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A FUZZY BASED MULTI-CRITERIA DECISION MAKING APPROACH ON TRANSPORTATION PROBLEM

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Abstract: In this study, both linear single and multi-objective transportation problems are formulated under fuzzy environment. Multi-objective transportation problem is a particular type of problem where all the constraints are of equality type and the objectives are conflicting in nature. Minimization of total transportation cost, total transportation times etc. are considered as different objectives for a multi-objective transportation model. Different approaches such as the step method, the utility function method, the lexicographic method and interactive methods have been developed by many researchers for multi-objective programming problem. For a multi-objective transportation problem with k number of objectives, the existing methods generates a set of K nondominated solutions and an optimal compromise solution. In first part, fuzzy linear programming technique is applied to linear single objective transportation problem and in second part multi-objective transportation problem which emphasizes on optimal compromise solution as well as efficient solution.

Keywords: Multi-objective problem, Transportation problem, fuzzy linear programming.

1. INTRODUCTION

A common problem involving distribution of goods from manufacturer to customers can be described as "Transportation Problem". It is a special type of linear programming problem which arises in many practical situations. In the earlier stage it was used for determining the optimal shipping pattern. The conventional transportation problem consists of transporting certain amount of product from different sources to different destinations. The transportation model was originally developed by Hitchcock (1941). Later Diaz (1979) developed an algorithm for finding the solution of multi-objective transportation problem.

Initial basic feasible solution of a transportation problem is obtained by using the North west corner rule, the Matrix minima method or the Vogel's Approximation Method (VAM). After obtaining initial basic feasible solution, to get optimal solution for the transportation problem we use MODI method (Modified Distribution

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Method). Charnes and Cooper (1954) developed the Stepping Stone Method (SSM) which is considered as an alternative method for getting optimal solution of the transportation problem.

The Linear Interactive and Discrete Optimization[LINDO](Schrage,1984), the General interactive Optimizer[GINO](Liebman and Scrage,1981) and TORA packages(Taha,1992) as well as other packages handles the transportation problem in explicit equation form and thus solve the problems as a standard linear programming problem. Diaz (1978) developed two compromise functions approach for solving multi objective transportation problem. Diaz (1979) and Isermann (1979) each developed algorithms for identifying all nondominated solutions for a multi objective transportation problem. The concept of decision making in a fuzzy environment was first proposed by Bellmann and Zadeh (1970). The application of fuzzy optimization technique to solve the linear programming problem with several objective functions was applied by Zimmermann (1978).

In this paper, we have discussed a Mamdani fuzzy inference approach with four input parameter such as (Transportation Time, Shipping distance, Mode of transportation and Service charges with two output max and min as a transportation cost. Secondly, a fuzzy programming approach with linear membership is used to find an optimal compromise solution for the multi-objective transportation problem. In first part the solution of multi-objective transportation problems with equal type constraints are discussed and in second part we discuss multi-objective transportation problem with mixed type of constraints.

1.2 Mathematical Formulation

Consider the m - sources S_1, S_2 …. Sm and n -destinations D_1, D_2 …Dn with k-objectives Z_1, Z_2 …Z_k. Without loss of generality let us assume all the objectives are of minimization type. Suppose the source S_i has an availability ai (i = 1, 2 …m) and the destination D_j has the requirement bj (j = 1, 2 ….n). Let for each objective Z_k, the penalty $Cij^{(k)}$ associated with transporting a single unit from source Si to destination Dj. The penalty could represent transportation cost, delivery time, quantity of products delivered and many others. A variable xij represents the unknown quantity transported from source Si to destination Dj.

It is usual to assume that the balance condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ holds i.e total supply unit and total demand unit are equal.

The mathematical model for a multi-objective transportation problem is stated as:

Min $$Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}{}^{k} x_{ij} \quad k = 1, 2... K \tag{1.1}$$

Subject to $$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i = 1, 2 ...m \tag{1.2}$$

$$\sum_{j=1}^{n} x_{ij} = b_i \text{ for j = 1, 2 ...n} \tag{1.3}$$

$$x_{ij} \geq 0 \tag{1.4}$$

1.3 Fuzzy Linear Programming

Fuzzy Linear programming occurs when fuzzy set theory is applied in linear multi criteria decision making problems. In fuzzy set theory an element X has a degree of membership in a set A, denoted by a membership function $\mu_A(X)$. The range of the membership function is [0, 1]. In multi criteria decision making problems, the objective functions are represented by fuzzy sets and the decision set is defined as the interaction of all the fuzzy sets and constraints. The decision rule is to select the solution having the highest membership of the decision set. Zadeh (1965) first introduced the concept of fuzzy set theory. For further discussion about fuzzy set theory, the reader may refer to Kaufmann (1976). Fuzzy linear programming with multiple objective functions was introduced by Zimmermann (1978). He first applied fuzzy set theory concept with suitable choices of membership functions and derived a fuzzy linear program which is identical to the maxmin program (Dyson (1980). Dyson (1980) has claimed that the fuzzy linear programming model is an innovation in the field of multi criteria decision making. Zimmermann (1978) presented the application of fuzzy linear programming approach to the linear vector maximum problem. He showed that solutions obtained by fuzzy linear programming are always efficient solutions and also gives an optimal compromise solution.

The multi-objective transportation problem is a vector minimum problem. In the case of the multi-objective fuzzy linear programming technique, only the objectives are fuzzy. The fuzzy linear programming technique for multi objective transportation problem gives an optimal compromise solution.

Currently, the fuzzy programming technique is applied to solve linear as well as non linear multi-objective programming problems. This approach is similar, in many respects, to the weighted linear goal programming method. Weighted linear goal programming depends on the development of weights whereas fuzzy programming uses fuzzy membership functions.

1.4 Fuzzy Rule based systems

A knowledge base (KB) and inference engine (IE) are two main components of fuzzy rule based systems (FRBS).There are various ways to represent knowledge .the KB generally represents the knowledge about the problem being solved in the form of fuzzy linguistic. IF- THEN rules, and the IE ,which puts in effect the fuzzy inference process, is needed to obtain an output from the FRBS, when an input is specified. This form of expression is commonly referred as the IF- THEN rule based form, like IF premise(antecedent),THEN conclusion(consequent) parameters. The schematic view of an FRBS is shown in Figure 1.

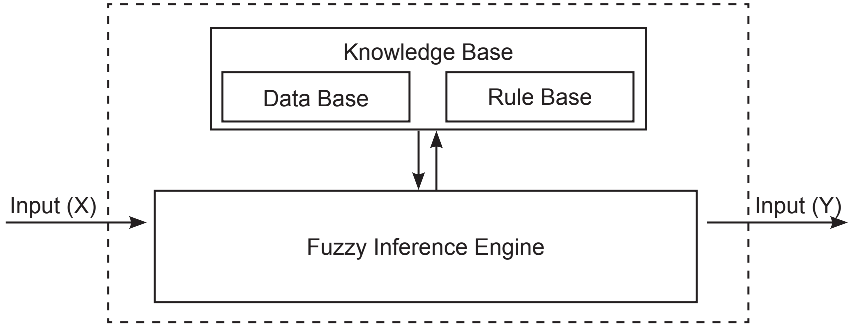

Fig 1: A schematic view of Fuzzy Rule Base System

An FRBS consists of three modules namely fuzzification, inference and defuzzification. Fuzzification is the process, in which the input parameters are converted into appropriate fuzzy sets to express measurement uncertainty. The fuzzified measurements are then used by the IE to evaluate the control rules stored in the fuzzy rule base and a fuzzified output is determined. This conversion is called as de-fuzzification.

1.5 Description of fuzzy input parameters

Each of the input parameters except mode of transportation(M) are represented in terms of linguistic variables like Low (L), Medium (M), High(H) and Very High (VH) whereas modes of transportation (M) is represented as Road (R), Train (T), Ship (S) and Flight(F). Each of the input parameters are represented in linguistic terms with their ranges shown in Figure 2

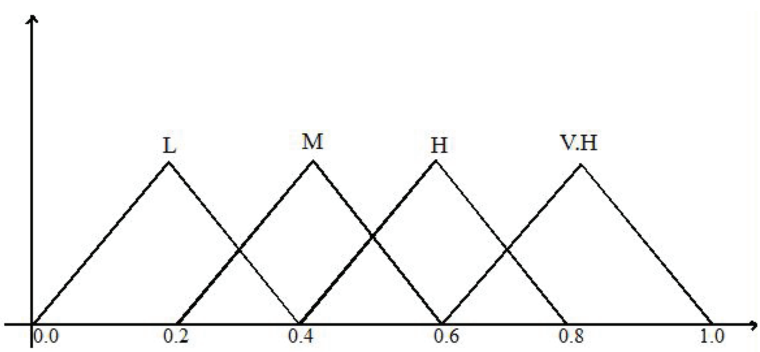

Fig. 2(a): Membership function for input fuzzy parameters
X_1= {Transportation Time}, X_2= {Shipping Distance}, X_3= {Service charge}

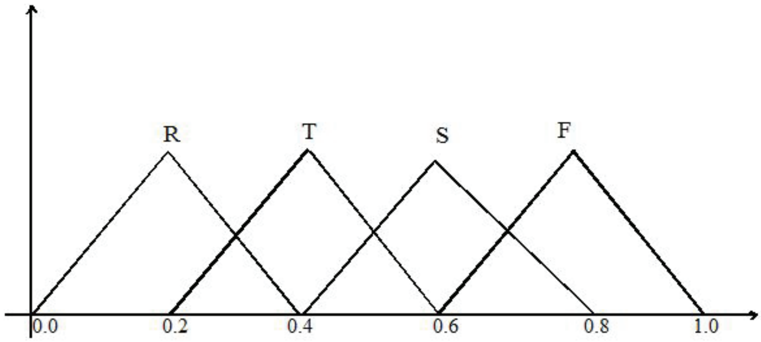

Fig.2 (b): Membership function for input fuzzy parameter X_4= {Mode of Transportation}

Table 1
Linguistic term and their range for the input parameters X_1 = {Transportation Time}, X_2 = {Shipping Distance}, X_3 = {Service charge}

Linguistic terms	*Membership function*	*Range of parameter*
Low (L)	Trimf	[0.0 , 0.4]
Medium (M)	Trimf	[0.2 , 0.6]
High (H)	Trimf	[0.4 , 0.8]
Very high(VH)	Trimf	[0.6 , 1.0]

Table 2
Linguistic term and their range for the input parameter X_4 = {Mode of Transportation}

Linguistic terms	*Membership function*	*Range of parameter*
Road (R)	Trimf	[0.0 , 0.4]
Train (T)	Trimf	[0.2 , 0.6]
Ship(S)	Trimf	[0.4 , 0.8]
Flight (F)	Trimf	[0.6 , 1.0]

1.6 Description of fuzzy output parameter

Two linguistic terms, namely Max and Min are used to represent output parameter, transportation cost. The Mamdani min-operator is used for aggregation and defuzzification is made using the Centre of the Sums (COS) method (Pratihar, 2008). Membership function distribution for output fuzzy parameter, Max and Min shown in Figure 3.

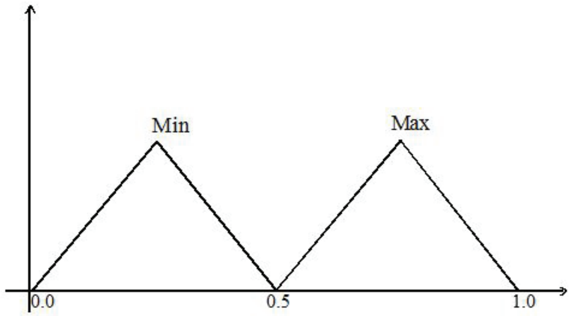

Fig. 3: Membership function for output fuzzy parameter

1.7 Determining fuzzy rule base from input and output variables

The traditional fuzzy reasoning tool is developed by using four input variables transportation time (T), shipping distance (D), Mode of transportation (M) and service charge (SC). Each of them except the input variable mode having four different responses like low (L), medium (M), high (H) and very high (VH) whereas the response of the input variable mode is considered as road(R), train (T), ship(S) and flight (F). A set of 256 rules are designed manually (shown in Appendix A) as follows.

Rule 1: if transportation time (T) is **Low**, shipping distance (D) is **Low**, Mode of transportation (M) is **road(R)** and service charge (SC) is **Low** then transportation cost is **Min.**

Rule 2: if transportation time (T) is **Low**, shipping distance (D) is **Low**, Mode of transportation (M) is **road(R)** and service charge (SC) is **Medium** then transportation cost is **Min.**

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Rule 256: if transportation time (T) is **Very High**, shipping distance (D) is **Very High**, Mode of transportation (M) is **flight (F)** and service charge (SC) is **Very High** then transportation cost is **Max.**

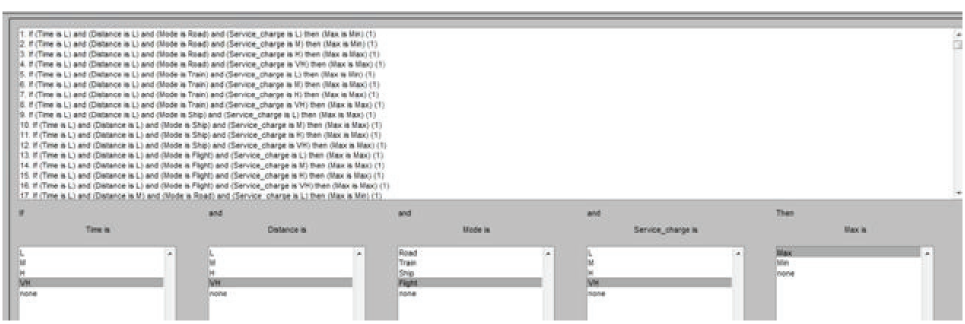

Figure 4: Number of rule fire out of 256 rules

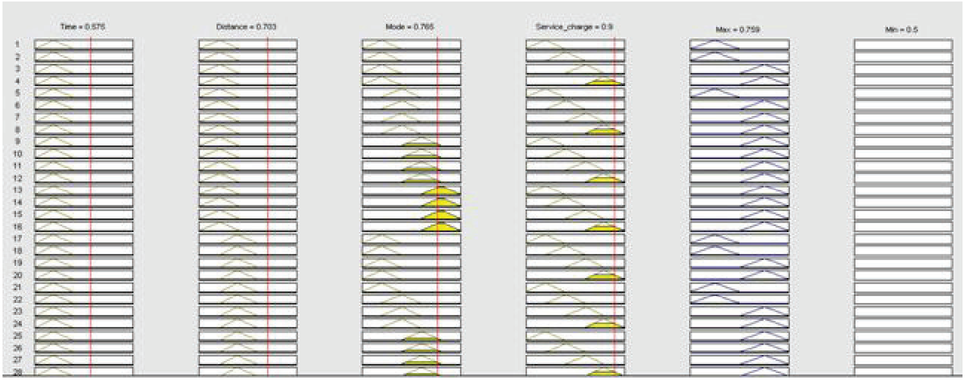

Figure 5: Fuzzy Encoding rules base on basis of antecedent and consequence

2.0 WORKING PRINCIPLE OF TRADITIONAL FLC (MAMDANI APPROACH)

An FLC consist of a set of rules presented in the form of IF (a set of conditions are satisfied) THEN (a set of consequences can be prepared). Here, the antecedent is a condition in its application domain and the consequent is a control action for the system under control. Both the antecedent and consequents of the IF-THEN rules are represented using some linguistic terms (Kriti, Mohanty, Mohanty, 2017). The inputs of FRBS should be given by fuzzy sets and therefore, to get the corresponding crisp value, a method of defuzzification is to be used. The fuzzification of input parameters involbs the followings:

(i) Measure of all input variables

(ii) Perform a scale mapping that transfers the ranges of values of input parameters into corresponding universe of discourse.

(iii) Perform the function of fuzzification that converts input data to suitable linguistic values, which may be viewed as the label of fuzzy sets.

The rule base comprises knowledge of the application domain by using the information of the database. Thus, the database provides necessary data to design the control rules involving linguistic terms. The rule base characterizes the control goals and policy of the domain experts by means of a set of linguistic control rules.

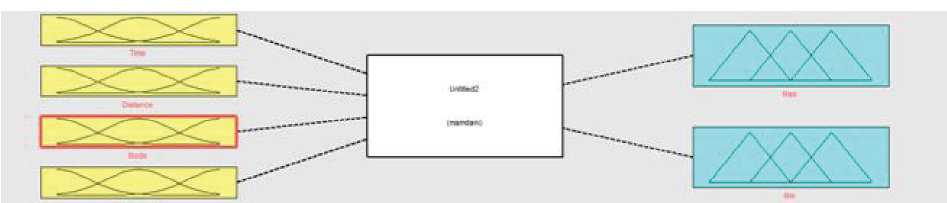

Figure 6: Four input and Two output Fuzzy Mamdani Approach

The IE of an FLC has the capability of simulating human DM based fuzzy concepts and of inferring fuzzy control actions by employing fuzzy implication and the rules. A method of defuzzification is used to obtain the crisp value corresponding to the fuzzified output. In this study, COS method of defuzzification was utilized, this is given below.

2.1 Results and Discussion

The results reveal that transportation cost directly depends upon shipping distance and transportation time. Transportation cost increase with increase of transportation time and shipping distance (see fig 7a).In transportation problem, mode of transportation and distance having major impact to compute the total transportation cost. In this study transportation cost is very high in case of Flight mode of transportation and more distance to deliver the product (see fig 7b). It is observed that for minimum transportation cost, in input value of service charges and distance amounts are to be less (see fig 7c). Similarly, wherever service charges high and shipping time less then cost of the transportation may be expensive(see fig 7d). It can be noticed that when service charges increases and mode of transportation is Flight then total transportation cost increase (see fig 7e).

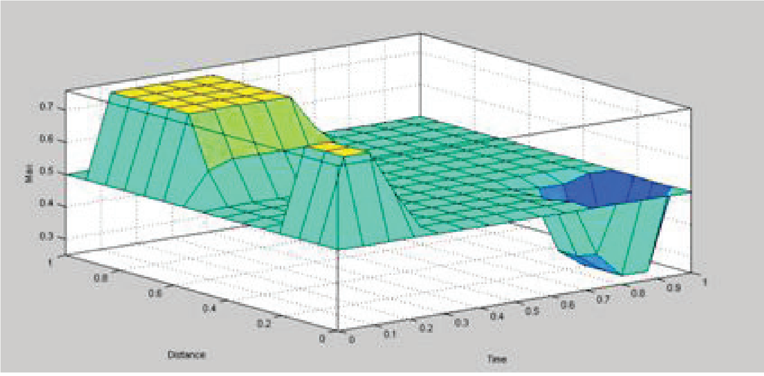

Figure 7(a): Transportation Time Vs. Shipping distance

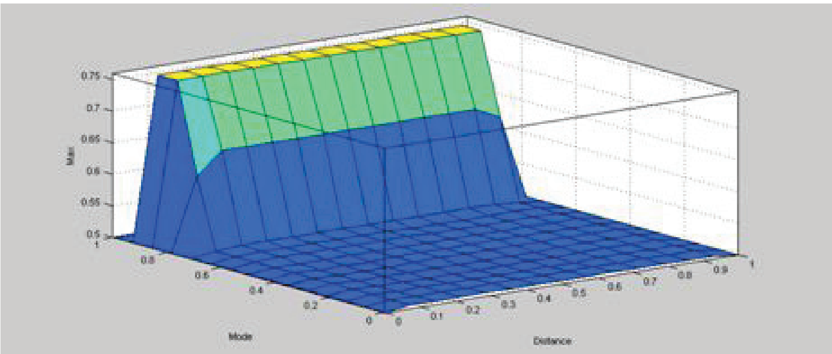

Figure 7(b): Mode of transportation Vs. Transportation cost

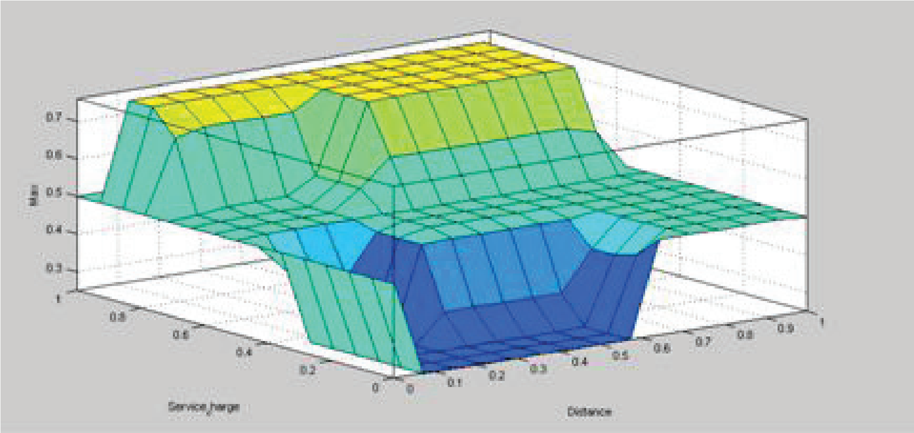

Figure 7(c): Service charges vs. Distance

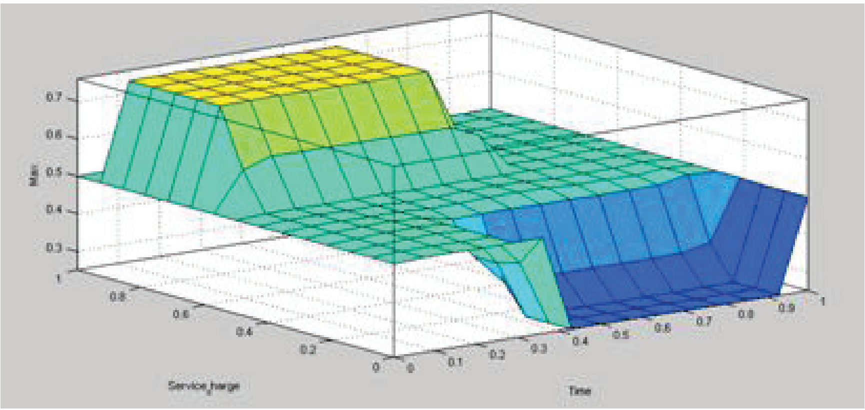

Figure 7(d): Service charges vs. Time

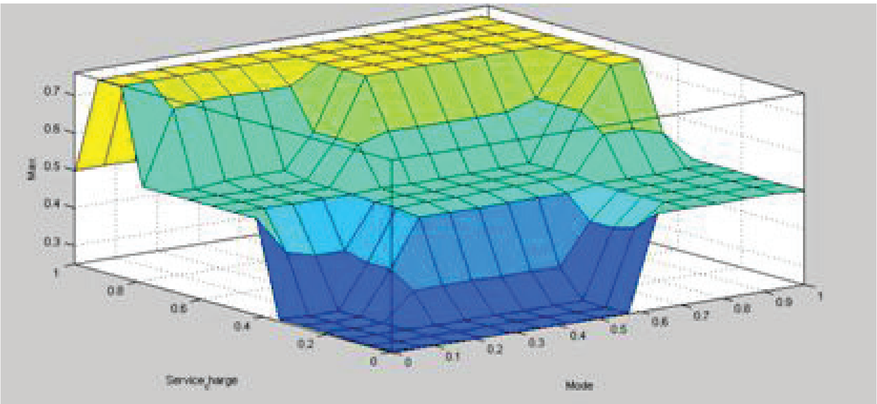

Figure 7(e): Service charge vs. mode of transportation

2.2 Fuzzy Logic System for Transportation Problem

Basic results associated to the development of fuzzy logic date back to Zadeh (1973) and Mamdani and Assilian (1975). Pappis and Mamdani (1977) were first solved the transportation problem by using fuzzy logic. In the mid of 1980 some of the Japanese authors made significant contribution to fuzzy set theory applications in transportation problem. Many of the problems in the field of transportation problem are often not well defined, ambiguous and vague. The phenomena and parameters used in transportation problem are characterized by subjectivity. It is very difficult to disregard the fact that subjective judgment present in the transportation problems dealing with choice of route, criteria to rank alternate transportation plan or project, mode of transportation, level of service etc. The purpose of the study is to minimize the objective functions Z_k (k = 1, 2 …K) along with the constraints using fuzzy logics connecting factors like transportation time (T), shipping distance (D), mode of transportation (M) and service charge (SC) respectively. The above factors are considered as input variables and are fuzzy in nature.

2.3 Fuzzy Programming technique for the multi-objective transportation problem

The first step is to assign for each objective, two values U_k and L_k as upper and lower bound for the k-th objective, where L_k = aspired level of achievement for the k-th objective U_k = highest acceptable level of achievement for the k-th objective, and $d_k = U_k - L_k$ = the degradation allowance for the k-th objective.

Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a crisp model i.e a conventional linear programming problem the steps of the fuzzy programming technique are as follows:

Step1. Solve the multi-objective transportation problem as a single objective transportation problem using each time, only one objective and ignoring all others.

Step2. From the results of step 1 determine the corresponding values for every objective at each solution derived.

Step3. From step 2 we may find, for each objective, the best (L_k) and Worst (U_k) values corresponding to the set of solutions. The initial fuzzy model is then given by the aspiration levels with each objective, as follows:

Find x_{ij}, i = 1, 2, ...m; j = 1, 2, ...n so as to satisfy

$$Z_k \le L_k \quad k = 1, 2, \ldots K$$

and constraints (1.2), (1.3), and (1.4).

For the multi-objective transportation problem, a membership function $\mu_k(x)$ corresponding to k-th objective is defined as

$$\mu_1(x) = \begin{cases} 1 & \text{if } Z_k \le L_k \\ 1 - \left(\dfrac{Z_k - L_k}{U_k - L_k} \right) & \text{if } L_k < Z_1 < U_k \\ 0 & \text{if } Z_k \ge U_k \end{cases}$$

We also sometimes use the symbol $M_k(Z_k)$ instead of $U_k(x)$

The equivalent linear programming problem for the vector minimum problem is as follow:

Maximize λ

subject to $$\lambda \le \left(\frac{U_k - Z_k}{U_k - L_k} \right) \text{ for all k}$$

and constraints (1.2), (1.3), (1.4), and $\lambda \le 0$,

where $$\lambda = \min_k \{\mu_k(x)\}$$

This linear programming problem can be further simplified as:

Maximize λ

Subjected to $$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij} + (U_k - L_k) \le U_k \quad \text{k = 1....K}$$

and constraints (1.2), (1.3), (1.4)

This problem gives a set of non dominated solution and an optimal compromise solution for linear vector maximum or minimum problems. The subroutine SSAR supplies the necessary slack, surplus and artificial variables to perform the simplex algorithm. The sub routines MAX and MIN calculate upper and lower values of each objective, respectively. The sub routine OBJ calculates the optimal compromise value of each objective. The payoff matrix in the main program gives the set of non dominated solutions. For the single-objective linear programming problems the computer code has been presented by Gillet (1976). This code has been extended for multi-objective linear programming problems by adding some suitable sub routines.

To illustrate the use of fuzzy programming technique for solving multi-objective transportation problems, we consider the following two numerical examples. Based on these two examples, we also compare the fuzzy programming technique with the algorithms proposed by Ringuest and Rinks (1987).

Example 1

This example is considered from Diaz (1978)

Minimize

$$Z_1 = 9X_{11} + 12\,X_{12} + 9X_{13} + 6X_{14} + 9X_{15} + 7X_{21} + 3X_{22} +$$

$$7X_{23} + 7X_{24} + 5X_{25} + 6X_{31} + 5X_{32} + 9X_{33} + 11X_{34} + 3X_{35} + 6x_{41} + 8x_{42} + 11X_{43} + 2X_{44} + 2X_{45} \quad (1.5)$$

Minimize

$$Z_2 = 2X_{11} + 9X_{12} + 8X_{13} + X_{14} + 4X_{15} + X_{21} + 9X_{22} + 9X_{23} + 5X_{24} + 2X_{25} + 8X_{31} + X_{32} + 8X_{33} + 4X_{34} + 5X_{35} + 2x_{41} + 8x_{42} + 6X_{43} + 9X_{44} + 8X_{45} \quad (1.6)$$

Minimize

$$Z_3 = 2X_{11} + 4X_{12} + 6X_{13} + 3X_{14} + 6X_{15} + 4X_{21} + 8X_{22} + 4X_{23} + 9X_{24} + 2X_{25} + 5X_{31} + 3X_{32} + 5X_{33} + 3X_{34} + 6X_{35} + 6x_{41} + 9x_{42} + 6X_{43} + 3X_{44} + X_{45} \quad (1.7)$$

$$\sum_{j=1}^{5} X_{1j} = 5 \quad \sum_{j=1}^{5} X_{2j} = 4 \quad \sum_{j=1}^{5} X_{3j} = 2 \quad \sum_{j=1}^{5} X_{4j} = 9$$

$$\sum_{i=1}^{4} X_{i1} = 4 \quad \sum_{i=1}^{4} X_{i2} = 4 \quad \sum_{i=1}^{4} X_{i3} = 6 \quad \sum_{i=1}^{4} X_{i4} = 2 \quad \sum_{i=1}^{4} X_{i5} = 4 \quad (1.8)$$

$$X_{ij} \geq 0 \quad i = 1, 2, 3, 4; \quad j = 1, 2, 3, 4, 5 \quad (1.9)$$

Based on their algorithm, Ringuest and Rinks (1987) have obtained the most preferred values of the objectives Z_1, Z_2 and Z_3 as 127, 104 and 76 respectively.

The fuzzy programming algorithm is illustrated in Example 1 is as follow:

Step 1 and Step 2:

Optimal Solutions for minimizing the first objective Z_1 subject to constraints (1.8), and (1.9) are as follows:

$$X_{13} = 5, X_{23} = 1, X_{31} = 1, X_{32} = 1, X_{41} = 3$$

$$X_{44} = 2, X_{45} = 4, \text{ with } Z_1x^*_1 = 102, Z_2x^*_1 = 141, Z_3x^*_1 = 94$$

Optimal solutions for minimizing the second objection Z2 subject to constraints (1.8) and (1.9) are as follows:

$$X_{11} = 3, X_{14} = 2, X_{25} = 4, X_{32} = 2, X_{41} = 1, X_{42} = 2,$$

$$X_{43} = 6 \text{ with } Z_1x^*_2 = 157, Z_2x^*_2 = 72, Z_3x^*_2 = 86$$

Optimal solutions for minimizing the third objective Z_3 subject to constraints (1.8) and (19) are as follows:

$$X_{11} = 3, X_{12} = 2, X_{21} = 1, X_{23} = 3, X_{32} = 2,$$

$$X_{43} = 3, X_{44} = 2, X_{45} = 4 \text{ with } Z_1x^*_3 = 134, Z_2x^*_3 = 122, Z_3x^*_3 = 64$$

Step 3:

$$U_1 = 157, U_2 = 141, U_3 = 94$$
$$L_1 = 102, L_2 = 72, L_3 = 64$$

The membership functions $\mu_1(x)$, $\mu_2(x)$ and $\mu_3(x)$ for objective $Z_1(x)$, $Z_2(x)$ and $Z_3(x)$ respectively, are defined as follows:

$$\mu_1(x) = \begin{cases} 1 & \text{if } Z_1 \le 102 \\ \left(\dfrac{157 - Z_1}{157 - 102}\right) & \text{if } 102 < Z_1 < 157 \\ 0 & \text{if } Z_1 \ge 157 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & \text{if } Z_2 \le 72 \\ \left(\dfrac{141 - Z_2}{141 - 72}\right) & \text{if } 72 < Z_2 < 141 \\ 0 & \text{if } Z_2 \ge 141 \end{cases}$$

$$\mu_3(x) = \begin{cases} 1 & \text{if } Z_3 \le 64 \\ \left(\dfrac{94 - Z_3}{94 - 64}\right) & \text{if } 64 < Z_3 < 94 \\ 0 & \text{if } Z_3 \ge 94 \end{cases}$$

Solving the foregoing linear programming problem by the simplex method, we get

$$X_{11} = 2.74, X_{13} = 0.26, X_{14} = 2.0, X_{22} = 2.0, X_{23} = 1.84$$
$$X_{25} = 0.16, X_{32} = 2.0, X_{41} = 1.26, X_{43} = 3.9, X_{45} = 3.84$$

Optimal compromise objective values of Z_1, Z_2 and Z_3 are 126.7930, 103.1039 and 77.5235, respectively. Also, we get set of non dominated solutions, i.e, {102, 141, 94}, {157, 72, 86} and {132, 122, 64}. The fuzzy programming technique is a straight forward method which leads to a set of non dominated (efficient) solutions and an optimal compromise solution.

Applying the fuzzy linear mixed integer programming technique we get A = 0.53623182

$$X_{11} = 3, X_{14} = 2, X_{22} = 2, X_{23} = 2, X_{32} = 2, X_{43} = 4, X_{41} = 1, X_{45} = 4$$
$$Z_1 = 127, Z_2 = 104, Z_3 = 76$$

The objective values coincide with those of Ringuest and Rinks (1987). In this method, we get three non dominated solutions as {102, 141, 94}, {157, 72, 86} and {132, 122, 64} and optimal compromise solutions of {127, 1045, 76}. Ringuest and

Rink (1987) have obtained six non dominated solutions and six dominated solutions. Using each non dominated solution, a decision maker has to find out the compromise function value. Then, the decision maker to decide which non dominated solution gives the best compromise solution.

Example 2

This example is considered from Aneja and Nair (1979).

Minimize

$$Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$$

Minimize

$$Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}$$

Subject to

$$\sum_{j=1}^{4} x_{1j} = 8 \qquad \sum_{i=1}^{3} x_{11} = 11$$

$$\sum_{j=1}^{4} x_{2j} = 19 \qquad \sum_{i=1}^{3} x_{12} = 3$$

$$\sum_{j=1}^{4} x_{3j} = 17 \qquad \sum_{i=1}^{3} x_{13} = 14$$

$$\sum_{j=1}^{3} x_{14j} = 16$$

$$X_{ij} \geq 0, \quad i = 1, 2, 3; \quad j = 1,2,3,4;$$

Ringuest and Rinks (1987) have obtained 156 and 200 as the most preferred values of the objectives Z_1 amd Z_2 respectively by applying their algorithms. Applying the fuzzy linear programming technique to Example 2, we get

$$x_{11} = 3.79, x_{12} = 3.00, x_{13} = 1.21, x_{21} = 7.21, x_{23} = 11.79, x_{33} = 1.00, x_{34} = 16.00,$$
$$Z_1 = 160.8591 \text{ and } Z_2 = 193.9260$$

Applying the fuzzy linear mixed integer programming technique, we get,

$$x_{11} = 4, x_{12} = 3, x_{13} = 1, x_{21} = 7, x_{23} = 12, x_{33} = 1, x_{34} = 16,$$
$$Z_1 = 160, Z_2 = 195.$$

The set of non dominated solutions is {143, 265}, and {208, 167}. Since the objectives could not be minimized at any point the "ideal solution" (at which the objective functions are at their individual minimum as given by the diagonal of the pay-off matrix) is not feasible. In the calculation phase, therefore, the feasible solution which is "nearest" to the ideal solution is searched. Here, the point {160, 195} or the point {160.859, 193.926} is nearer to the ideal solution point {143, 167}, than the point {156, 200}. Hence, the best optimal compromise solution is obtained by fuzzy linear programming.

In general Z_j amd Z_k have different physical dimensions (e.g., cost and time). The ideal point $\underline{Z}^* = \{Z_1^*, Z_2^*\}$ in the two dimensional criterion space corresponds to the vector $\underline{\mu}^*$ in the membership space whose each component is unity (Viz., $\underline{\mu}^* = \{1, 1\}$). Any solution $\underline{Z} = \{Z_1, Z_2\}$ in the criterion space corresponds to the vector $\underline{\mu}^*$ in the membership space is as $\underline{\mu} = \{\mu_1(Z_1), \mu_2(Z_2)\}$. The membership functions $\mu_1(Z_1)$ and $\mu_2(Z_2)$ are commensurable. Distance between two points $\underline{Z}$ and $\underline{Z}^*$, (viz., distance between two points $\underline{\mu}$ and $\underline{\mu}^*$) is defined as positive square root of the expression $1 - \mu_1(Z_1)^2 + 1 - \mu_1(Z_1)^2$.

2.4 Variants of multi-objective transportation problem

A generalization of the multi-objective transportation problem is considered where the origin and destination constraints are not only of equality type but also are of inequality type. Appa (1973) discussed about single objective transportation problem and its variants. He considered the problems by taking all combinations of the form of the coefficients of objective, supply constraints, demand constraints, and relation of total supply and total demand.

We consider the multi-objective transportation problems by taking all combinations of the row constraints, column constraints, and the relation of total supply and total demand of the multi objective transportation problem in this problem.

We define the following terms:

$$(A_1)\, c_{ij}^k \geq 0,\ (A_2)\, c_{ij}^k \leq 0,\ (A_3)\, c_{ij}^k \geq\leq 0,\ \text{for } k = 1,\ 2, \ldots K$$

Row constraints:

$$(\beta_1)\ \Sigma_j x_{ij} \geq a_i,\ \ (\beta_2)\ \Sigma_j x_{ij} = a_i,\ \ (\beta_3)\ \Sigma_j x_{ij} \leq a_i$$

Column constraints:

$$(\beta_1')\ \Sigma_i x_{ij} \geq b_j,\ \ (\beta_2')\ \Sigma_i x_{ij} = b_j,\ \ (\beta_3')\ \Sigma_i x_{ij} \leq b_j$$

Relation of total Supply and total demand:

$$(C_1)\ \Sigma_i a_i > \Sigma_j b_j,\ \ (C_2)\ \Sigma_i a_1 = \Sigma_j b_j,\ \ (C_3)\ \Sigma_i a_i < \Sigma_j b_j$$

For all the cases, we assume $a_i \geq 0$, $b_j \geq 0$, $x_{ij} \geq 0$ for all i, j and the objective functions are minimized.

Some of the repetitive cases of these 81 variants are eliminated and 54 problems are considered in detail. Preliminary analysis of the 54 problems has been discussed by Appa (1973) in detail. In the case of variants of multi objective transportation problem, all the analysis of Appa (1973) holds well because the constraints remain unchanged.

Example 3

To illustrate the fuzzy programming approach to the variants of multi-objective transportation problem, we have adopted a multi-objective transportation problem from Isermann (1979) having the following characteristics

Supplies: $a_1 = 100, a_2 = 125, a_3 = 75$

Demand: $b_1 = 60, b_2 = 80, b_3 = 160$

Penalties:

$$C_1 = \begin{bmatrix} 3 & 1 & -1 \\ 4 & 2 & 5 \\ -1 & 6 & 4 \end{bmatrix}, C_2 = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 6 & 4 \\ 3 & -1 & 3 \end{bmatrix} \text{ and } C^3 = \begin{bmatrix} 7 & 7 & 5 \\ 1 & 7 & 1 \\ 5 & 7 & 8 \end{bmatrix}$$

Isermann (1979) obtained the following seven non dominated solutions for the problem {285, 1185, 1525}, {360, 1095, 1420}, {685, 1030, 1160}, {900, 795, 1180}, {925, 790, 1160}, {1200, 675, 1300}, and {1225, 670, 1280}

Applying the fuzzy programming technique to the problem, we get an optimal compromise solution as:

$$X_{13} = 100, X_{21} = 38.658, X_{22} = 26.342$$

$$X_{23} = 60, X_{31} = 21.342, X_{32} = 53.658$$

$$Z_1 = 707.9245, Z_2 = 901.7086, \text{ and } Z_3 = 1265.3669$$

And three non dominated solutions as:

{285,670, 1160}, {1225, 670, 1280} and {925, 790, 1160}

The efficient solution which is nearer to the ideal solution {285, 670, 1160}, is the best compromise solution.

2.5 Multi-objective transportation problem with mixed constraints

Appa (1973) considered variants of the transportation problem in which the availability and / or demand constraints are in equations as opposed to the usual equations. However, he did not consider the case where availability and demand constraints are of mixed type. The purpose of Brigden (1974) was to demand some of Appa's ideas to cater for the mixed case Klingman and Russel (1974) transformed the transportation problem with mixed constraints into a transshipment problem and then converted the resulting transshipment problem into an equivalent transportation problem. From the optimal solution of the equivalent transportation problem, the optimal solution of the original problem was found out. On the other hand, Brigden (1974) converted the original problem into a related transportation problem with equality type of constraints by augmenting the original problem with the addition of two sources and two destinations. He obtained the optimal solution of the original problem from the optimal solution of the related transportation problem from the optimal solution table of the related transportation problem. Isermann (1982) has shown that there is no need to augment the original problem by the addition of two more sources and two more destinations. Only one more source and one more destination are sufficient to establish the result. The model is useful for many practical purposes such as to investigate the effect of increasing and decreasing the availability at various origins and / or increasing and decreasing the requirement at various destinations.

Mathematically; a multi objective transportations problem with mixed constraints can be stated

$$\text{Min} \quad Z_k = \Sigma_{i=1}^{m} \Sigma_{j=1}^{n} c_{ij}^{k} x_{ij},\ k = 1,\ 2, \ldots K \tag{2.1}$$

Subjected to

$$\Sigma_{j=1}^{n} x_{ij} = a_1,\ i \in I_1 = \{1, 2, ...m_1\} \tag{2.2}$$

$$\geq a_1,\ i \in I_2 = \{m_1 + 1, 2, ...m_2\} \tag{2.3}$$

$$\leq a_1,\ i \in I_3 = \{m_2 + 1...m\} \tag{2.4}$$

$$\Sigma_{i=1}^{m} x_{ij} = b_1, j \in J_1 = \{1, 2, ...n_1\} \tag{2.5}$$

$$\geq b_1, j \in J_2 = \{n_1 + 1...n_2\} \tag{2.6}$$

$$\leq b_1, j \in J_3 = \{n_2 + 1...n\} \tag{2.7}$$

$$x_{ij} \geq 0 \tag{2.8}$$

For $i \in I_1 \cup I_2 \cup I_3 =$ I, the index set of sources,

$j \in J_1 \cup J_2 \cup J_3 = J$, the index set of destinations.

Where $a_i > 0$ for all $i \in I$, $b_j > 0$ for all $j \in J$ and $c_{ij}^{k} \geq 0$ foe all $i \in I, j \in J$ and $k = 1, 2 ...K$

The necessary and sufficient condition for the existence of feasible solution for single objective transportation problem with mixed constraints has been discussed by Brigden (1974). This is also applicable to multi objective transportation problem with mixed constraints.

Example 4

For the multi-objective transportation problem with mixed constraints, the following numerical example is considered.

Min $$Z_k = \Sigma_{i=1}^{3} \Sigma_{j=1}^{3} c_{ij}^{k} x_{ij},\ k = 1,\ 2,\ ...K$$

Subject to $$\Sigma_{j=1}^{4} x_{1j} = 4,\ \Sigma_{j=1}^{4} x_{2j} \geq 6,\ \Sigma_{j=1}^{4} x_{3j} \geq 6,\ \Sigma_{j=1}^{4} x_{4j} \leq 5$$

$$\Sigma_{i=1}^{4} x_{11} = 10,\ \Sigma_{i=1}^{4} x_{12} \geq 2,\ \Sigma_{i=1}^{4} x_{13} \leq 6,\ \Sigma_{i=1}^{4} x_{14} \leq 3$$

$$x_{ij} \geq 0, \text{ for } i = 1, 2, 3, 4; \quad j = 1, 2, 3, 4$$

Where the three penalties are as follows:

$$C^1 = \begin{bmatrix} 9 & 6 & 6 & 4 \\ 8 & 5 & 9 & 2 \\ 1 & 6 & 2 & 2 \\ 2 & 1 & 7 & 5 \end{bmatrix} C^2 = \begin{bmatrix} 9 & 12 & 9 & 6 \\ 7 & 3 & 7 & 7 \\ 6 & 5 & 9 & 11 \\ 6 & 8 & 11 & 2 \end{bmatrix} \text{and } C^2 = \begin{bmatrix} 2 & 2 & 8 & 1 \\ 1 & 9 & 9 & 5 \\ 8 & 1 & 8 & 4 \\ 2 & 8 & 6 & 8 \end{bmatrix}$$

Using the fuzzy programming technique to the problem we get an optimal compromise solution as:

$$x_{13} = 1, x_{14} = 3, x_{21} = 5.217$$

$$x_{22} = 0.7831, x_{31} = 4.7831, x_{32} = 1.217, \quad \lambda = 0.681$$

$$Z_1 = 75.7352, Z_2 = 100.6507, \text{ and } Z_3 = 62.7465$$

2.6 Conclusion

We have formulated single objective transportation problem with the objective to minimize the transportation cost. During this we analyze the problem by considering different combinations. It is observed that shipping distance and transportation time both are directly related to compute the transportation cost. Service charges depend upon the mode of transportation. In this study we conclude, transportation cost mainly depends upon the transportation time and mode of transportation. It is also observed that, Fuzzy programming algorithm is a more convenient and feasible method for finding an optimal compromise solution for the multi-objective transportation problem. For a larger problem, it is not easy to find the compromise solution by using the interactive algorithms. But, using the fuzzy programming method, one can easily find a compromise solution. Interactive algorithms by Ringust and Rinks (1987) and Diaz (1978) are only applicable to a particular type of the multi objective transportation problem where the constraints are of equality type. But the fuzzy programming algorithms are applicable to all types of multi-objective transportation problems, the vector minimum problem and the vector maximum problem. On the whole, fuzzy linear programming is a more suitable method for the multi objective transportation problem.

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APPENDIX A

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
1	L	L	R	L	Min
2	L	L	R	M	Min
3	L	L	R	H	Max
4	L	L	R	VH	Max
5	L	L	T	L	Min
6	L	L	T	M	Max
7	L	L	T	H	Max
8	L	L	T	VH	Max
9	L	L	S	L	Max
10	L	L	S	M	Max
11	L	L	S	H	Max
12	L	L	S	VH	Max
13	L	L	F	L	Max
14	L	L	F	M	Max
15	L	L	F	H	Max
16	L	L	F	VH	Max
17	L	M	R	L	Min
18	L	M	R	M	Min
19	L	M	R	H	Max
20	L	M	R	VH	Max
21	L	M	T	L	Min
22	L	M	T	M	Min
23	L	M	T	H	Max
24	L	M	T	VH	Max
25	L	M	S	L	Max
26	L	M	S	M	Max
27	L	M	S	H	Max
28	L	M	S	VH	Max
29	L	M	F	L	Max
30	L	M	F	M	Max
31	L	M	F	H	Max
32	L	M	F	VH	Max
33	L	H	R	L	Min
34	L	H	R	M	Max
35	L	H	R	H	Max
36	L	H	R	VH	Max

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
37	L	H	T	L	Min
38	L	H	T	M	Max
39	L	H	T	H	Max
40	L	H	T	VH	Max
41	L	H	S	L	Max
42	L	H	S	M	Max
43	L	H	S	H	Max
44	L	H	S	VH	Max
45	L	H	F	L	Max
46	L	H	F	M	Max
47	L	H	F	H	Max
48	L	H	F	VH	Max
49	L	VH	R	L	Max
50	L	VH	R	M	Max
51	L	VH	R	H	Max
52	L	VH	R	VH	Max
53	L	VH	T	L	Max
54	L	VH	T	M	Max
55	L	VH	T	H	Max
56	L	VH	T	VH	Max
57	L	VH	S	L	Max
58	L	VH	S	M	Max
59	L	VH	S	H	Max
60	L	VH	S	VH	Max
61	L	VH	F	L	Max
62	L	VH	F	M	Max
63	L	VH	F	H	Max
64	L	VH	F	VH	Max
65	M	L	R	L	Min
66	M	L	R	M	Min
67	M	L	R	H	Max
68	M	L	R	VH	Max
69	M	L	T	L	Min
70	M	L	T	M	Min
71	M	L	T	H	Max
72	M	L	T	VH	Max
73	M	L	S	L	Min
74	M	L	S	M	Min

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
75	M	L	S	H	Max
76	M	L	S	VH	Max
77	M	L	F	L	Min
78	M	L	F	M	Max
79	M	L	F	H	Max
80	M	L	F	VH	Max
81	M	M	R	L	Min
82	M	M	R	M	Min
83	M	M	R	H	Max
84	M	M	R	VH	Max
85	M	M	T	L	Min
86	M	M	T	M	Min
87	M	M	T	H	Max
88	M	M	T	VH	Max
89	M	M	S	L	Min
90	M	M	S	M	Min
91	M	M	S	H	Max
92	M	M	S	VH	Max
93	M	M	F	L	Max
94	M	M	F	M	Max
95	M	M	F	H	Max
96	M	M	F	VH	Max
97	M	H	R	L	Min
98	M	H	R	M	Min
99	M	H	R	H	Max
100	M	H	R	VH	Max
101	M	H	T	L	Min
102	M	H	T	M	Max
103	M	H	T	H	Max
104	M	H	T	VH	Max
105	M	H	S	L	Min
106	M	H	S	M	Max
107	M	H	S	H	Max
108	M	H	S	VH	Max
109	M	H	F	L	Max
110	M	H	F	M	Max
111	M	H	F	H	Max
112	M	H	F	VH	Max

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
113	M	VH	R	L	Min
114	M	VH	R	M	Min
115	M	VH	R	H	Max
116	M	VH	R	VH	Max
117	M	VH	T	L	Min
118	M	VH	T	M	Max
119	M	VH	T	H	Max
120	M	VH	T	VH	Max
121	M	VH	S	L	Max
122	M	VH	S	M	Max
123	M	VH	S	H	Max
124	M	VH	S	VH	Max
125	M	VH	F	L	Max
126	M	VH	F	M	Max
127	M	VH	F	H	Max
128	M	VH	F	VH	Max
129	H	L	R	L	Min
130	H	L	R	M	Min
131	H	L	R	H	Min
132	H	L	R	VH	Min
133	H	L	T	L	Min
134	H	L	T	M	Min
135	H	L	T	H	Min
136	H	L	T	VH	Max
137	H	L	S	L	Min
138	H	L	S	M	Min
139	H	L	S	H	Max
140	H	L	S	VH	Max
141	H	L	F	L	Min
142	H	L	F	M	Max
143	H	L	F	H	Max
144	H	L	F	VH	Max
145	H	M	R	L	Min
146	H	M	R	M	Min
147	H	M	R	H	Min
148	H	M	R	VH	Max
149	H	M	T	L	Min
150	H	M	T	M	Min
151	H	M	T	H	Max
152	H	M	T	VH	Max

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
154	H	M	S	M	Max
155	H	M	S	H	Max
156	H	M	S	VH	Max
157	H	M	F	L	Min
158	H	M	F	M	Max
159	H	M	F	H	Max
160	H	M	F	VH	Max
161	H	H	R	L	Min
162	H	H	R	M	Min
163	H	H	R	H	Min
164	H	H	R	VH	Max
165	H	H	T	L	Min
166	H	H	T	M	Min
167	H	H	T	H	Max
168	H	H	T	VH	Max
169	H	H	S	L	Min
170	H	H	S	M	Min
171	H	H	S	H	Max
172	H	H	S	VH	Max
173	H	H	F	L	Min
174	H	H	F	M	Max
175	H	H	F	H	Max
176	H	H	F	VH	Max
177	H	VH	R	L	Min
178	H	VH	R	M	Min
179	H	VH	R	H	Max
180	H	VH	R	VH	Max
181	H	VH	T	L	Min
182	H	VH	T	M	Min
183	H	VH	T	H	Max
184	H	VH	T	VH	Max
185	H	VH	S	L	Min
186	H	VH	S	M	Max
187	H	VH	S	H	Max
188	H	VH	S	VH	Max
189	H	VH	F	L	Min
190	H	VH	F	M	Max
191	H	VH	F	H	Max
192	H	VH	F	VH	Max

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
193	VH	L	R	L	Min
194	VH	L	R	M	Min
195	VH	L	R	H	Min
196	VH	L	R	VH	Max
197	VH	L	T	L	Min
198	VH	L	T	M	Min
199	VH	L	T	H	Min
200	VH	L	T	VH	Max
201	VH	L	S	L	Min
202	VH	L	S	M	Min
203	VH	L	S	H	Max
204	VH	L	S	VH	Max
205	VH	L	F	L	Min
206	VH	L	F	M	Min
207	VH	L	F	H	Max
208	VH	L	F	VH	Max
209	VH	M	R	L	Min
210	VH	M	R	M	Min
211	VH	M	R	H	Min
212	VH	M	R	VH	Max
213	VH	M	T	L	Min
214	VH	M	T	M	Min
215	VH	M	T	H	Min
216	VH	M	T	VH	Max
217	VH	M	S	L	Min
218	VH	M	S	M	Min
219	VH	M	S	H	Max
220	VH	M	S	VH	Max
221	VH	M	F	L	Min
222	VH	M	F	M	Min
223	VH	M	F	H	Max
224	VH	M	F	VH	Max
225	VH	H	R	L	Min
226	VH	H	R	M	Min
227	VH	H	R	H	Min
228	VH	H	R	VH	Max
229	VH	H	T	L	Min
230	VH	H	T	M	Min
231	VH	H	T	H	Max

Rule base used by traditional fuzzy reasoning for predicting outputs

Sl no.	*Time*	*Distance*	*Mode*	*Service charges*	*Output*
232	VH	H	T	VH	Max
233	VH	H	S	L	Min
234	VH	H	S	M	Max
235	VH	H	S	H	Max
236	VH	H	S	VH	Max
237	VH	H	F	L	Min
238	VH	H	F	M	Max
239	VH	H	F	H	Max
240	VH	H	F	VH	Max
241	VH	VH	R	L	Min
242	VH	VH	R	M	Min
243	VH	VH	R	H	Max
244	VH	VH	R	VH	Max
245	VH	VH	T	L	Min
246	VH	VH	T	M	Min
247	VH	VH	T	H	Max
248	VH	VH	T	VH	Max
249	VH	VH	S	L	Min
250	VH	VH	S	M	Max
251	VH	VH	S	H	Max
252	VH	VH	S	VH	Max
253	VH	VH	F	L	Max
254	VH	VH	F	M	Max
255	VH	VH	F	H	Max
256	VH	VH	F	VH	Max