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Adaptive Control and Synchronization Design of a Seven-Term Novel Chaotic System with a Quartic Nonlinearity

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Abstract: First, this paper announces a seven-term novel 3-D chaotic system with a quartic nonlinearity and also a quadratic nonlinearity. The phase portraits of the novel chaotic system are displayed and the mathematical properties are discussed. We show that the novel chaotic system has three unstable equilibrium points. The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 0.64357$, $L_2 = 0$ and $L_3 = -1.74274$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as$L_1 = 0.64357$ and Lyapunov dimension as $D_L = 2.3693$. Since the sum of the Lyapunov exponents of the novel chaotic system is negative, it follows that the novel chaotic system is dissipative. Next, we derive new results for the adaptive control design of the novel chaotic system with unknown parameters. We also derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters. The adaptive control and synchronization results for the novel chaotic system have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper.

Keywords: Chaos, chaotic systems, chaos synchronization, adaptive control, stability.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1]. In other words, a chaotic system is a nonlinear dynamical system with at least one positive Lyapunov exponent. Some paradigms of chaotic systems can be listed as Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering.

Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

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Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-132], adaptive control method [133-149], sampled-data feedback control method [150-151], time-delay feedback approach [152], backstepping method [153-164], sliding mode control method [165-173], etc.

In this paper, we derive a seven-term novel 3-D chaotic system with a quartic nonlinearity and also a quadratic nonlinearity. The phase portraits of the novel chaotic system are displayed and the mathematical properties are discussed. We show that the novel chaotic system has three unstable equilibrium points. Explicitly, we show that one equilibrium point of the novel chaotic system is a saddle point, while the other two equilibrium points are saddle-foci.

The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 0.64357$, $L_2 = 0$ and $L_3 = -1.74274$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 0.64357$ and Lyapunov dimension as $D_L = 2.3693$.

Next, we derive new results for the adaptive control design of the novel chaotic system with unknown parameters. We also derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters. The adaptive control and synchronization results for the novel chaotic system have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper.

2. A NOVEL 3-D CHAOTIC SYSTEM

In this section, we propose a novel 3-D chaotic system modelled by the dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = x_2 + x_1 x_3 \\ \dot{x}_3 = b - c x_2^4 - p x_3 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b, c, p are constant, positive parameters of the system.

The system (1) describes a *strange chaotic attractor* for the parameter values

$$a = 2, \quad b = 5, \quad c = 2, \quad p = 0.1 \tag{2}$$

For numerical simulations, we take the initial values of the system (1) as

$$x_1(0) = 0.8, \quad x_2(0) = 0.8, \quad x_3(0) = 0.8 \tag{3}$$

Figure 1 shows the strange chaotic attractor of the system (1).

Figures 2-4 show the 2-D view of the chaotic attractor of the system (1) in (x_1, x_2), (x_2, x_3), and (x_1, x_3) planes respectively.

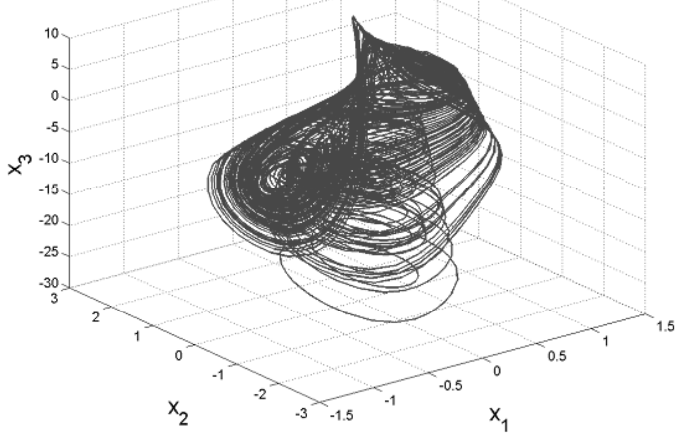

Figure 1: Strange attractor of the novel chaotic system

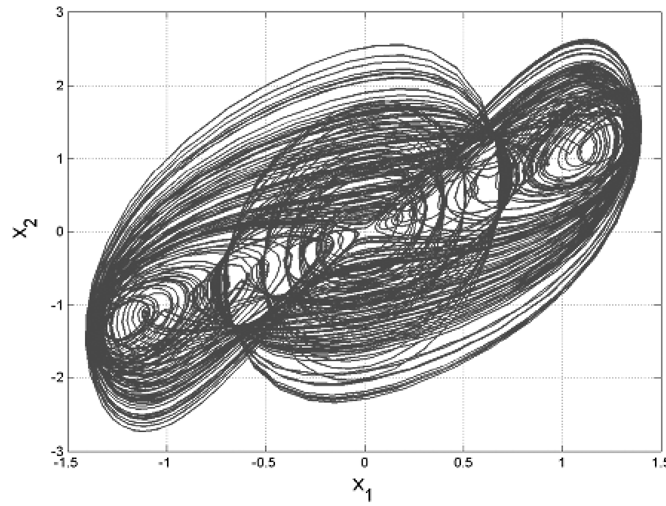

Figure 2: 2-D view of the novel chaotic system in (x_1, x_2) plane

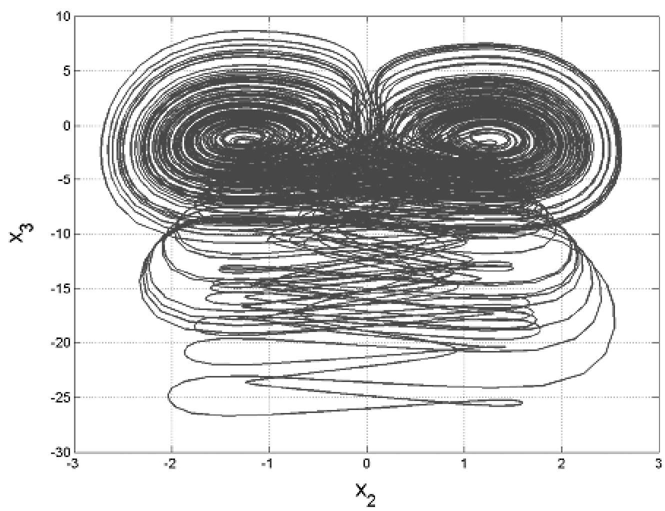

Figure 3: 2-D view of the novel chaotic system in (x_2, x_3) plane

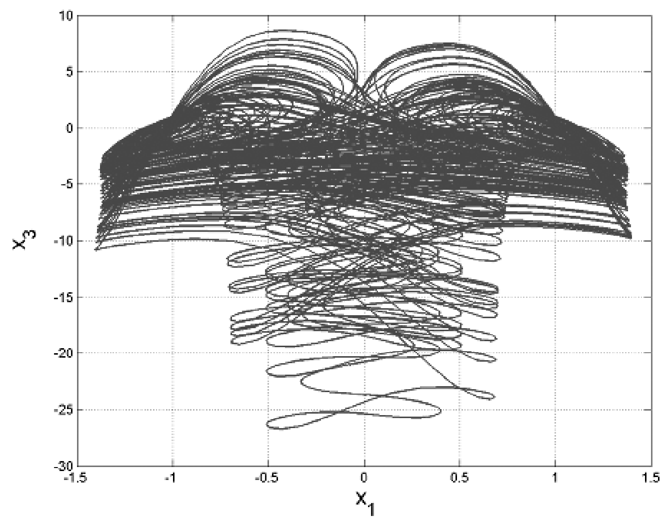

Figure 4: 2-D view of the novel chaotic system in (x_1, x_3) plane

3. PROPERTIES OF THE NOVEL 3-D CHAOTIC SYSTEM

In this section, we detail the qualitative properties of the novel 3-D chaotic system (1), which is described in Section 2.

3.1. Dissipativity

We write the system (1) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{4}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) \\ f_2(x_1, x_2, x_3) = x_2 + x_1 x_3 \\ f_3(x_1, x_2, x_3) = b - c x_2^4 - p x_3 \end{cases} \tag{5}$$

We take the parameter values as

$$a = 2, \quad b = 5, \quad c = 2, \quad p = 0.1 \tag{6}$$

The divergence of the vector field f on R^3 is obtained as

$$\text{div } f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a + 1 - p = -\mu, \tag{7}$$

where

$$\mu = a - 1 + p = 1.1 > 0 \tag{8}$$

Let Ω be any region in R^3 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV}{dt} = \int_{\Omega(t)} (\text{div } f)\, dx_1\, dx_2\, dx_3 = \int_{\Omega(t)} (-\mu)\, dx_1\, dx_2\, dx_3 = -\mu V \tag{9}$$

Integrating the linear differential equation (9), we get the solution as

$$V(t) = V(0) \exp(-\mu t) \tag{10}$$

From Eq. (10), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel 3-D chaotic system (1) is dissipative.

Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

3.2. Symmetry

The 3-D novel chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{11}$$

Since the transformation (11) persists for all values of the system parameters, the novel 3-D chaotic system (1) has rotation symmetry about the x_3–axis and that any non-trivial trajectory must have a twin trajectory.

3.3. Invariance

The x_3–axis ($x_1 = 0$, $x_2 = 0$, $x_4 = 0$) is invariant for the system (4). Hence, all orbits of the system (1) starting on the axis stay in the axis for all values of time. Also, this invariant motion is bounded but not asymptotically stable.

3.4. Equilibrium Points

The equilibrium points of the novel 3-D chaotic system (1) are obtained by solving the following nonlinear system of equations

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) = 0 \\ f_2(x_1, x_2, x_3) = x_2 + x_1 x_3 = 0 \\ f_3(x_1, x_2, x_3) = b - c x_2^4 - p x_3 = 0 \end{cases} \tag{12}$$

We take the parameter values as in the chaotic case, *viz.*

$$a = 2, \quad b = 5, \quad c = 2, \quad p = 0.1 \tag{13}$$

Solving the equations (12) using the values (13), we obtain three equilibrium points:

$$E_1 = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1.2637 \\ 1.2637 \\ -1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} -1.2637 \\ -1.2637 \\ -1 \end{bmatrix} \tag{14}$$

The Jacobian matrix of the novel chaotic system (1) at any point $X \in R^3$ is obtained as

$$J(x) = \begin{bmatrix} -a & a & 0 \\ x_3 & 1 & x_1 \\ 0 & -4cx_2^3 & -p \end{bmatrix} = \begin{bmatrix} -2 & 2 & 0 \\ x_3 & 1 & x_1 \\ 0 & -8x_2^3 & -0.1 \end{bmatrix} \tag{15}$$

The Jacobian of the system (1) at E_1 is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} -2 & 2 & 0 \\ 50 & 1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix} \tag{16}$$

The eigenvalues of J_1 are numerically obtained as

$$\lambda_1 = -0.1, \quad \lambda_2 = -10.6119, \quad \lambda_3 = 9.6119 \tag{17}$$

This shows that the equilibrium E_1 is a saddle point, which is unstable.

The Jacobian of the system (1) at E_2 is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -2 & 2 & 0 \\ -1 & 1 & 1.2637 \\ 0 & -16.1444 & -0.1 \end{bmatrix} \tag{18}$$

The eigenvalues of J_2 are numerically obtained as

$$\lambda_1 = -1.8615, \quad \lambda_{2,3} = 0.3809 \pm 4.6663i \tag{19}$$

This shows that the equilibrium E_2 is a saddle-focus, which is unstable.

The Jacobian of the system (1) at E_3 is obtained as

$$J_3 = J(E_3) = \begin{bmatrix} -2 & 2 & 0 \\ -1 & 1 & -1.2637 \\ 0 & 16.1444 & -0.1 \end{bmatrix} \tag{20}$$

The eigenvalues of J_3 are numerically obtained as

$$\lambda_1 = -1.8615, \quad \lambda_{2,3} = 0.3809 \pm 4.6663i \tag{21}$$

This shows that the equilibrium E_3 is a saddle-focus, which is unstable.

Thus, all the three equilibrium points of the novel 3-D chaotic system (1) are unstable.

3.5. Lyapunov Exponents

We take the parameter values of the novel system (1) as

$$a = 2, \quad b = 5, \quad c = 2, \quad p = 0.1 \tag{22}$$

We take the initial conditions of the novel system (1) as

$$x_1(0) = 0.8, \quad x_2(0) = 0.8, \quad x_3(0) = 0.8 \tag{23}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 0.64357, \quad L_2 = 0, \quad L_3 = -1.74274 \tag{24}$$

Thus, the system (1) is chaotic, since it has a positive Lyapunov exponent.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 0.64357$.

Since $L_1 + L_2 + L_3 = -1.0992 < 0$, it is immediate that the system (1) is dissipative.

The MATLAB plot of the Lyapunov exponents of the novel chaotic system (1) is depicted in Figure 5.

3.6. Lyapunov Dimension

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.3693 \tag{25}$$

which is fractional.

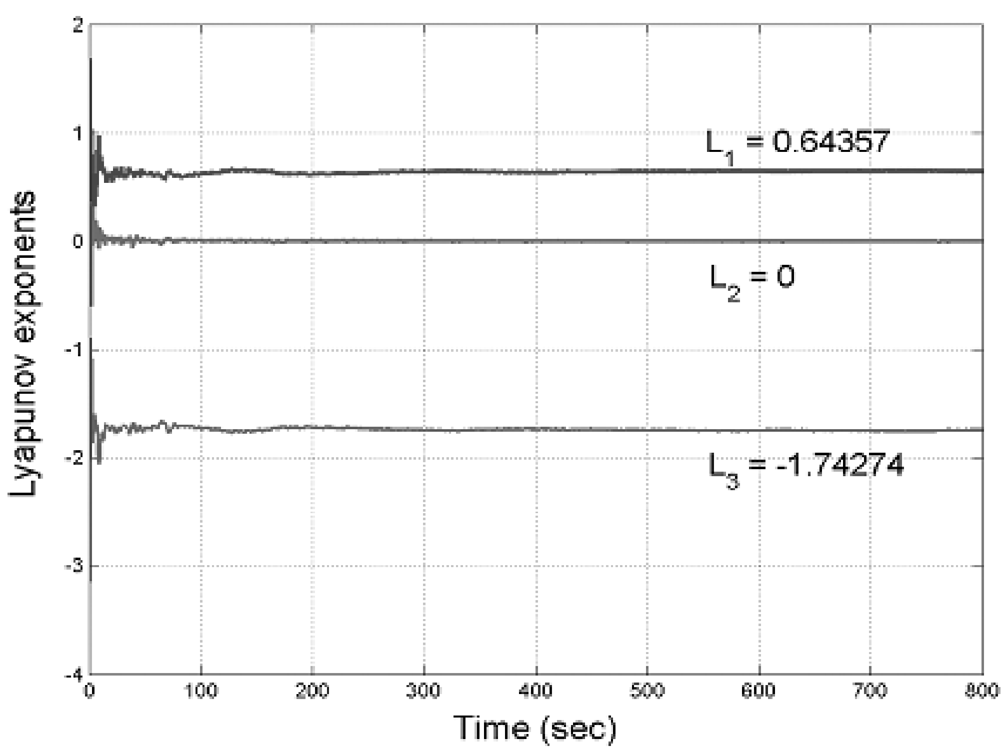

Figure 5: Lyapunov exponents of the novel 3-D chaotic system

4. ADAPTIVE CONTROL OF THE NOVEL 3-D CHAOTIC SYSTEM WITH UNKNOWN PARAMETERS

In this section, we design new results for the adaptive controller to stabilize the novel 3-D chaotic system with unknown parameters for all initial conditions.

Thus, we consider the novel 3-D chaotic system with controls given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + u_1 \\ \dot{x}_2 = x_2 + x_1 x_3 + u_2 \\ \dot{x}_3 = b - c x_2^4 - p x_3 + u_3 \end{cases} \tag{26}$$

where x_1, x_2, x_3 are state variables, a, b, c, p are constant, unknown, parameters of the system and u_1, u_2, u_3 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - k_1 x_1 \\ u_2 = -x_2 - x_1 x_3 - k_2 x_2 \\ u_3 = -\hat{b}(t) + \hat{c}(t) x_2^4 + \hat{p}(t) x_3 - k_3 x_3 \end{cases} \tag{27}$$

where $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{p}(t)$ are estimates for the unknown system parameters a, b, c, p respectively, and k_1, k_2, k_3 are positive gain constants.

The closed-loop system is obtained by substituting (27) into (26) as

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)](x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = -k_2 x_2 \\ \dot{x}_3 = [b - \hat{b}(t)] - [c - \hat{c}(t)] x_2^4 - [p - \hat{p}(t)] x_3 - k_3 x_3 \end{cases} \tag{28}$$

To simplify (28), we define the parameter estimation error as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \tag{29}$$

Substituting (29) into (28), we obtain

$$\begin{cases} \dot{x}_1 = e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = -k_2 x_2 \\ \dot{x}_3 = e_b - e_c x_2^4 - e_p x_3 - k_3 x_3 \end{cases} \tag{30}$$

Differentiating the parameter estimation error (29) with respect to t, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \end{cases} \tag{31}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_p) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right), \tag{32}$$

which is positive definite on R^7.

Differentiating V along the trajectories of (30) and (31), we obtain

$$\begin{aligned} \dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_2 - \dot{\hat{b}}\right] + e_c\left[-x_2^4 x_3 - \dot{\hat{c}}\right] \\ + e_p\left[-x_3^2 - \dot{\hat{p}}\right] \end{aligned} \tag{33}$$

In view of (33), we define an update law for the parameter estimates as

$$\begin{cases} \dot{\hat{a}} = x_1(x_2 - x_1) \\ \dot{\hat{b}} = x_2 \\ \dot{\hat{c}} = -x_2^4 x_3 \\ \dot{\hat{p}} = -x_3^2 \end{cases} \tag{34}$$

Theorem 1. The novel chaotic system (26) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (27) and the parameter update law (34), where k_i, (i = 1, 2, 3) are positive constants.

***Proof*.** The result is proved using Lyapunov stability theory [174]. We consider the quadratic Lyapunov function V defined by (32), which is a positive definite function on R^7.

Substituting the parameter update law (34) into (33), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{35}$$

which is a negative semi-definite function on R^7.

Therefore, it can be concluded that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[x_1(t) \quad x_2(t) \quad x_3(t) \quad e_a(t) \quad e_b(t) \quad e_c(t) \quad e_p(t)\right]^T \in L_\infty. \tag{36}$$

We define

$$k = \min\{k_1, k_2, k_3\}. \tag{37}$$

Then it follows from (34) that

$$\dot{V} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\dot{V}. \tag{38}$$

Integrating the inequality (38) from 0 to t, we get

$$k\int_o^t \|x(\tau)\|^2 \, d\tau \;\le\; -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{39}$$

From (39), it follows that $x(t) \in L_2$

Using (30), we can conclude that $\dot{x}(t) \in L_\infty$.

Hence, using Barbalat's lemma [174], we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$. This completes the proof. ■

4.1. Numerical Results

For the novel chaotic system (26), the parameter values are taken as in the chaotic case (2).

We take the feedback gains as $k_i = 6$ for $i = 1, 2, 3$.

The initial values of the chaotic system (13) are taken as

$$x_1(0) = 3.1, \; x_2(0) = 5.4, \; x_3(0) = 7.6 \tag{40}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 3.4, \; \hat{b}(0) = 10.1, \; \hat{c}(0) = 6.9, \; \hat{p}(0) = 4.2 \tag{41}$$

Figure 6 depicts the time-history of the controlled novel chaotic system.

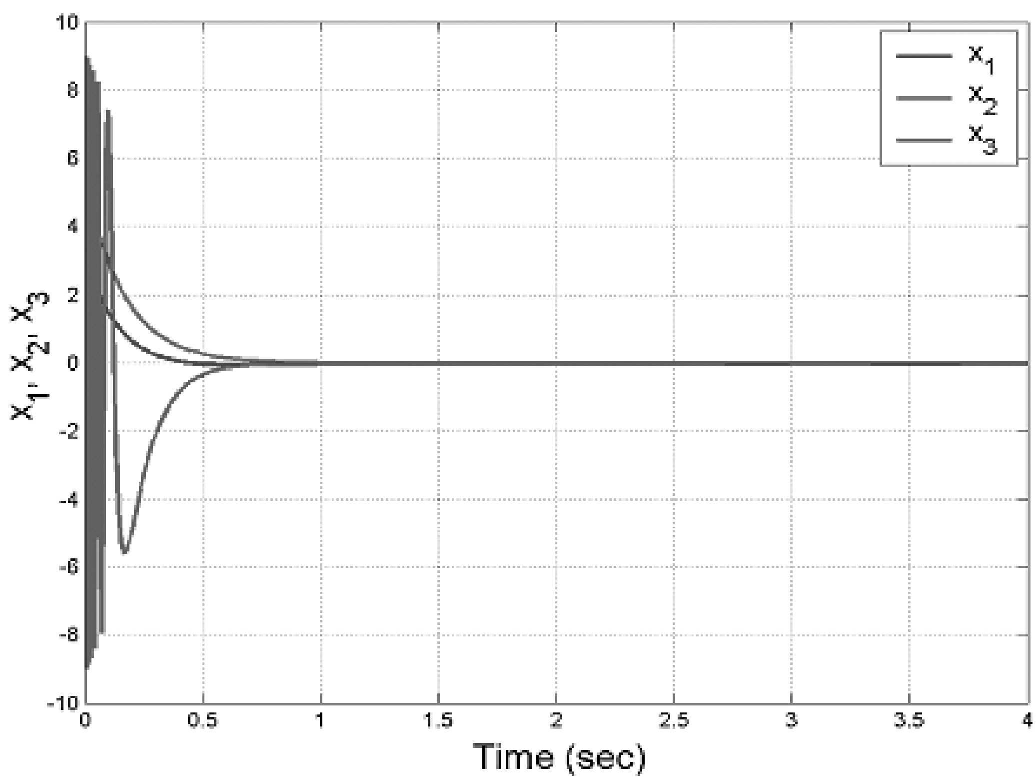

Figure 6: Time history of the controlled novel chaotic system

5. ADAPTIVE SYNCHRONIZATION OF THE NOVEL 3-D CHAOTIC SYSTEMS WITH UNKNOWN PARAMETERS

In this section, we derive new results for the adaptive synchronization of the identical novel chaotic systems with unknown parameters.

As the master system, we take the novel 3-D chaotic system

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = x_2 + x_1 x_3 \\ \dot{x}_3 = b - c x_2^4 - p x_3 \end{cases} \tag{42}$$

where x_1, x_2, x_3 are state variables and a, b, c, d, p are constant, unknown, parameters of the system.

As the slave system, we take the controlled novel 3-D chaotic system

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = y_2 + y_1 y_3 + u_2 \\ \dot{y}_3 = b - c y_2^4 - p y_3 + u_3 \end{cases} \tag{43}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controllers to be designed.

The synchronization error is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{44}$$

The error dynamics is easily obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1 \\ \dot{e}_2 = e_2 + y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 = -c(y_2^4 - x_2^4) - p e_3 + u_3 \end{cases} \tag{45}$$

We consider the adaptive control law defined by

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) - k_1 e_1 \\ u_2 = -e_2 - y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 = \hat{c}(t)(y_2^4 - x_2^4) + \hat{p}(t) e_3 - k_3 e_3 \end{cases} \tag{46}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (46) into (45), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = [a - \hat{a}(t)](e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -[c - \hat{c}(t)](y_2^4 - x_2^4) - [p - \hat{p}(t)] e_3 - k_3 e_3 \end{cases} \tag{47}$$

To simplify the error dynamics (47), we define the parameter estimation error as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \tag{48}$$

Using (48), we can simplify the error dynamics (35) as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -e_c(y_2^4 - x_2^4) - e_p e_3 - k_3 e_3 \end{cases} \tag{49}$$

Differentiating the parameter estimation error (36) with respect to t, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \end{cases} \tag{50}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_c, e_p) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_c^2 + e_p^2\right), \tag{51}$$

which is positive definite on R^6.

Differentiating along the trajectories of (49) and (50), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_c\left[-e_3(y_2^4 - x_2^4) - \dot{\hat{c}}\right] + e_p\left[-e_3^2 - \dot{\hat{p}}\right] \tag{52}$$

In view of (40), we define an update law for the parameter estimates as

$$\begin{cases} \dot{\hat{a}} = e_1(e_2 - e_1) \\ \dot{\hat{c}} = -e_3(y_2^4 - x_2^4) \\ \dot{\hat{p}} = -e_3^2 \end{cases} \tag{53}$$

Theorem 2. The identical novel chaotic systems (42) and (43) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (46) and the parameter update law (53), where k_i, (i = 1, 2, 3) are positive constants.

Proof. The result is proved using Lyapunov stability theory [174]. We consider the quadratic Lyapunov function V defined by (51), which is a positive definite function on R^6.

Substituting the parameter update law (53) into (52), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{54}$$

which is a negative semi-definite function on R^6.

Thus, it can be concluded that the synchronization vector $e(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[e_1(t) \quad e_2(t) \quad e_3(t) \quad e_a(t) \quad e_c(t) \quad e_p(t)\right]^T \in L_\infty. \tag{55}$$

We define

$$k = \min\{k_1, k_2, k_3\}. \tag{56}$$

Then it follows from (42) that

$$\dot{V} \le -k\|e\|^2 \text{ or } k\|e\|^2 \le -\dot{V}. \tag{57}$$

Integrating the inequality (57) from 0 to t, we get

$$k\int_o^t \|e(\tau)\|^2 \, d\tau \;\le\; -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{58}$$

Therefore, we can conclude that $e(t) \in L_2$.

Using (49), we can conclude that $\dot{e}(t) \in L_\infty$.

Hence, using Barbalat's lemma [174], we can conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

5.1. Numerical Results

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.*

$$a = 2, \; b = 5, \; c = 2, \; p = 0.1 \tag{59}$$

We take the feedback gains as $k_i = 6$ for $i = 1, 2, 3$.

The initial values of the master system (42) are taken as

$$x_1(0) = 1.2, \; x_2(0) = 5.7, \; x_3(0) = 2.1 \tag{60}$$

The initial values of the slave system (43) are taken as

$$y_1(0) = 7.8, \; y_2(0) = 3.6, \; y_3(0) = 1.7 \tag{61}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 6.4, \; \hat{b}(0) = 9.5, \; \hat{p}(0) = 5.9 \tag{62}$$

Figures 7-9 depicts the complete synchronization of the identical novel chaotic systems.

Figure 10 depicts the time-history of the synchronization errors.

6. CONCLUSIONS

In this paper, we have derived a seven-term novel 3-D chaotic system with two nonlinearities – a quadratic nonlinearity and a quartic nonlinearity. We gave a qualitative analysis of the mathematical properties of the novel 3-D chaotic system. We determined the Lyapunov exponents and Lyapunov dimension of the chaotic

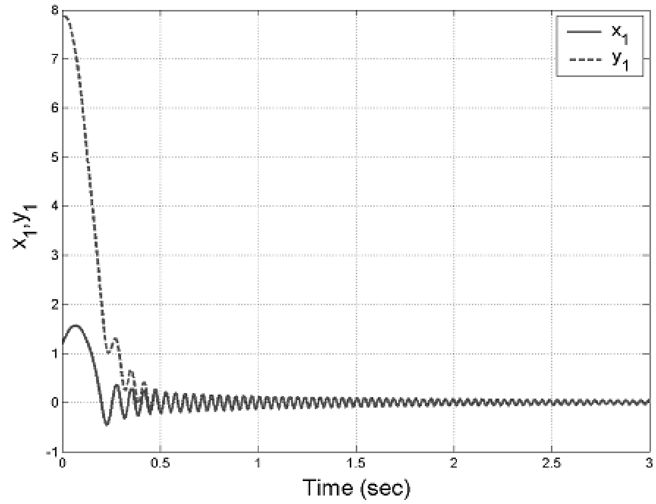

Figure 7: Complete synchronization of the states x_1 and y_1

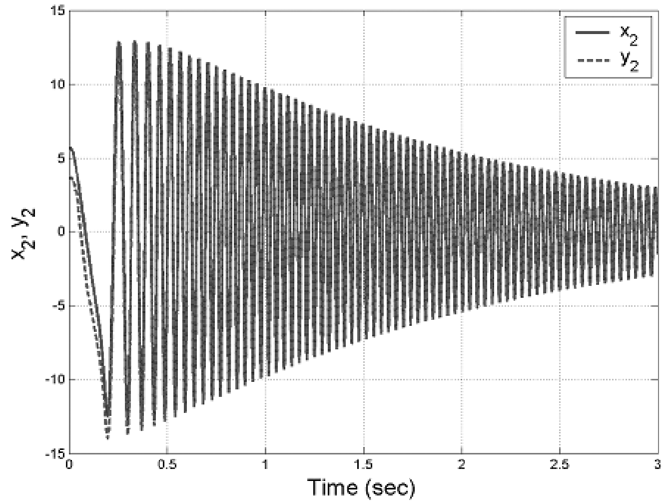

Figure 8: Complete synchronization of the states x_2 and y_2

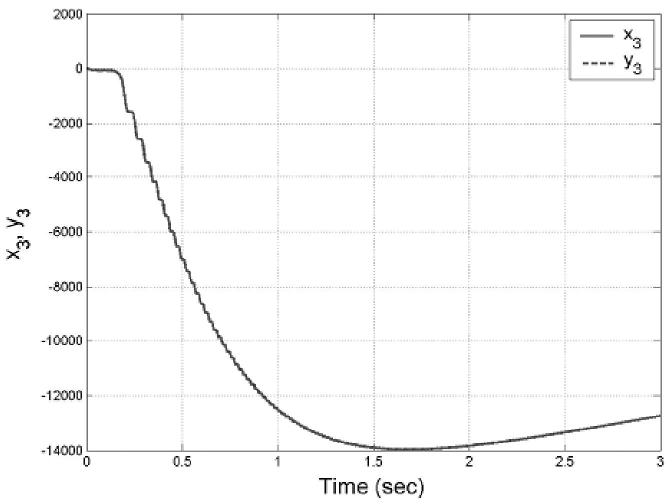

Figure 9: Complete synchronization of the states x_1 and y_1

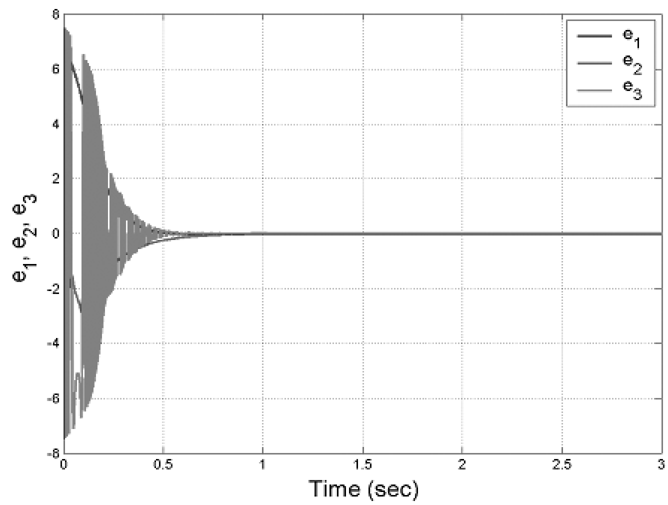

Figure 10: Time history of the chaos synchronization errors e_1, e_2, e_3

system. Next, we have derived adaptive control and synchronization results for the novel chaotic system with unknown parameters, which have been established using Lyapunov stability theory. Numerical simulations with MATLAB were exhibited to demonstrate the phase portraits of the novel chaotic system and the adaptive results derived in this paper.

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