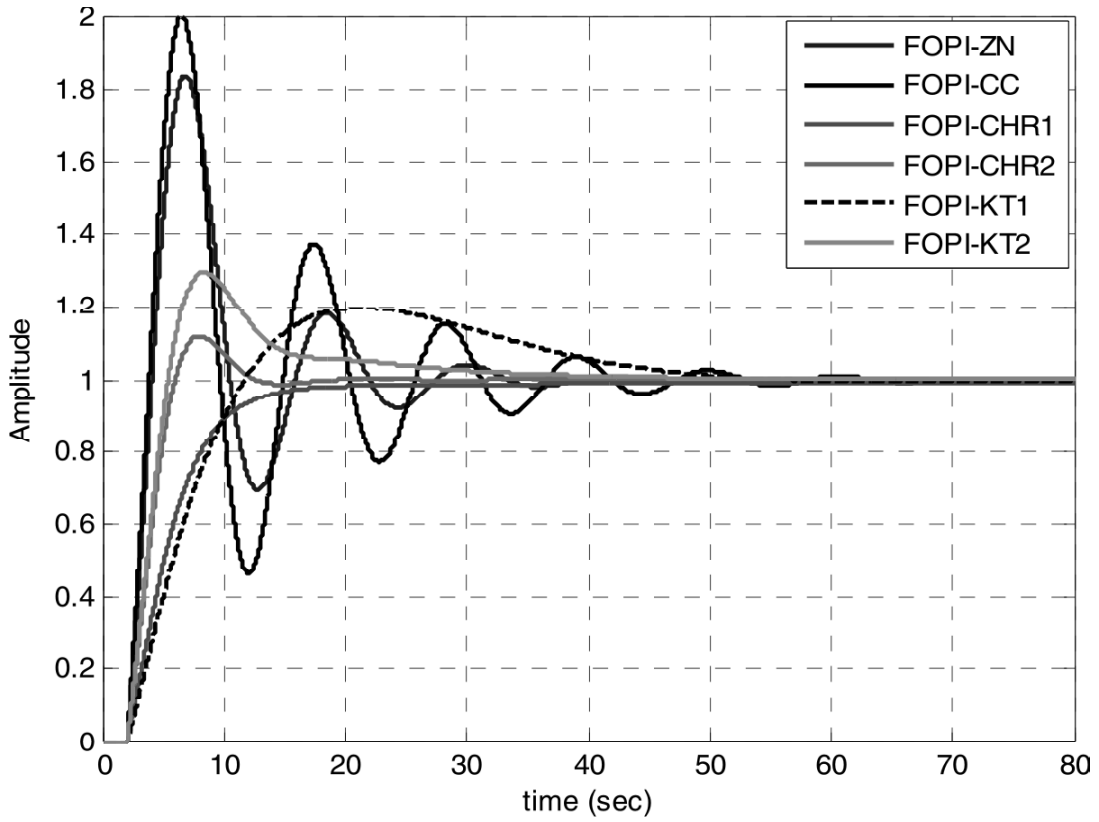


Figure 6: Step response of the process with  $\lambda = 1$

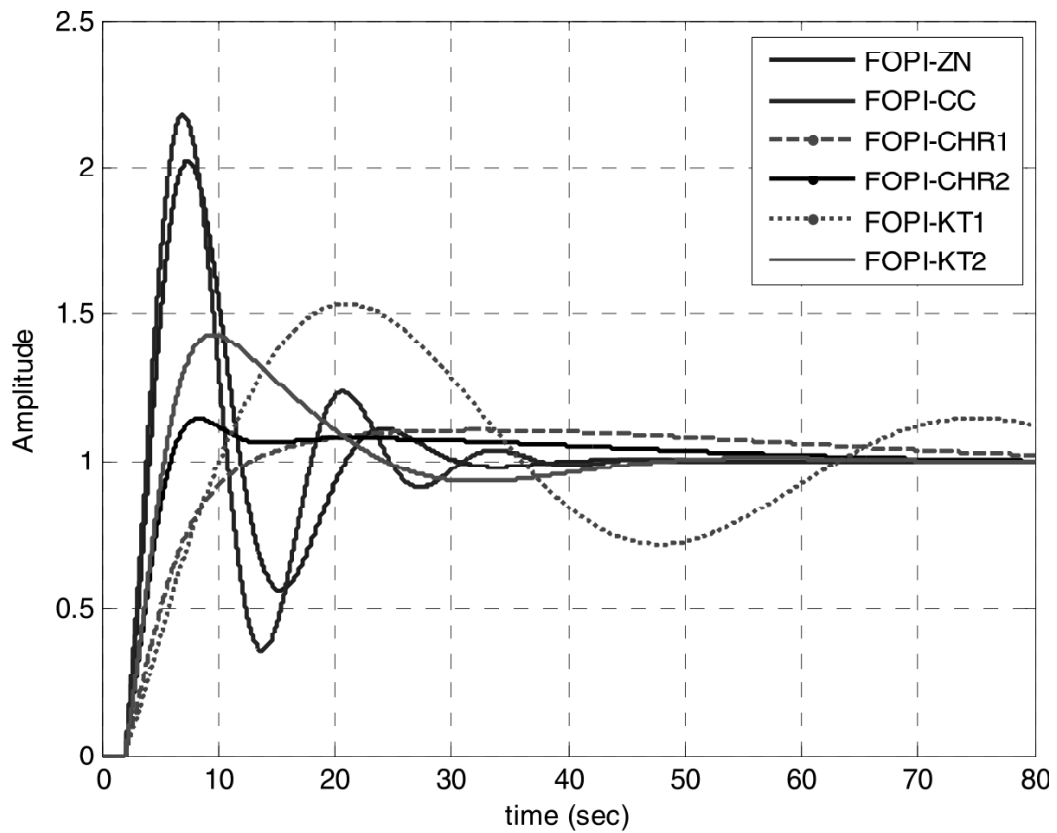


Figure 7: Step response of the process with  $\lambda = 1.3$

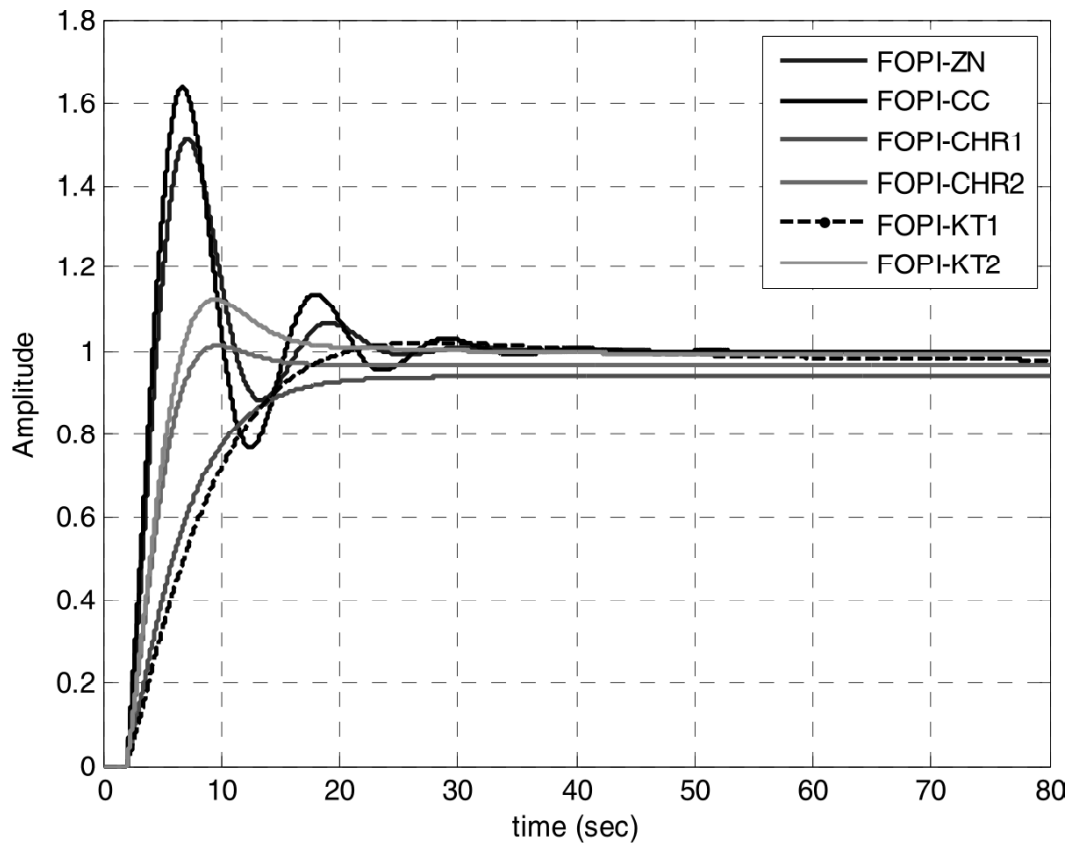


Figure 8: Step response of the process with 25% change in time constant for  $\lambda = 0.7$

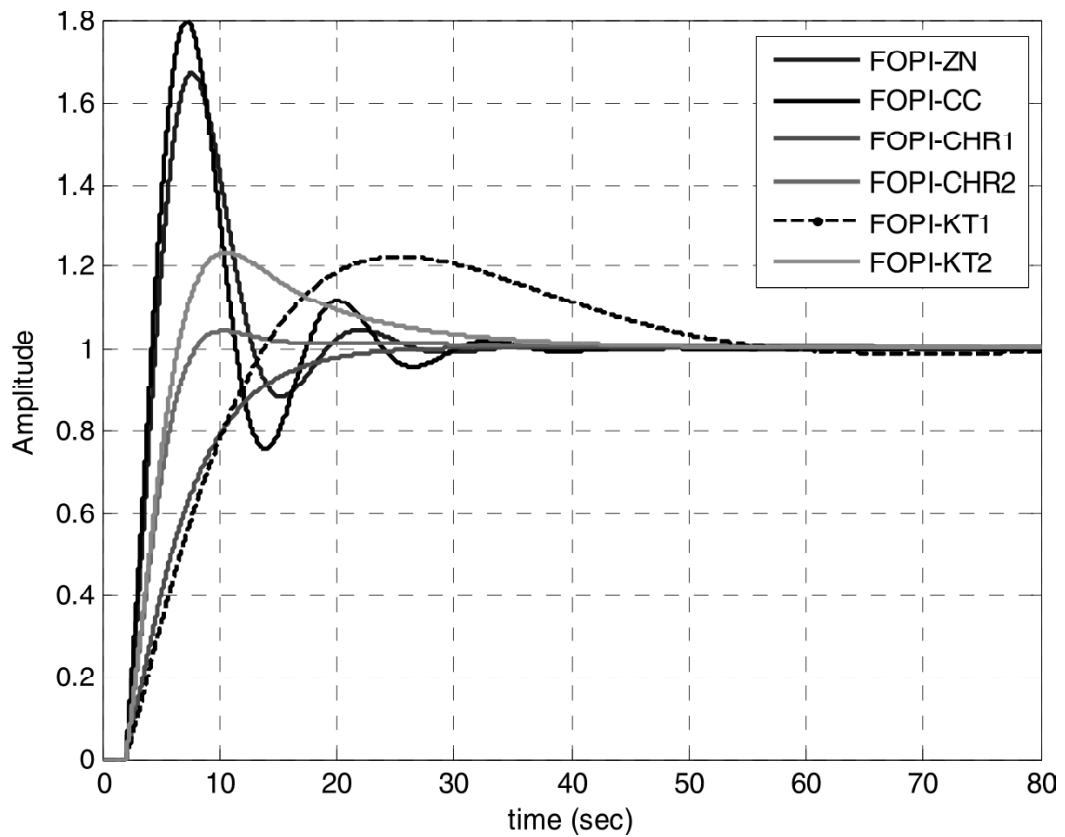


Figure 9: Step response of the process with 25% change in time constant for  $\lambda = 1$

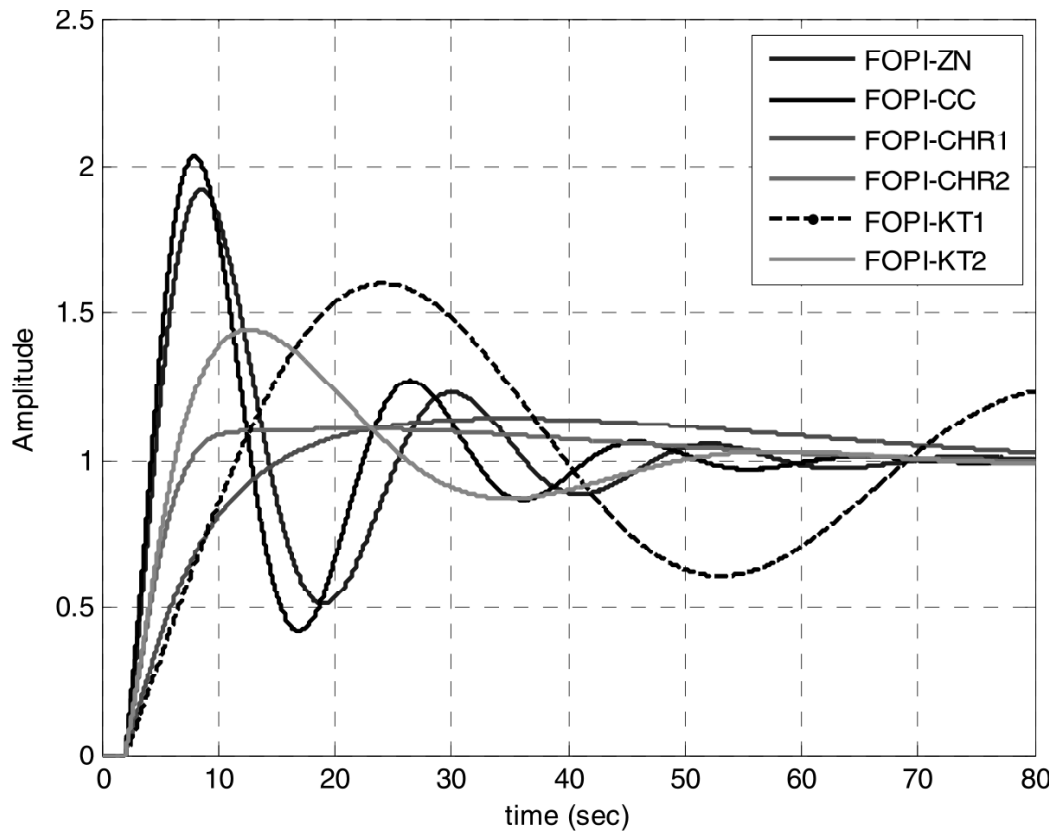


Figure 10: Step response of the process with 25% change in time constant for  $\lambda = 1.3$

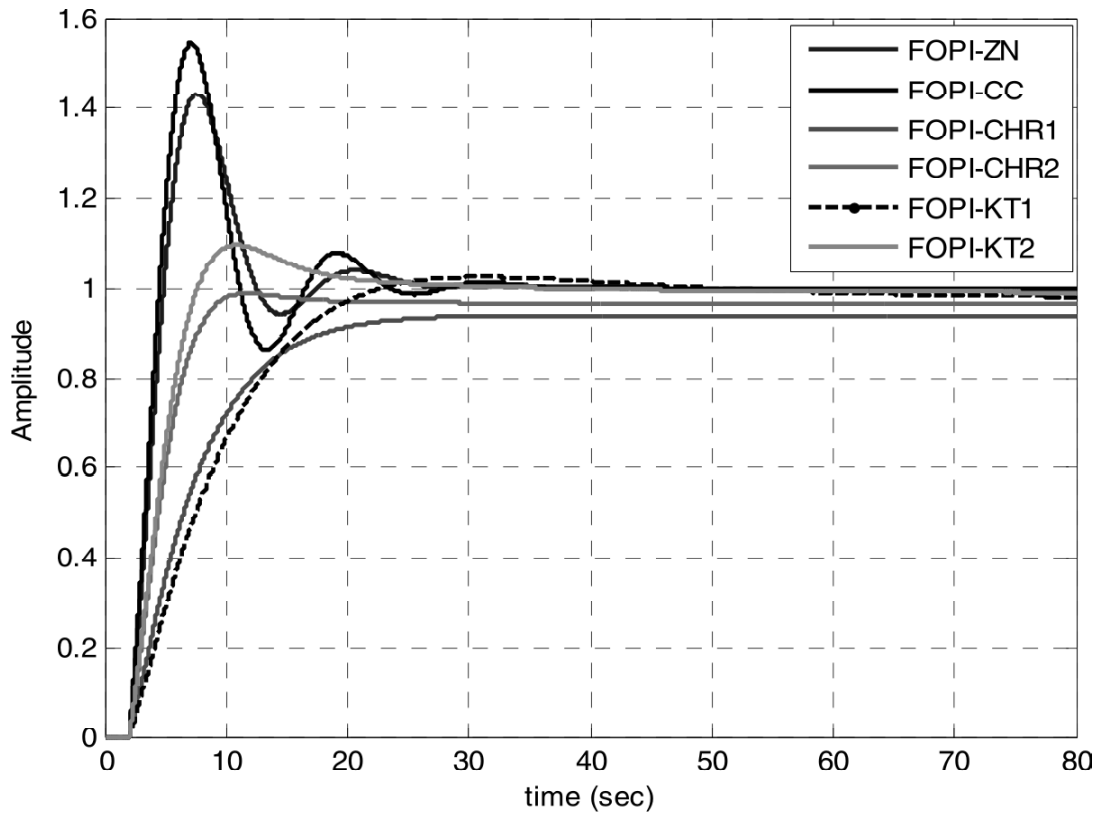


Figure 11: Step response of the process with 40% change in time constant for  $\lambda = 0.7$

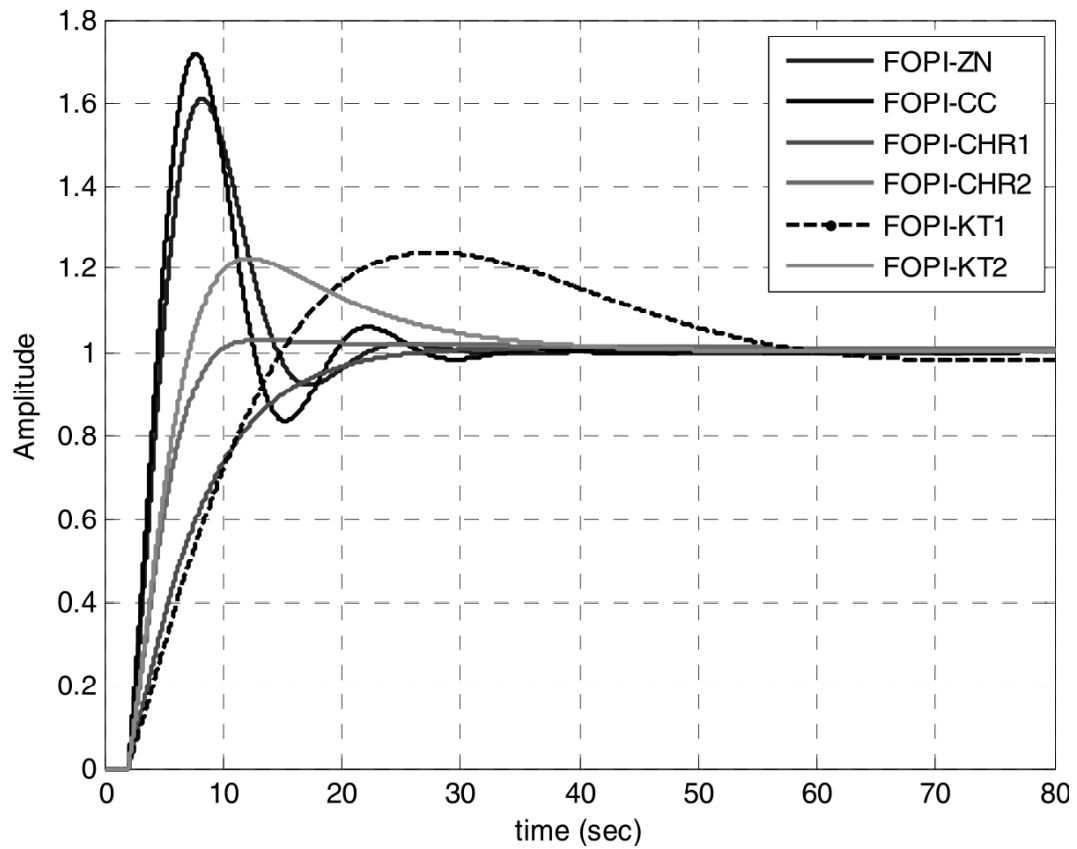


Figure 12: Step response of the process with 40% change in time constant for  $\lambda = 1$

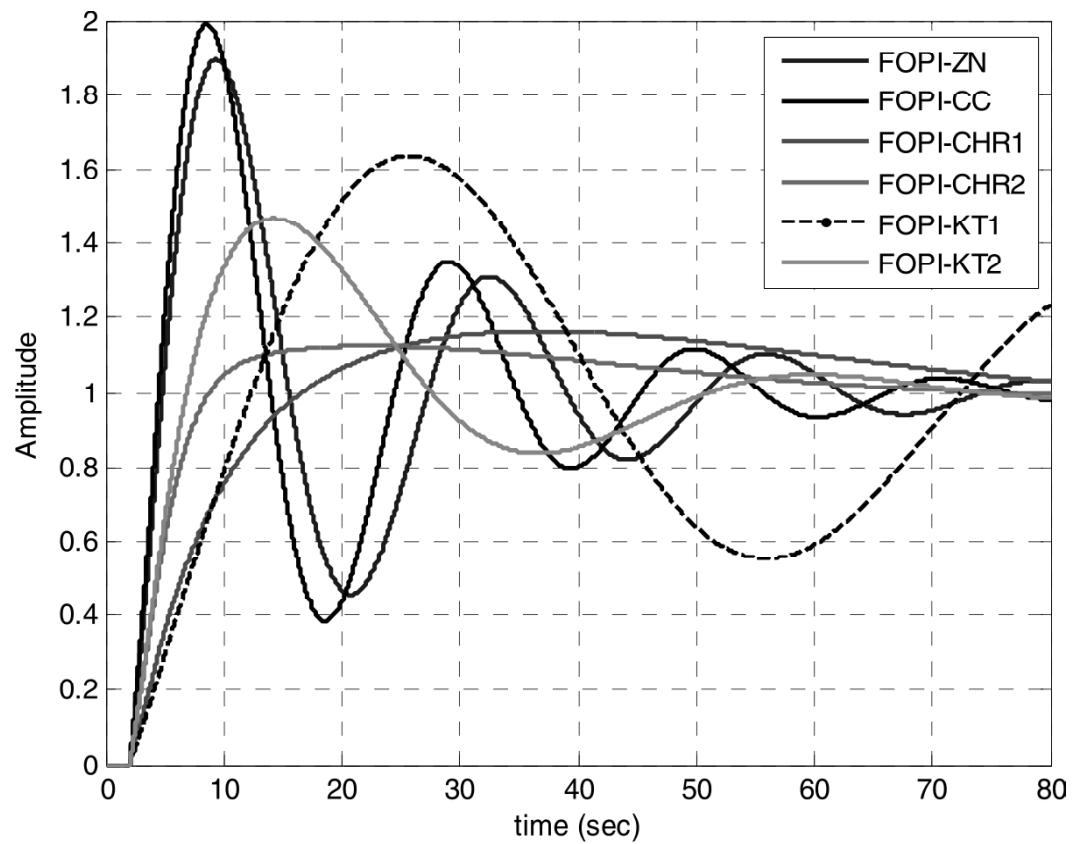


Figure 13: Step response of the process with 40% change in time constant for  $\lambda = 1.3$

**Table IV**  
**Time Domain Analysis for Servo Operation**

	ZN			CC			CHR (0% Overshoot)			CHR (20% Overshoot)			KT (Ms=1.4)			KT (Ms = 2.0)		
	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$
ITAE	64.7	70.77	102	129.1	133	122.2	100.9	36.78	125	54.69	12	81.55	42.24	129.6	339.5	24.56	47.93	97.39
IAE	7.927	8.697	11.23	11.02	11.72	12.6	8.444	6.387	8.848	5.575	4.314	6.795	7.021	10.21	17.43	4.881	6.405	9.128
ISE	4.479	5.229	7.599	5.94	6.907	9.072	4.409	4.219	4.471	3.276	3.239	3.426	4.746	5.206	8.879	3.282	3.575	4.6
Rise Time (sec)	1.6	1.61	1.632	1.388	1.469	1.461	6.62	7.524	9.074	2.819	2.965	3.108	8.433	7.384	4.314	2.607	2.627	2.609
Settling Time (sec)	34.25	32.3	35.19	48.78	47.67	35.5	12.72	15.605	36.814	11.628	11.662	38.571	30.69	44.29	44.4	12.65	29.8	44.54
Peak Overshoot (%)	69.76	83.81	102.2	90.51	95.02	117.7	0.084	0	2.206	13.724	11.637	10.575	3.406	18.13	111.8	22.58	29.06	42.69
Peak Time (sec)	6.463	6.828	7.4	6.214	6.521	7.002	20.211	50	32.203	7.846	8.04	8.385	21.72	21.31	21.1	7.846	8.396	9.617

**Table V**  
**Time Domain Analysis for Servo Operation with 25% Change in Time Constant**

	ZN			CC			CHR (0% Overshoot)			CHR (20% Overshoot)			KT (Ms=1.4)			KT (Ms = 2.0)		
	<i>K=12; L=2.05; T=67 (25% change in Time constant)</i>																	
	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$
ITAE	39.11	47.39	241.5	53.68	64.25	220.7	229.2	43.04	293	116.3	34.37	139.2	85.01	205.8	799.9	40.07	66.84	182.8
IAE	6.221	7.417	15.38	7.218	8.424	15.38	11.35	7.35	12.75	6.65	5.104	8.46	8.72	12.61	26.21	5.306	7.308	11.79
ISE	3.782	4.698	8.645	4.203	5.243	9.378	5.181	4.85	5.44	3.58	3.521	3.845	5.47	6.142	12.68	3.477	3.861	5.466
Rise Time (sec)	1.957	1.962	1.976	1.735	1.746	1.763	9.63	10.8	9.87	3.91	4.13	4.01	10.934	8.78	9.60	3.47	3.38	3.27
Settling Time(Sec)	22.585	25.04	65.97	31.12	29.01	57.92	18.60	20.82	71.9	13.22	13.96	63.13	43.7	54.81	77.65	19.47	35.72	67.5
Peak Overshoot (%)	51.365	67	91.719	64.11	79.84	103.68	0	0	14.2	1.2	4.2	10.62	1.99	22.52	60.17	12.31	23.49	44.4
Peak Time (sec)	7.141	7.69	8.59	6.748	7.221	7.964	79.98	42.95	34.88	9.69	10.41	22.02	27.88	25.5	24.14	9.51	10.61	12.77

**Table VI**  
**Time Domain Analysis for Servo Operation with 40% Change in Time Constant**

	ZN			CC			CHR (0% Overshoot)			CHR (20% Overshoot)			KT (Ms=1.4)			KT (Ms = 2.0)		
	<i>K=12; L=2.05; T=75.04 (40% change in time constant)</i>																	
	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$	$\lambda=0.7$	$\lambda=1$	$\lambda=1.3$
ITAE	33.94	47.2	369.6	41.31	55.32	343.1	231.3	69.06	343.3	114	49.36	160.9	87.42	250.7	911.3	39.41	76.92	241.9
IAE	5.932	7.423	18.57	6.476	7.945	18.47	11.92	8.38	14.17	6.93	5.72	9.28	9.39	13.95	28.72	5.54	7.839	13.47
ISE	3.674	4.656	9.854	3.912	4.996	10.44	5.576	5.243	6.00	3.78	3.72	4.13	5.89	6.708	14.62	3.631	4.074	6.095
Rise Time (sec)	2.188	2.18	2.25	1.924	1.929	1.907	11.37	12.6	11.18	4.7	5	4.72	12.42	9.57	10.27	4.02	3.87	3.66
Settling Time(Sec)	24.27	21.07	75.42	22.65	25.513	76.64	21.5	23.25	73.41	13	15.34	65.27	49.40	59.35	78.93	24.76	37.28	72.53
Peak Overshoot (%)	43.595	60.77	83.33	54.83	71.937	103.3	0	1.1	16.3	0	2.8	12.46	2.51	24.14	63.7	9.5	22.4	46.8
Peak Time (sec)	7.588	8.244	9.37	7.121	7.691	8.595	79.99	42.29	36.9	11.86	13.25	22.66	31.19	27.77	25.78	10.92	12.29	14.32

**IX. CONCLUSIONS**

The response of the conical tank process was simulated using MATLAB/SIMULINK for  $\lambda = 0.7$ ,  $\lambda = 1$  and  $\lambda = 1.3$ . The performance of the controller using different tuning methods are compared for  $\lambda = 0.7$ ,  $\lambda = 1$  and  $\lambda = 1.3$ . It is observed that for  $\lambda = 0.7$ , Kappa-Tau (Ms=2.0) method is more suitable for conical tank process for servo operation. For  $\lambda = 1$  and  $\lambda = 1.3$ , CHR (20% overshoot) method is more appropriate. These methods have least values of ISE, IAE and ITAE.

### References

- [1] I. Podlubny, "Fractional-order systems and PI <sup>$\alpha$</sup> D <sup>$\beta$</sup>  controllers," IEEE Trans. on Automatic Control, vol.44, No.1, pp. 208-213, 1999.
- [2] K. Miller and B. Ross, "An introduction to the fractional calculus and fractional differential equations", Wiley, 1993
- [3] Valerio Duarte, Jose Sa da Costa, "Tuning rules for fractional PID controllers", in Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and its applications, Porto, Portugal, July 19-21, 2006.
- [4] Yang Quan Chen, Tripti Bhaskaran and Dingyu Xue, "Practical Tuning Rule Development for Fractional Order Proportional and Integral Controllers", Journal of Computational and Nonlinear Dynamics, Vol 3, 2008.
- [5] Fabrizio Padula, Antonio Visioli, "Tuning rules for optimal PID and fractional order PID controllers," Journal of Process Control, pp.69-81 2011.
- [6] C. A. Monje, Y. Chen, B. Vinagre, D. Xue, and V. Feliu, "Fractional-order Systems and Controls: Fundamentals and Applications", Springer, 2010.
- [7] S.Nithya, N.Sivakumaran, T.K.Radhakrishnan and N.Anantharaman, "Soft Computing Based Controllers Implementation for Non-linear Process in Real Time", Proceedings of the World Congress on Engineering and Computer Science 2010, Vol II.
- [8] Sundaresan, K.R.; Krishnaswamy, R.R., Estimation of time delay, Time constant parameters in Time, Frequency and Laplace Domains, Can.J.Chem.Eng, Vol.56, pp. 257, 1978
- [9] P.Aravind, M.Valluvan and S.Ranganathan, "Modelling and Simulation of Non Linear Tank", IJAREEIE, Vol 2, Issue 2, 2013.
- [10] B. Wayne Bequette, "Process Control: Modeling, Design and Simulation", Pearson Education Inc., 2003.
- [11] G. Sreenivasulu and S.N.Reddy, "Performance Evaluation of Superheated Steam Temperature Control System based on Tuning methods of Analog Controllers", IETE Journal of Research, Vol 49, No.6, 2003, pp. 399-404.
- [12] J.G.Ziegler and N.B.Nichols and Rochester.N.Y, "Optimum Settings for Automatic Controllers," Transactions of ASME, vol 64, 1942, pp. 759-765.
- [13] G. H. Cohen, and G. A. Coon, "Theoretical considerations of retarded control," Transactions of ASME, vol. 75, 1953, pp. 827-834.
- [14] Dr. Satya Sheel and Omhari Gupta, "New Techniques of PID Controller Tuning of DC Motor-Development of a Toolbox," MIT IJEIE, Vol 2, No 2, 2012, pp. 65-69.
- [15] K. Astrom and T. Hagglund, "PID Controllers: Theory, Design and Tuning", 2nd Edition, Instrument Society of America, 1995.
- [16] Aidan O'Dwyer, "Handbook of PI and PID Controller Tuning Rules" 3rd Edition, Imperial College Press, 2009.
- [17] N. Gireesh and G. Sreenivasulu, "Robustness of PI Controller for various tuning methods for a nonlinear process", Australian Journal of Basic and Applied Sciences, 8(17), pp. 491-497, 2014.
- [18] Karanjkar D. S., Chatterji. S. and Venkateswaran P.R. "Trends in Fractional Order Controllers" International Journal of Emerging Technology and Advanced Engineering, Vol. 2, Issue 3, pp. 383-389, 2012.