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Performance Analysis of Multi User MIMO BC System with Reduced Feedback and Selection Scheme

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Abstract: In this paper, the performance of the MU-MIMO (Multi-user Multiple Input Multiple Output) system is analyzed using the SUS (Semi-orthogonal User Selection) scheme combined with the two-stage feedback. The proposed system is applied on the selected set of users for finding the BER and the ergodic capacity of the system. For this the CSIT (Channel State Information) is requires to obtain throughput using the feedback bits only. The MESC (Maximum Estimated SINR combiner) is calculated for the users to convey their feedback. The analytical result shows that the BER decreases with the increase in number of antennas which helps in increasing the efficiency of the system proposed. The ergodic capacity of the system increases with an increase in the number of feedback bits thus helping in accommodating more number of users.

Index Terms: MU-MIMO (Multi-user Multiple Input Multiple Output), CSIT (Channel State Information), MESC (Maximum Estimated SINR combiner).

1. INTRODUCTION

MU-MIMO [1](Multi-user Multiple Input Multiple Output) provides higher access rates for voice, video and data cannels. This helps in coping up with the challenges that the increasing demand for services poses. For this multiple antennas are deployed at the transmission and reception ends, known as the MIMO channel. With the potential for providing spatial multiplexing gain and diversity gain MU-MIMO communication systems have lured consideration. It is considered as an optimistic approach for future wireless network.DPC (Dirty Paper Coding) [2] the transmission scheme for capacity-achieving MU-MIMO downlink i.e. broadcast channel [3] has computational complication. Thus preventing enforcement in real-time situations. So it should be exploited strategically as it should be reliable for high data-rate transmission. For multi-user scheduling, the base station employs CSI [4] (Channel State Information). The CSIT at the transmitter should be known perfectly, the performance upgradation can be done for achieving utmost capacity, with increasing number of transmission and receiving antennas. Capacity gain is hampered by factors such as information available at the transmitter and the receiver side, channel SINR and co-relation between channel gains. Not only the CSIT but also the

CDI[5] (Channel Directional Information) plays a vital role. The CDI characterizes the performance of MESC (Maximum estimated SINR Combiner) combination with ZFBF-SUS [6](Zero forcing Beam Forming Semi-orthogonal Selection).

In the SUS scheme, the users are selected on the orthogonality criterion. The favored users are authorized to transmit their information to the receiver. This helps in diminishing the complication from the ZFBF transmission. For a large number of users this an effective method that leads to ultra efficient use of bare resources. This paper contributes in deducing the BER and increasing the capacity of the MU-MIMO system using the SUS algorithm.

2. SYSTEM MODEL

A MU-MIMO system with N_t transmitting antennas and N_r receiving antennas is shown in Figure 1.

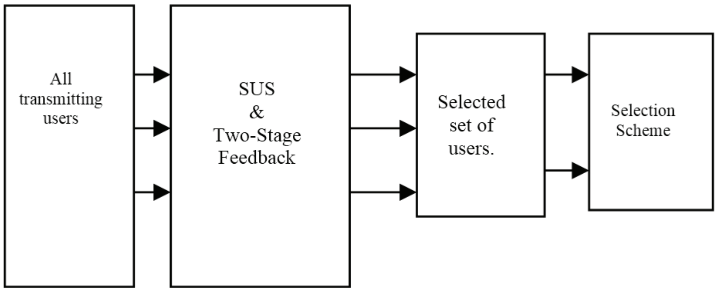

Figure 1: The system model of the proposed system

It is a narrow band time-invariant wireless channel represented by $N_r \times N_t$ determinant matrix H. The transmitted signal x is composed of N_t independent input symbols $x_1, x_2, x_3, \ldots, x_{Nt}$. The received signal is as:

$$Y = Hx + N; \tag{1}$$

where, N denotes the noise factor affecting the signal, which is Zero-mean circular symmetric complex Gaussian vector (ZMCSCG).

For deploying the SUS algorithm with the two-stage feedback, the expected SINR value is calculated for all users. The users with orthogonal CDI are selected at the transmitter and the effective MESC for the k^{th} user is considered. The pdf (probability density function) and the cdf (cumulative density function) respectively for the best codeword is given in (2) and (3). For deploying the SUS algorithm with the two-stage feedback [7], the expected SINR value is calculated for all users. The users with orthogonal CDI are selected at the transmitter and the effective MESC for the k^{th} user is considered. The pdf (probability density function) and the cdf (cumulative density function) [8] respectively for the best codeword is

$$f_{\gamma_k}(z) = \binom{N_t - 1}{N_r - 1} 2^B e^{-z/\rho} (-1/\rho) + \binom{N_t - 1}{N_r - 1} 2^B e^{(=z/\rho)} (1 + z)^{N_t - N_r - 2} \tag{2}$$

$$F_{\gamma_k}(z) = 1 - \frac{\binom{N_t - 1}{N_r - 1} 2^B e^{(-z/\rho)}}{(1 + z)^{N_t - N_r}} \tag{3}$$

where, B represents the number of feedback bits and $\rho = P/N_t$, P = power transmitted.

3. PERFORMANCE ANALYSIS

From the given pdf and cdf the best transmitting antenna and the best user can be found out using the selection scheme [9].

A. Best Transmitting Antenna using the Selection Scheme

In order to select best transmitting antenna, by using the theory of order statistics, the pdf and cdf of best transmitting antenna is

$$f_{\gamma_{bt,\,t}}(\gamma) = \mathrm{L} f_{\gamma_k}(z)[\mathrm{F}_{\gamma_k}(z)]^{\mathrm{L}-1} \tag{4}$$

$$\mathrm{F}_{\gamma_{bt,\,t}}(\gamma) = [\mathrm{F}_{\gamma_k}(z)]^{\mathrm{L}} \tag{5}$$

Substituting the values of (2) and (3) in (4) and (5); the cdf and the pdf for the best transmitting antenna obtained are:

$$\mathrm{F}_{\gamma_{bt,\,t}}(z) = \left[1 - \frac{\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(-z/\rho)}}{(1+z)^{\mathrm{N}_t - \mathrm{N}_r}}\right]^{\mathrm{L}} \tag{6}$$

$$f_{\gamma_{bt,\,t}}(z) = \mathrm{L}\left[\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{-z/\rho}(-1/\rho) + \begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(=z/\rho)}(1+z)^{\mathrm{N}_t - \mathrm{N}_r - 2}\right]\left[1 - \frac{\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(-z/\rho)}}{(1+z)^{\mathrm{N}_t - \mathrm{N}_r}}\right]^{\mathrm{L}-1} \tag{7}$$

where, L is the number of users in the multi-user system.

B. Best User using Antenna Selection

The best user from the best transmitting antenna can be found again by using the theory of order statics

$$\mathrm{F}_{\gamma_{bu,\,u}}(z) = [\mathrm{F}_{\gamma_{bt,\,t}}]^{k} \tag{8}$$

$$f_{\gamma_{bu,\,u}}(z) = k f_{\gamma_{bt,\,t}}(z)[\mathrm{F}_{\gamma_{bt,\,t}}(z)]^{k-1} \tag{9}$$

On substituting the substituting the values of (6) and (7) in (8) and (9), the equations for best user are as follows:

$$\mathrm{F}_{\gamma_{bu,\,u}}(z) = \left[1 - \frac{\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(-z/\rho)}}{(1+z)^{\mathrm{N}_t - \mathrm{N}_r}}\right]^{\mathrm{L}\times k} \tag{10}$$

$$\begin{aligned} f_{\gamma_{bu,\,u}}(z) = {} & k\mathrm{L}\left[\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{-z/\rho}(-1/\rho) + \begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(=z/\rho)}(1+z)^{\mathrm{N}_t - \mathrm{N}_r - 2}\right] \\ & \left[1 - \frac{\begin{pmatrix}\mathrm{N}_t - 1)\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(-z/\rho)}}{(1+z)^{\mathrm{N}_t - \mathrm{N}_r}}\right]^{\mathrm{L}-1}\left(\left[1 - \frac{\begin{pmatrix}\mathrm{N}_t - 1\\ \mathrm{N}_r - 1\end{pmatrix} 2^{\mathrm{B}} e^{(-z/\rho)}}{(1+z)^{\mathrm{N}_t - \mathrm{N}_r}}\right]^{\mathrm{L}}\right)^{k-1} \end{aligned} \tag{11}$$

C. Bit Error Rate (BER)

The average bit error rate is used for accessing telecomm, radio and network systems which transmit data from one location to another. The general expression for BER [9] is as follows:

$$\mathrm{P}_e = \frac{1}{2}\int_0^{\infty} erfc\left(\sqrt{z}\right) f_{\gamma_{bu,u}}(z)\,dz \tag{12}$$

It qualifies the channel carrying data by counting the number of errors present in the data string. The factors affecting BER are transmit power, interference, modulation and bandwidth of the signal. This identifies the performance of system which is directly proportional to operational performance. Using the pdf for the best user the bit error rate is as follows:

$$P_e = \frac{1}{2} \times k \times L \int_0^{\infty} erfc\left(\sqrt{z}\right)\left[\binom{N_t - 1}{N_r - 1} 2^B e^{-z/\rho}(-1/\rho) + \binom{N_t - 1}{N_r - 1} 2^B e^{(=z/\rho)}(1+z)^{N_t - N_r - 2}\right]$$

$$\left[1 - \frac{\binom{N_t - 1}{N_r - 1} 2^B e^{(-z/\rho)}}{(1+z)^{N_t - N_r}}\right]^{L-1}\left(\left[1 - \frac{\binom{N_t - 1}{N_r - 1} 2^B e^{(-z/\rho)}}{(1+z)^{N_t - N_r}}\right]^{L}\right)^{k-1} dz \tag{13}$$

D. Ergodic Capacity

Maximum mutual information averaged over all channel states is known as Ergodic Capacity. This can be achieved by varying the transmit power. To derive the expression for capacity we need to find out the first moment, therefore

$$E\{f_{\gamma_{bu,u}}\} = \int_0^{\infty} \gamma f_{\gamma_{bu,u}}(\gamma)\, d\gamma \tag{14}$$

where, E{.} is an expectation operator and the value of $f_{\gamma_{bu,u}}$.The above evaluation is evaluated using Wolfram Mathematica. The ergodic capacity $C_{\gamma_{bu,u}}$ of the system model is determined by averaging the instantaneous capacity over all possible channels

$$C_{\gamma_{bu,u}} \cong \log_2(1 + \gamma E\{f_{\gamma_{bu,u}}\}) - \frac{\gamma^2(\sigma_{\gamma_{bu,u}})^2}{2(1 + \gamma E\{f_{\gamma_{bu,u}}\})^2} \tag{15}$$

where, σ^2 is variance.

4. RESULTS AND CONCLUSIONS

The analysis of the derived results has been validated using monte carlo simulations for varying number of transmit and receive antennas. It is also observed that by changing the number of feedback bits, the system performs better. Results obtained with varying the number of feedback bits from B = 2, 4 and 8 are shown in Figure 2-7.

A. Bit Error Rate for Varying Transmit and Receive Antennas

$N_t = 2$ and $N_r = 2$

From the Figure 2 it is evident that the BER increases with a symbolic increase in the number of feedback bits. When the numbers of feedback bits are less i.e 2 and 4 then the difference between the error rate is less.

$N_t = 4$ and $N_r = 2$

From the Figure 3 it is evident, the BER increases with a symbolic increase in the number of feedback bits from 2 to 8. With the increase in number of antennas it can be seen that the error rate decreases for increasing SNR values.

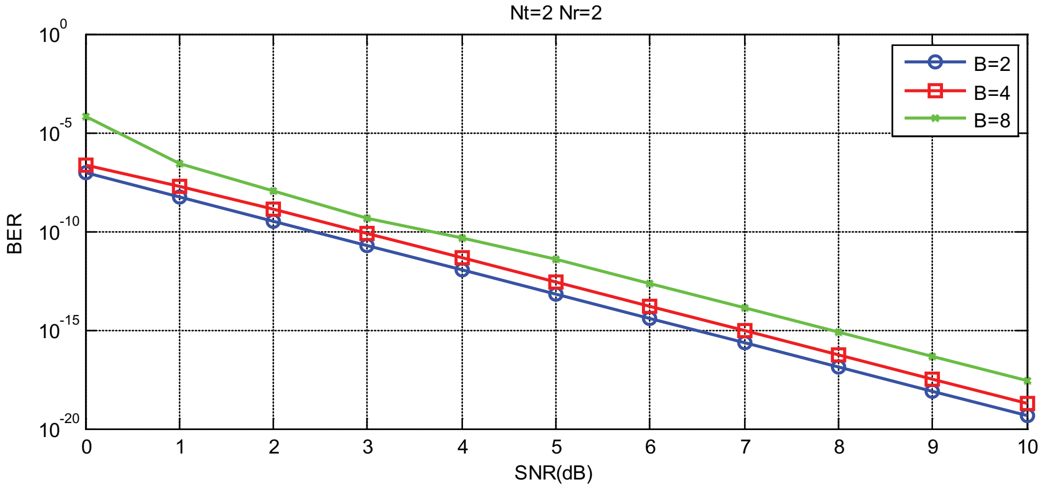

Figure 2: The graph shows the BER for $N_t = 2$ and $N_r = 2$ are constant and changing number of feedback bits

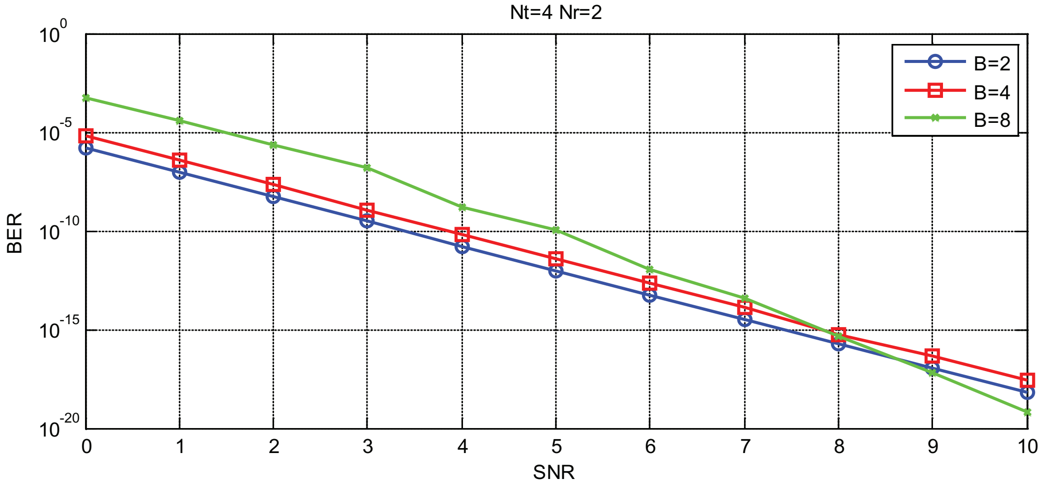

Figure 3: The graph shows the BER for $N_t = 4$ and $N_r = 2$ are constant and changing number of feedback bits

$N_t = 4$ and $N_r = 4$

From Figure 4, it is clearly seen that the error rate is less for less number of feedback bits. With the increase in number of antennas the error rate decreases for less number of feedback bits. This helps in increasing the efficiency of the system.

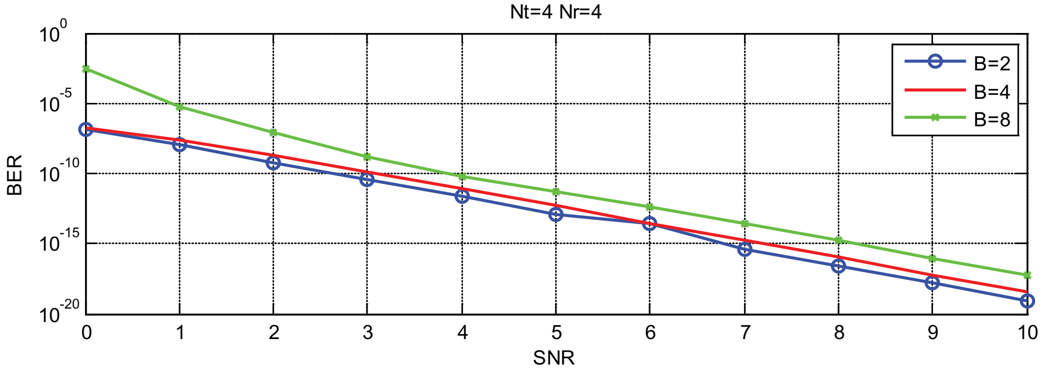

Figure 4: The graph shows the BER for $N_t = 4$ and $N_r = 4$ are constant and changing number of feedback bits

B. Ergodic Capacity for Varying Transmit and Receive Antennas

$N_t = 2$ and $N_r = 2$

The Figure 5 shows us varying capacity for varying feedback bits when the numbers of transmitting and receiving antennas are 2. From the Table 1 it is clearly evident that there is a significant increase in the capacity of the system from 6.61741 for B = 2 to 32.7952 for B = 8.

Table 1
When $N_t = 2$ and $N_r = 2$ are fixed and the number of feedback bits are changing from 2 to 8

N_t *(Transmitting Antennas)*	N_r *(Receiving Antennas)*	*B (Number of feedback bits)*	*Capacity*
2	2	2	6.61741
2	2	4	16.0833
2	2	8	32.7952

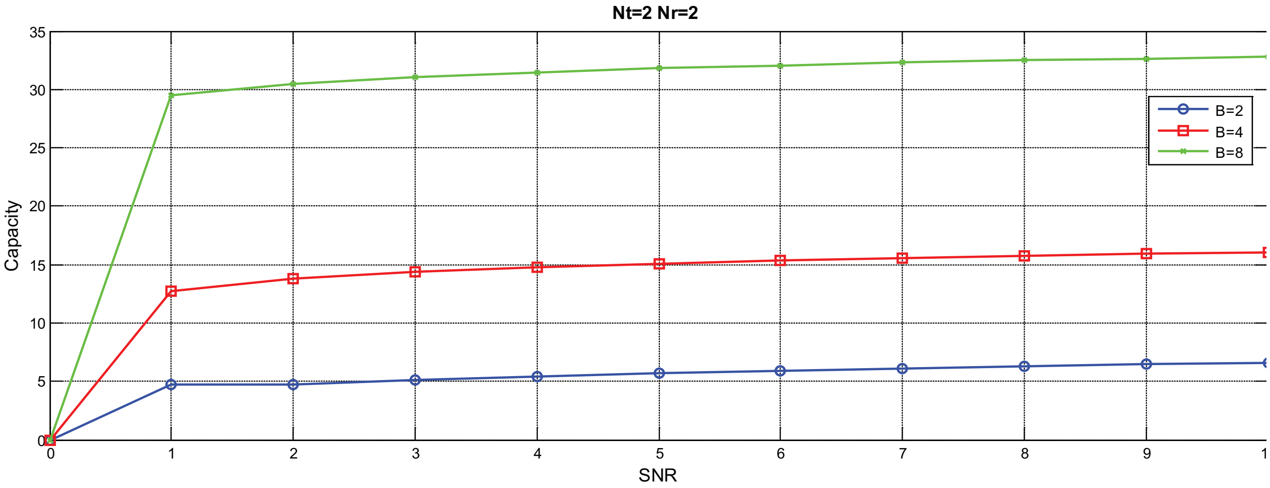

Figure 5: The graph shows the capacity for $N_t = 2$ and $N_r = 2$ are fixed and changing number of feedback bits

$N_t = 4$ and $N_r = 2$

The Figure 6 shows varying capacity for varying feedback bits when there are 4 transmitting antennas and 2 receiving antennas. From the Table 2 is clearly evident that there is a significant increase from 25.225 for B = 2 to 74.4320 for B = 8.

Table 2
When $N_t = 4$ and $N_r = 2$ are fixed and the number of feedback bits are changing from 2 to 8

N_t *(Transmitting Antennas)*	N_r *(Receiving Antennas)*	*B (Number of feedback bits)*	*Capacity*
4	2	2	25.2225
4	2	4	41.1504
4	2	8	74.4320

$N_t = 4$ and $N_r = 4$

The Figure 7 shows variable capacity for variable feedback bits for 4 transmitting antennas and 4 receiving antennas. From the table we can interpret that the capacity has increased from 12.3273 for B = 2 to 62.6618 for B = 8.

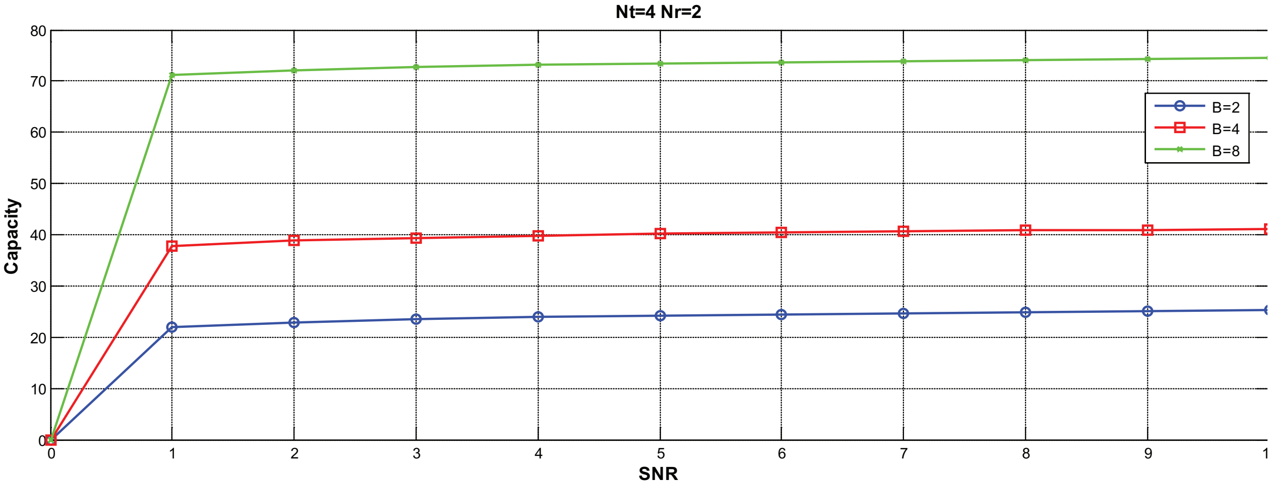

Figure 6: The graph shows the capacity for $N_t = 4$ and $N_r = 2$ are fixed and changing number of feedback bits

Table 3
When $N_t = 4$ and $N_r = 4$ are fixed and the number of feedback bits are changing from 2 to 8

N_t *(Transmitting Antennas)*	N_r *(Receiving Antennas)*	*B* *(Number of feedback bits)*	*Capacity*
4	4	2	12.3273
4	4	4	30.2982
4	4	8	62.6618

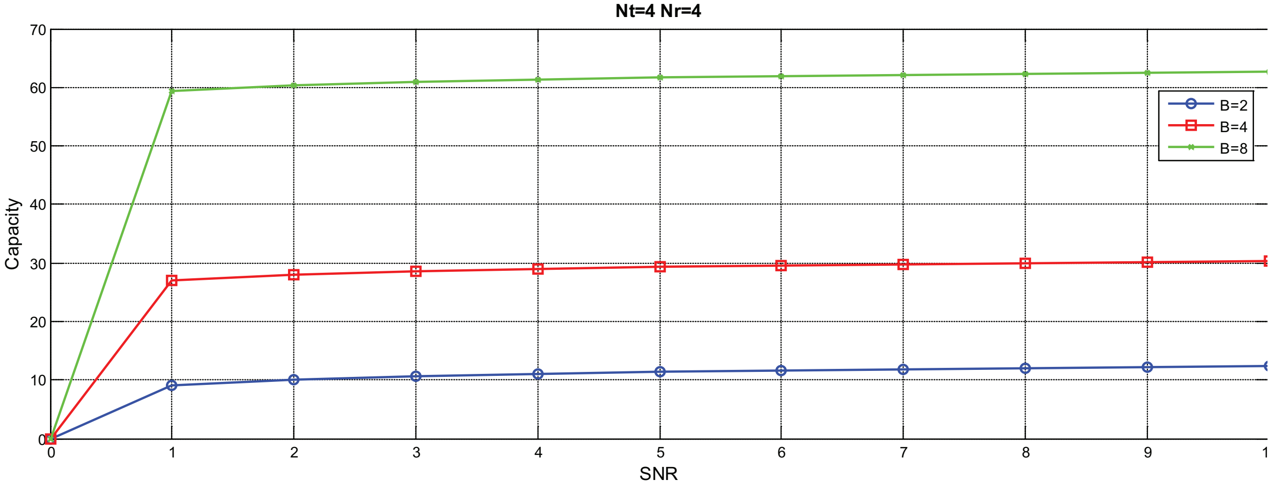

Figure 7: The graph shows the capacity for $N_t = 4$ and $N_r = 4$ are constant and changing number of feedback bits

5. CONCLUSION

In this paper, using the SUS scheme and the two-stage feedback MU-MIMO system has been studied. The BER and the capacity of the system have been analytically analyzed using the pdf and the cdf of the best codeword. This is mainly done keeping in mind the number of feedback bits. The increase in number of feedback bits when increase lead to a significant increase in the capacity of the system, which is needed for the increasing number of users in the present time. This scenario leads to the evident need of MU-MIMO system to be implemented as the number of users in the radio spectrum is increasing day-by-day.

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