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# SOME RESULTS OF INTUITIONISTIC FUZZY e-CONNECTEDNESS

***Abstract:*** *In this paper, the Intuitioninistic fuzzy e-connectedness between Intuitioninistic fuzzy semi opensets are introduced.*

***Keywords****: IF e-closed sets, IF e-connectedness between IF semi open sets* $\alpha$ *and* $\beta$*, IF P-connectedness.*

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## 1. INTRODUCTION AND PRELIMINARIES

In Zadeh [6 ] introduced the fundamental concept of a fuzzy set.Chang extended the concept of pointset topology to fuzzy sets. Atanassov [1] introduced intuitionistic fuzzy set. After the introduction of intuitionistic fuzzy topology by Cocker [2] in 1997. V. Chandraseker, A. Vadivel and D. Sobana [3,4] introduced the concept of Intuitionistic fuzzy e-open set in IFTS. The concept of connectedness between sets was first introduced by kuratowski [5] in Topology. A space X is said to be connectedness between subset A and B if and only if there is no clopen set F in X such that $A \subseteq F$ and $A \cap F$ is empty [Kuratowski 1968].

## 2. INTUITIONISTIC FUZZY E-CONNECTEDNESS BETWEEN INTUITIONISTIC FUZZY SEMI OPEN SETS

### Definition 2.1

An IFTS X to be IF connected between IF semi open sets $\alpha$ and $\beta$ if and only if there exists no IF clopen $\gamma$ of $X$ suchthat $\alpha \subseteq \gamma$ and $\urcorner \gamma q \beta$.

### Definition 2.2

An IFTS $X$ is said to be IF e-connected between intuitionistic semi open sets $\alpha$ and $\beta$ of $X$ if and only if there exists no IF e-clopen subset $\gamma$ of $X$ suchthat $\alpha \subseteq \gamma$ and $\urcorner \gamma q \beta$.

**Theorem 2.1**

If an IFTS $X$ is IF e-connected between IF semi open sets $\alpha$ and $\beta$, then $\alpha$ and $\beta$ are non empty.

**Proof:** If any IF set $\alpha$ is empty, then $\alpha$ being an IF e-clopen set of $X$, $X$ cannot be IF e-connected between IF semi open sets $\alpha$ and $\beta$. This proves the theorem.

**Theorem 2.2**

If X is IF e-connected between IF semi open sets $\alpha$ and $\beta$ and if $\alpha \subseteq \alpha_1$ and $\beta \subseteq \beta_1$, then $X$ is IF e-connected between IF semi open sets $\alpha_1$ and $\beta_1$.

**Proof:** Suppose $X$ is not IF e-connected between semi open sets $\alpha$ and $\beta$, then there is an IF e- clopen subset $\gamma$ of $X$ suchthat $\alpha_1 \leq \gamma$ and $\rceil \gamma q \beta_1$. Then $X$ is not IF e-connected between IF semi open sets $\alpha_1$ and $\beta_1$.

**Example 2.1**

Let $X = \{a, b\}$, $\alpha = \langle x,(0.2, 0.1), (0.7, 0.5)\rangle$, $\beta = \langle x,(0.3,0.5), (0.7,0.2)\rangle$, $\gamma = \langle x, (0.3, 0.2), (0.2, 0.5)\rangle$, $\delta = \langle x,(0.6, 0.1), (0.4, 0.5)\rangle$, then the family $\sigma = \{0^{\sim}, 1^{\sim}, \alpha\}$ is an IF topology on $X$. Then $(X, \tau)$ is IF e-connected between IF semi open sets $\gamma$ and $\delta$.

**Theorem 2.3**

An IFTS $(X, \tau)$ is IF e-connected between IF semi open sets $\gamma$ and $\delta$ if and only if there is no IF e-clopen set Fin X such that $\gamma \subseteq F \subseteq \delta^c$.

**Proof:** It follows from the definition 2.1

**Theorem 2.4**

An IFTS $(X, \tau)$ is Intuitionistic fuzzy e-connected between IF semi open sets $\alpha$ and $\beta$ if and only if it is IF e-connectedness betweencl$_e(\alpha)$ and cl$_e(\beta)$.

**Proof:** Necessity: Follows from Theorem 2.1 because $\alpha \subseteq cl_e(\alpha)$ and $\beta \subseteq cl_e(\beta)$.

Sufficiently: Suppose $(X, \tau)$ is not IF e-connected between IF semi open sets $\alpha$ and $\beta$ then there is an IF e-clopen set $F$ of $X$ suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta)$ which implies that $F \subseteq \beta^c$. Therefore, $F = int_e(F) \subseteq int_e(\beta^c) = (cl_e(\beta))^c$. Hence $\rceil(Fqcl_e\beta)$ and $X$ is not IF e-connected betweencl$_e(\alpha)$ and cl$_e$ $(\beta)$.

**Theorem 2.5**

If $X$ is IF e-connected between IF semi open sets $\alpha$ and $\beta$ then it is IF e-connected between cl $(\alpha)$ and cl $(\beta)$.

**Proof:** Suppose $x$ is not IF e-connected between $\alpha$ and $\beta$ then there is an IF e-clopen set $F$ of $X$ suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta)$. But cl$(\alpha) \subseteq F$ and $\rceil(Fqcl\ \beta)$. Hence X is IF e-connected betweencl $(\alpha)$ and cl $(\beta)$.

**Theorem 2.6**

X is not IF e-connected between IF semi open sets $\alpha$ and $\beta$ if and only if there exists IF e-clopen disjoint sets $F_1$ and $F_2$ suchthat $X = F_1 \cup F_2$ and $\alpha_i \subseteq F_i$ for $i = 1, 2$.

**Proof:** Obvious.

**Theorem 2.7**

If an IFTS $(X, \tau)$ is IF e-connectedness between IF semi open sets $\alpha$ and $\beta$, then it is IF connected between IF semi open sets $\alpha$ and $\beta$.

**Proof:** If $(X, \tau)$ is not IF connected between semi open sets $\alpha$ and $\beta$, there exists an IF clopen set $F$ in $X$ suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Since every IF set is an IF e-open set then F is an IF e-clopen set in X suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Hence $(X, \tau)$ is not IF e-connected IF semi open sets $\alpha$ and $\beta$. which contradicts the hypothesis.

**Example 2.2**

From Example 2.1, $(X, \tau)$ is IF p - Connected between IF semi open sets $\gamma$ and $\delta$. $0^{\sim}$ is IF pre clopen set satisfying $\gamma \nleq 0^{\sim}$ and $0 \leq d^c$.

**Theorem 2.8**

If an IFTS $(X, \tau)$ is Intuitionistic fuzzy P-connected between IF semi open sets $\alpha$ and $\beta$, then it is IF e-connected between IF semi open sets $\alpha$ and $\beta$.

**Proof:** If $(X, \tau)$ is not IF P-connected Intuitionistic semi open sets $\alpha$ and $\beta$. Then there exists an IF P-clopen set $F$ in $X$ such that $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Since every IF P-open set is Intuitionistic fuzzy e-open set on $X$. Hence $(X, \tau)$ is not IF e-connected between $\alpha$ and $\beta$.

**Theorem 2.9**

Let $(Y, \tau_Y)$ be an IF clopen subspace of a IFTS $(X, \tau)$ and A,B be IF subsets of Y. If $(X, \tau)$ is IF e-connected between IF semi open sets $\alpha$ and $\beta$, where $\alpha \subseteq A$, $\beta \subseteq B$, then so does $(Y, \tau_Y)$.

**Proof:** If $(Y, \tau_Y)$ is not IF e-connected between $\alpha$ and $\beta$, then there exists an IF e-clopen set $F$ of $Y$ such that $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Since $Y$ is IF in $X$, $F$ is an IF e-clopen set in $X$. Hence $X$ cannot be IF e-connected between $\alpha$ and $\beta$.

**Theorem 2.10**

Let $(Y, \tau_Y)$ be a subspace of a IFTS $(X, \tau)$ and A, B be IF subsets of $Y$. If $(Y, \tau_Y)$ is IF e-connected between IF semi open sets $\alpha$ and $\beta$, where $\alpha \subseteq A$, $\beta \subseteq B$, then so does $(X, \tau)$.

**Proof:** Suppose $(X, \tau)$ is not IF e-connected between $\alpha$ and $\beta$, then there exists an IF e-clopen set $F$ in $X$ suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta)$.Put $F_Y = F \cap Y$. Then $F_Y$ is IF e-clopen set $F$ of $Y$ such that $\alpha \subseteq F_Y$ and $\rceil(F_Y q\, \beta)$. Hence $(Y, \tau_Y)$ is not IF e-connected between $\alpha$ and $\beta$.

**Theorem 2.11**

If an IFTS $(X, \tau)$ is Intuitionistic fuzzy semi connected IF open sets $\alpha$ and $\beta$ then it is IF e-connected $\alpha$ and $\beta$.

**Proof:** If $(X, \tau)$ is not IF P-connected between IF $\alpha$ and $\beta$. Then there exists an IF P-clopen set $F$ in $X$ such that $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Since every IF semi-open set is IF e-open set on $X$. Hence $(X, \tau)$ is not IF e-connected between $\alpha$ and $\beta$.

**Theorem 2.12**

Let $(X, \tau)$ be an IFTS and $\alpha$ and $\beta$ be two intuitionistic fuzzy sets in $X$. If $\alpha q\beta$, then $(X, \tau)$ is IF e-connected between $\alpha$ and $\beta$.

**Proof:** If $F$ is any Intuitionistic fuzzy e-clopen set of $X$ suchthat $\alpha \subseteq F$ and since $\alpha q\beta$. Hence $Fq\beta$. There is no intuitionistic fuzzy e-clopen set $F$ in $X$ such that $\alpha \subseteq F$ and $\rceil(Fq\beta)$. Hence $(X, \tau)$ is Intuitionistic fuzzy e-connected between $\alpha$ and $\beta$.

**Theorem 2.13**

If IFTS $(X, \tau)$ is neither IF connected between $\alpha$ and $\beta_0$, nor IF e- connected in $\alpha$ and $\beta_1$, then $(X, \tau)$ is not intuitionistic fuzzy e-connected $\alpha$ and $\beta_0 \cup \beta_1$.

**Proof:** $(X, \tau)$ is not IF connected $\alpha$ and $\beta_0$, there is a IF clopen set $F$ in $X$ suchthat $\alpha \subseteq F$ and $\rceil(Fq\beta_0)$ and $(X, \tau)$ is not IF e-connected $\alpha$ and $\beta_1$, there exists a clopen set $F_1$ in $X$ such that $\alpha \subseteq F$ and $\rceil(Fq\beta_1)$. Define $F = F_0 \cap F_1$. Since intersection of IF clopen $F$ is IF clopen in $X$ and satisfies $\alpha \subseteq F$ and $\rceil(Fq\beta_0 \cup \beta_1)$.

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