International Journal of Mathematical Sciences
Vol. 17, Nos. 3-4, July-December 2018, pp. 227-240

CASP-CUSUM SCHEMES BASED ON TRUNCATED LOMAX FAMILY OF DISTRIBUTION

G. Venkatesulu, P. Mohammed Akhtar, B. Sainath and B.R. Narayana Murthy

Abstract: In the present paper we obtained the measure of information energy corresponding to measures of Kapur's [1], [2] family, Havrda-Charvat's [3] measure of entropy, two parametric measures of entropy, generalized measure of entropy, three parametric Bi-measures of entropy respectively and also discussed the particular cases for each information measure of entropy. Entropy, generalized measure of entropy, three parametric Bi-measures of entropy respectively and also discussed the particular cases for each information measure of entropy.

Keywords: Measure of Entropy, Directed Divergence Information Energy, Bi-Measure of Entropy, Generlised Measures of Entropy, Concavity.

1. INTRODUCTION

Quality is a relative term and it is generally used with reference to the end use of a product. The quality of a product depends on the perception of the customer in a given situation. The situation can be user-oriented, cost-oriented or supplier-oriented. Quality has to be planned, achieved, controlled and improved continuously.

Acceptance sampling is concerned with inspection and decision making regarding products, one of the oldest aspects of quality assurance. In the 1930's and 1940's, acceptance sampling was one of the major components of the field of statistical quality control and was used primarily for incoming or receiving inspection. Acceptance sampling plans do not provide any direct form of quality control. Acceptance sampling simply accepts or rejects lots. Even if all lots are of the same quality, sampling plan will accept some lots and reject others, the accepted lots being no better than the rejected ones. Process controls are used to control and systematically improve quality, but acceptance sampling is not. The most effective use of acceptance sampling is not to inspect the quality of the product but rather as an audit tool to ensure that the output of a process conforms to requirements.

Acceptance sampling plan is an essential tool in the Statistical Quality Control. In most of the statistical quality control experiment, it is not possible to perform hundred percent inspections, due to various reasons. The acceptance sampling plan was first applied in the US military for testing the bullets during World War II. For instance, if every bullet tested in advance, no bullets are available for shipment, and on the other hand, if no bullets are tested, then disaster may occur in the battlefield at the crucial time.

Acceptance sampling is a middle ground between the extremes of 100% inspection and no inspection. Generally, 100% used in some situations where the component is extremely critical and passing any defectives would result is an unacceptably high failure cost at subsequent stages.

The truncated distributions mean deleting some values of the domain of the probability density function (pdf). It is a statistical tool to describe phenomena that occur in its loss of the part of the observations or data and describe phenomena that depend on time do not start from $t = 0$. Truncation occurs in various situations. For example in life testing and reliability, truncated Exponentiated Lomax distribution is proposed for modeling the lifetime distributions of items such as electronic components, light bulbs, batteries and so on.

A truncated distribution is a conditional distribution resulting when the domain of the parent distribution is restricted to a smaller region. A truncated distribution occurs when there is no ability to know about or record events occurring above or below a set threshold or outside a certain range. Truncated distributions have achieved a large number of applications in various real-life fields like economics, biochemistry, biology, chemistry, engineering, networking and other fields.

Gupta et.al[6] introduced a class of exponentiated distributions based on cumulative distribution function for the exponential distribution. In a similar manner, Nadaragah and Kotz[12] proposed the Exponentiated Gamma and Exponentiated Gumbel distributions.

Hawkins, D. M.[7] Proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's of location and scale CUSUM Charts.

Alzaatrch et.al[2] proposed another method for generating many new distributions. Gauss and Cordeiro[5] proposed a new method of adding two parameters to a continuous distribution that extends the idea of Nadarajah and Kotz.

Recently, exponentiated Lomax distribution, Lomax-logarithm, and extended Lomax Poisson distributions have been given, respectively by, Ramos et al.[15], Al-Zaharani and Sagor[3.], and Al-Zaharani.

Kakoty. S., Chakravarthy A.B.[9] determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally, truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production of engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable is first approximated as an exponential variable.

Vardeman.S, Di-ou Ray[18] has introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further, the phenomena under study are the occurrence of the rate of rare events and the inter-arrival times for a homogenous poison process are identically independently distributed exponential random variables.

Lonnie. C. Vance[10] consider Average Run Length of Cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are considered.

Mohammed Riaz, Nasir Abbas, and Ronald J.M.M Does[11] propose two Runs rules schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Mohammed Akhtar. P and Sarma K.L.A.P[1] analyzed and Optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluate L (0), L' (O) and the probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results.

Narayana Murthy, B.R. and Mohammed Akhtar.P[13] proposed an Optimization of CASP CUSUM Schemes based on Truncated log-logistic distribution and evaluate the probability of acceptance for different parameter values.

Sainath.B and Mohammed Akhtar.P[16] studied an Optimization of CASP-CUSUM Schemes based on truncated Burr distribution and the results were analyzed at different values of the parameters.

Venkatesulu.G and Mohammed Akhtar.P[19] Determined Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally critical comparisons his based o the obtained numerical results.

In the present paper, it is determined CASP-CUSUM Chart when the variable under study follows truncated Exponentiated Lomax Distribution. Thus it is more worthwhile to study some interesting characteristics of this distribution.

Exponentiated Lomax Distribution

The Lomax distribution can also be called Pareto Type II distribution and its application can be found in many fields like actuarial science, economics, and so on. The distribution was defined by Lomax and it is a heavy-tailed distribution. We generalize the Lomax distribution by powering a positive real number α to the cumulative distribution function (CDF). This new type of distribution called as Exponentiated Lomax Distribution[17].

Definition

A continuous random variable X assuming non-negative values is said to have Exponentiated Lomax Distribution with parameters α, λ and θ > 0, its probability density function is given by

$$f(x) = \alpha\theta\lambda[1-(1+\lambda x)^{-\theta}]^{\alpha-1}(1+x)^{-(\theta+1)}, x > 0, \theta, \alpha \text{ and } \lambda > 0 \qquad (1.1)$$

The Probability density function of the Exponentiated Lomax distribution can be reduced to the Exponentiated Pareto distribution with parameters (θ, α) at a particular value of λ=1. When λ=α=1, the probability density function of the Exponentiated Lomax distribution reduces to the standard Lomax distribution with single parameter θ.

Truncated Exponentiated Lomax Distribution

It is the ratio of probability density function of the Lomax distribution to their cumulate distribution function at the point B.

The random variable X is said to follow a Truncated Exponentiated Lomax Distribution as

$$f_B(x) = \frac{\alpha\theta\lambda[1-(1+\lambda x)^{-\theta}]^{\alpha-1}}{[1-(1+\lambda B)^{-\theta}]^{\alpha}} \quad \lambda > 0, \alpha \text{ and } \theta > 0 \qquad (1.2)$$

where, ‘**B**’ is the truncated point of the Exponentiated Lomax Distribution.

2. DESCRIPTION OF THE PLAN AND TYPE- C OC CURVE

Beattie[4] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, “Return interval” with the length h’, above the decision line is taken. We plot on the chart the sum $S_m = \sum(X_i - k_1)X_i\text{'}s(i = 1,2,3........)$ are distributed independently and k_1 is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., *h* + *h*’
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure, in brief, is given below.

1. Start plotting the CUSUM at 0.

2. The product is accepted $S_m = \sum(X_i - k) < h$; when $S_m < 0$, return cumulative to 0.
3. When $h < S_m < h + h'$ the product is rejected: when S_m crossed h, i.e., when $S_m > h + h'$ and continue rejecting product until $S_m > h + h'$ return cumulative to $h + h'$

The type-C, OC function, which is defined as the probability of acceptance of an item as a function of incoming quality, when the sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0)+L'(0)} \tag{2.1}$$

where, $L(0)$ = Average Run Length in acceptance zone and

$L'(0)$ = Average Run Length in rejection zone.

Page E.S.[14] has introduced the formulae for $L(0)$ and $L'(0)$ as

$$L(0) = \frac{N(0)}{1-P(0)} \tag{2.2}$$

$$L'(0) = \frac{N'(0)}{1-P'(0)} \tag{2.3}$$

where, P(0) = Probability for the test starting from zero on the normal chart,

N(0) = ASN for the test starting from zero on the normal chart,

P'(0) = Probability for the test on the return chart and

N'(0) = ASN for the test on the return chart

He further obtained integral equations for the quantities P(0), N(0), and P'(0), N'(0) as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z) dy, \tag{2.4}$$

$$N(z) = 1 + \int_0^h N(y) f(y + k_1 - z) dy, \tag{2.5}$$

$$P'(z) = \int_{k_1+z}^{B} f(y) dy + \int_0^h P'(y) f(-y + k_1 + z) dy \tag{2.6}$$

$$N'(z) = 1 + \int_0^h N'(y) f(-y + k_1 + z) dy, \tag{2.7}$$

$$F(x) = 1 + \int_A^h f(x) dx :$$

$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y) dy$$

and z is the distance of the starting of the test in the normal chart from zero.

3. METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t) F(t) dt \tag{3.1}$$

where,

$$F(X) = P(z),$$

$$Q(X) = F(k - z),$$

$$R(X,t) = f(y + k - z)$$

Let the integral $I = \int_c^d f(x) dx$ be transformed to

$$I = \frac{d-c}{2} \int_c^d f(y) dy = \frac{d-c}{2} \sum a_i f(t_i) \tag{3.2}$$

$$y = \frac{2x - (c-d)}{d-c}$$

where, a_i's and t_i's respectively the weight factor and abscissa for the Gauss-Chebyshev polynomial, given in Jain M.K. and et al[8] using (3.1) and (3.2),(2.4) can be written as

$$F(X) = Q(X) \frac{d-c}{2} \sum a_i R(x, t_i) F(t_i) \tag{3.3}$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for $x = t_i$, $i = 0$ (1) n then obtain.

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i R(t_j, t_i) F(t_i) \tag{3.4}$$

$$j = 0(1)n$$

Substituting

$$F(t_i) \;=\; F_i, Q(t_i) = Q_i, i = 0(1)n, in\ (3.4),\ \text{we get}$$

$$F_0 \;=\; Q_0 + \frac{d-c}{2}[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \ldots a_n R(t_0,t_n)F_n)]$$

$$F_1 \;=\; Q_1 + \frac{d-c}{2}[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \ldots a_n R(t_1,t_n)F_n)]$$

……………. ……………. ……………. …………….

……………. ……………. ……………. …………….

$$F_n \;=\; Q_n + \frac{d-c}{2}[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots a_n R(t_n,t_n)F_n)] \tag{3.5}$$

In the system of equations except for F_i, i = 0, 1, 2 … n are known and hence can be solved for F_i, we solved the system of equations by the method of Iteration. For this, we write the system (3.5) as

$$[1 - Ta_0 R(t_0,t_0)]F_0 \;=\; Q_0 + T[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \ldots a_n R(t_0,t_n)F_n)]$$

$$[1 - Ta_1 R(t_1,t_1)]F_1 \;=\; Q_1 + T[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \ldots a_n R(t_1,t_n)F_n)]$$

……………. ……………. ……………. …………….

……………. ……………. ……………. …………….

$$[1 - Ta_n R(t_n,t_n)]F_n \;=\; Q_n + T[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots a_n R(t_n,t_n)F_n)] \tag{3.6}$$

where, $T = \dfrac{d-c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = ... = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = ... = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i i = 0, 1, 2, ..., n which are just the refined values of F_i i = 0, 1, 2, …, n. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P'(0), N(0), N'(0) can be obtained.

4. COMPUTATION OF ARL'S AND P (A)

We developed computer programs to solve the equations (2.4), (2.5), (2.6) and (2.7) and we got the following results given in the Tables (4.1) to (4.24).

Table 4.1
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1, h = 0.10, h' = 0.10$

B	*L(0)*	*L'(0)*	*P(A)*
1.4	9560.27246	0.9955152	0.9998958707
1.3	10692.57031	0.9955151	0.9999068975
1.2	13140.67188	0.9955150	0.9999242425
1.1	20877.24023	0.9955149	0.9999523163
1.0	3340357.50000	0.9955147	0.9999997020

Table 4.2
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1, h = 0.12, h' = 0.12$

B	*L(0)*	*L'(0)*	*P(A)*
1.4	9540.76172	0.9946278	0.9998957515
1.3	10669.30371	0.9946277	0.9999067783
1.2	13097.95020	0.9946276	0.9999240637
1.1	20806.46680	0.9946275	0.9999521971
1.0	2085847.87500	0.9946272	0.9999995232

Table 4.3
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1, h = 0.15, h' = 0.15$

B	*L(0)*	*L'(0)*	*P(A)*
1.4	9500.76074	0.9933026	0.9998954535
1.3	10620.99023	0.9933026	0.9999064803
1.2	13039.38379	0.9933025	0.9999238253
1.1	20675.34570	0.9933023	0.9999519587
1.0	1388693.62500	0.9933020	0.9999992847

Table 4.4
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1, h = 0.18, h' = 0.18$

B	*L(0)*	*L'(0)*	*P(A)*
1.4	9461.02246	0.9919847	0.9998951554
1.3	10573.02148	0.9919846	0.9999061823
1.2	12971.11035	0.9919845	0.9999235272
1.1	20494.98828	0.9919842	0.9999516010
1.0	978936.68750	0.9919839	0.9999989867

Table-4.5
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.4, h = 0.10, h' = 0.10$

B	*L(0)*	*L'(0)*	*P(A)*
1.8	55487.69922	0.9955152	0.9999820590
1.7	63747.31641	0.9955152	0.9999843836
1.6	80685.00781	0.9955152	0.9999876618
1.5	132553.93750	0.9955152	0.9999924898
1.4	3340359.25000	0.9955152	0.9999997020

Table-4.6
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1, h = 0.25, h' = 0.25$

B	*L(0)*	*L'(0)*	*P(A)*
1.4	9346.42578	0.9889369	0.9998942018
1.3	10433.90234	0.9889368	0.9999052286
1.2	12761.46191	0.9889366	0.9999225140
1.1	20036.10547	0.9889363	0.9999506474
1.0	487937.84375	0.9889358	0.9999979734

Table 4.7
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.4, h = 0.15, h' = 0.15$

B	*L(0)*	*L'(0)*	*P(A)*
1.8	54104.99219	0.9933028	0.9999816418
1.7	61949.20703	0.9933028	0.9999839664
1.6	77870.73438	0.9933028	0.9999872446
1.5	125295.75781	0.9933027	0.9999920726
1.4	1388694.625000	0.9933026	0.9999992847

Table 4.8
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.4, h = 0.18, h' = 0.18$

B	*L(0)*	*L'(0)*	*P(A)*
1.8	53169.13672	0.9919848	0.9999813437
1.7	60737.00781	0.9919848	0.9999836683
1.6	75990.58594	0.9919848	0.9999869466
1.5	120593.75781	0.9919848	0.9999917746
1.4	978937.50000	0.9919847	0.9999989867

Table 4.9
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.4, h = 0.20, h' = 0.20$

B	*L(0)*	*L'(0)*	*P(A)*
1.8	52451.23828	0.9911101	0.9999811053
1.7	59809.50000	0.9911101	0.9999834299
1.6	74560.71875	0.9911101	0.9999867082
1.5	117091.82813	0.9911101	0.9999915361
1.4	791763.75000	0.9911101	0.9999987483

Table 4.10
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.4, h = 0.25, h' = 0.25$

B	*L(0)*	*L'(0)*	*P(A)*
1.8	50425.25391	0.9889371	0.9999803901
1.7	57206.58203	0.9889371	0.9999827147
1.6	70595.35156	0.9889370	0.9999859929
1.5	107726.67188	0.9889370	0.9999908209
1.4	502724.43750	0.9889369	0.9999980330

Table 4.11
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.8, h = 0.10, h' = 0.10$

B	*L(0)*	*L'(0)*	*P(A)*
2.2	214125.60938	0.9955153	0.9999953508
2.1	249280.56250	0.9955153	0.9999960065
2.0	315128.25000	0.9955153	0.9999968410
1.9	506115.09375	0.9955153	0.9999980330
1.8	3340359.50000	0.9955152	0.9999997020

Table 4.12
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.8, h = 0.12, h' = 0.12$

B	*L(0)*	*L'(0)*	*P(A)*
2.2	208584.92188	0.9946279	0.9999952316
2.1	241837.59375	0.9946279	0.9999958873
2.0	303396.25000	0.9946279	0.9999967217
1.9	476765.53125	0.9946279	0.9999979138
1.8	2085849.25000	0.9946279	0.9999995232

Table 4.13
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.8, h = 0.20, h' = 0.20$

B	*L(0)*	*L'(0)*	*P(A)*
2.2	176883.42188	0.9911101	0.9999943972
2.1	200325.81250	0.9911101	0.9999950528
2.0	240971.62500	0.9911101	0.9999958873
1.9	339327.37500	0.9911101	0.9999970794
1.8	791763.87500	0.9911101	0.9999987483

Table 4.14
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 1.8, h = 0.25, h' = 0.25$

B	*L(0)*	*L'(0)*	*P(A)*
2.2	156508.57813	0.9889371	0.9999936819
2.1	174630.62500	0.9889371	0.9999943376
2.0	204813.70313	0.9889371	0.9999951720
1.9	271965.75000	0.9889371	0.9999963641
1.8	502724.50000	0.9889371	0.9999980330

Table 4.15
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.10, h' = 0.10$

B	*L(0)*	*L'(0)*	*P(A)*
2.4	371151.06250	0.9955153	0.9999973178
2.3	428251.21875	0.9955153	0.9999976754
2.2	538767.68750	0.9955153	0.9999981523
2.1	835089.87500	0.9955153	0.9999988079
2.0	3340359.50000	0.9955153	0.9999997020

Table 4.16
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.12, h' = 0.12$

B	*L(0)*	*L'(0)*	*P(A)*
2.4	355038.15625	0.9946279	0.9999971986
2.3	406994.96875	0.9946279	0.9999975562
2.2	505660.43750	0.9946279	0.9999980330
2.1	758490.62500	0.9946279	0.9999986887
2.0	2085849.25000	0.9946279	0.9999995232

Table 4.17
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.15, h' = 0.15$

B	*L(0)*	*L'(0)*	*P(A)*
2.4	320468.03125	0.9933028	0.9999969006
2.3	370318.62500	0.9933028	0.9999973178
2.2	450387.50000	0.9933028	0.9999977946
2.1	617197.68750	0.9933028	0.9999983907
2.0	1388694.87500	0.9933028	0.9999992847

Table 4.18
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.18, h' = 0.18$

B	*L(0)*	*L'(0)*	*P(A)*
2.4	291963.87500	0.9919848	0.9999966025
2.3	326312.56250	0.9919848	0.9999969602
2.2	387021.87500	0.9919848	0.9999974370
2.1	520060.65625	0.9919848	0.9999980927
2.0	978937.68750	0.9919848	0.9999989867

Table 4.19
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.20, h' = 0.20$

B	*L(0)*	*L'(0)*	*P(A)*
2.3	302309.84375	0.9911101	0.9999967217
2.2	353766.84375	0.9911101	0.9999971986
2.1	461862.28125	0.9911101	0.9999978542
2.0	791763.87500	0.9911101	0.9999987483
1.9	8313521.0000	0.9911101	0.9999998808

Table 4.20
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2, h = 0.25, h' = 0.25$

B	*L(0)*	*L'(0)*	*P(A)*
2.3	247610.59375	0.9889371	0.9999960065
2.2	281184.90625	0.9889371	0.9999964833
2.1	345623.12500	0.9889371	0.9999971390
2.0	502724.53125	0.9889371	0.9999980330
1.9	1184993.62500	0.9889371	0.9999991655

Table 4.21
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2.2, h = 0.15, h' = 0.15$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	537559.31250	0.9933028	0.9999981523
2.4	640936.06250	0.9933028	0.9999984503
2.3	833216.87500	0.9933028	0.9999988079
2.2	1388694.87500	0.9933028	0.9999992847
2.1	16664338.00000	0.9933028	0.9999999404

Table 4.22
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2.2, h = 0.18, h' = 0.18$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	462276.12500	0.9919848	0.9999978542
2.4	520060.65625	0.9919848	0.9999980927
2.3	665677.62500	0.9919848	0.9999985099
2.2	978937.68750	0.9919848	0.9999989867
2.1	2377420.25000	0.9919848	0.9999995828

Table 4.23
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2.2, h = 0.20, h' = 0.20$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	415676.06250	0.9911101	0.9999976158
2.4	461862.28125	0.9911101	0.9999978542
2.3	554234.75000	0.9911101	0.9999982119
2.2	791763.87500	0.9911101	0.9999987483
2.1	1511549.25000	0.9911101	0.9999993443

Table 4.24
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 4, \theta = 6, k = 2.2, h = 0.25, h' = 0.25$

B	*L(0)*	*L'(0)*	*P(A)*
2.4	345623.12500	0.9889371	0.9999971390
2.3	394997.84375	0.9889371	0.9999974966
2.2	502724.53125	0.9889371	0.9999980330
2.1	721300.43750	0.9889371	0.9999986291
2.0	2073738.75000	0.9889371	0.9999995232

5. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters α, λ, θ, *k*, *h* and *h*' are given at the top of each table, we determine optimum truncated point B at which P(A) the probability of accepting an item is maximum and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P(A) are given in columns I, II, III, and IV respectively.

From the above Tables 4.1 to 4.24 we made the following conclusions:

(i) From the Table 4.1 to 4.24, it is observed that the values of P (A) are increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

(ii) From the Table 4.1 to 4.24, we observe that it can be maximized the truncated point B by increasing value of *k*.

(iii) From Table 4.1 to 4.24, it is observed that at the maximum level of probability of acceptance P(A) the truncated point '**B**' from 5.0 to 1.0 as the value of ***h*** changes from 0.10 to 0.25.

(iv) From the Table 4.1 to 4.24, it was observed that the truncated point '**B**' changes from 5.0 to 1 .1 and P (A) are as $h \rightarrow 0.15$ maximum **i.e. 0.9999999404.** Thus truncated point B and k are inversely related.

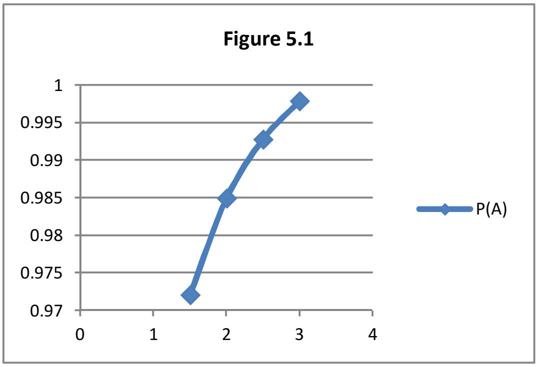
(v) From Table 4.1 to 4.24 it is observed that the optimal truncated point changes from 2.5 to 2.1 as $h \rightarrow 0.15$.

(vi) It is observed that the Table -5.1 values of Maximum Probabilities increased as the increased values of '**k**' as shown below the Figure 5.1.

Table 5.1

α = 2, λ = 4, *h* = 0.10, *h*' = 0.10

k	*P(A)*
1.5	0.972061
2	0.984949
2.5	0.992802
3	0.997891

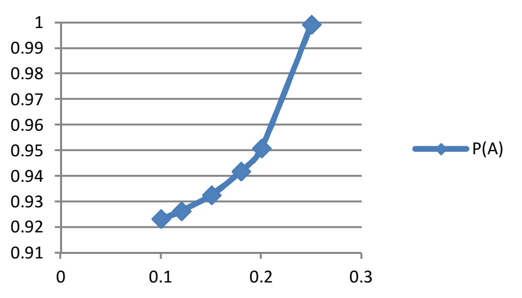


(vii) It is observed that the Table 5.2 values of Maximum Probabilities increased as the increased values of *h* and *h*' as shown below the Figure 5.2.

Table 5.2
α = 2, λ = 4, B = 3.9, *k* = 3

h and h'	*P(A)*
0.10	0.923217
0.12	0.926376
0.15	0.932649
0.18	0.941885
0.20	0.950871
0.25	0.999331

Figure 5.2



(viii) The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above Table 4.1 to 4.24 are observed from the following Table.

Table 5.3
Consolidated Table from the Tables (4.1) to (4.24)

B	*α*	*λ*	*θ*	*k*	*h*	*h'*	*P(A)*
2.1	2	4	6	1.8	0.12	0.12	0.9999958873
1.8	2	4	6	1.8	0.15	0.15	0.9999992847
2.5	2	4	6	1.8	0.18	0.18	0.9999935031
1.4	2	4	6	1	0.15	0.15	0.9998954535
1.0	2	4	6	1	0.18	0.18	0.9999989867
1.1	2	4	6	1	0.20	0.20	0.9999513626
2.6	2	4	6	2	0.25	0.25	0.9999951720
2.2	2	4	6	1.8	0.25	0.25	0.9999936819
2.7	2	4	6	2	0.10	0.10	0.9999966621
1.5	2	4	6	1	0.25	0.25	0.9998869896
1.7	2	4	6	1.4	0.10	0.10	0.9999843836
1.3	2	4	6	1	0.12	0.12	0.9999067783
2.2	2	4	6	2.2	0.12	0.12	0.9999995232
2.1	2	4	6	2.2	0.15	0.15	0.9999999404
1.8	2	4	6	1.8	0.10	0.10	0.9999997020
2.0	2	4	6	1.8	0.20	0.20	0.9999958873
2.7	2	4	6	2.2	0.20	0.20	0.9999971986
2.0	2	4	6	2.2	0.25	0.25	0.9999995232
1.7	2	4	6	1.4	0.18	0.18	0.9999836683
1.8	2	4	6	1.4	0.20	0.20	0.9999811053
2.5	2	4	6	2.2	0.18	0.18	0.9999978542

By observing the Table 5.3, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL and P (A) reach their maximum i.e., ***3340359.5, 0.9999997020*** respectively, is

$$\begin{bmatrix} B = 1.8 \\ \alpha = 2 \\ \lambda = 4 \\ \theta = 6 \\ k = 1.8 \\ h = 0.10 \\ h' = 0.10 \end{bmatrix}$$

REFERENCES

[1] Akhtar, P. Md. and Sarma, K.L.A.P. (2004). "Optimization of CASP-CUSUM Schemes based on Truncated Gamma Distribution". *Bulletin of Pure and applied sciences*, Vol. 23E, No. 2, pp. 215-223.

[2] Alzaatreh A., Lee C. & Felix F. (2013). "A new method for generating families of continuous distributions". *METRON*. 71, pp. 63-79.

[3] Al-Zahrani.B., H. Sagor. (2014). "The Poisson-Lomax distribution", *Revista Colombiana de Estadística*, Vol. 37, No. 1, pp. 225-245.

[4] Beattie, B.W. (1962). "A Continuous Acceptance Sampling procedure based upon a Cumulative Sums Chart for a number of defective". *Applied Statistics*, Vol. 11, No. 2, pp. 137-147.

[5] Gauss M. & Cordeiro M (2013). "The Exponentiated Generalized Class of Distributions". *Journal of Data Science*. 11. pp. 1-12.

[6] Gupta C., Gupta P., & Gupta D. (1998). "Modeling failure time data by Lehmann alternatives". *Communications in Statistics-Theory and Methods*. 27. pp. 887-904.

[7] Hawkins, D.M. (1992). "A Fast Accurate Approximation for Average Lengths of CUSUM Control Charts". *Journal of Quality Technology*, Vol. 24, No. 1, pp. 37-43.

[8] Jain, M.K., Iyengar, S.R.K. and Jain, R.K. "Numerical Methods of Scientific and Engineering Computations", *Willy Eastern Ltd.*, New Delhi.

[9] Kakoty, S and Chakravaborthy, A.B., (1990). "A Continuous Acceptance Sampling Plan for Truncated Normal distribution based on Cumulative Sums", *Journal of National Institution for Quality and Reliability*, Vol. 2, No. 1, pp. 15-18.

[10] Lonnie, C. Vance. (1986). "Average Run Length of CUSUM Charts for Controlling Normal means". *Journal of Quality Technology*, Vol. 18, pp. 189-193.

[11] Muhammad Riaz, Nasir, Abbas and Ronald, J.M.M. Does, (2011). "Improving the performance of CUSUM charts", *Quality and Reliability Engineering International*, Vol. 27, pp. 415-424.

[12] Nadarajah S. (2006). "The exponentiated Gumbel distribution with climate application", *Environmetrics*, Vol. 17: pp. 13-23.

[13] Narayana Muthy, B.R, Akhtar, P. Md and Venkataramudu, B. (2012). "Optimization of CASP-CUSUM Schemes based on Truncated Log-Logistic Distribution". *Bulletin of Pure and Applied Sciences*, Vol. 31E (Math & Stat.): Issue (No. 2), pp. 243-25.

[14] Page, E.S., (1954). “Continuous Inspection Schemes”, *Biometrika*, Vol. XLI, pp. 104-114.

[15] Ramos, M.W.A., Marinho, P.R.D., da Silva R.V. and Cordeiro, G.M. (2013). “The exponentiated Lomax Poisson distribution with an application to lifetime data”, *Advances and Applications in Statistics* 34, pp. 107-135.

[16] Sainath.B., P. Mohammed Akhtar, G. Venkatesulu, and Narayana Muthy, B.R, (2016). “CASP CUSUM Schemes based on Truncated Burr Distribution using Lobatto Integration Method”. IOSR *Journal of Mathematics (IOSR-JM)*, Vol. 12, Issue 2, pp. 54-63.

[17] Salem H. (2014). “The exponential Lomax distribution: Different estimation Methods” *American Journal of Mathematics and Statistics*. 2014, Vol. 2, No. 6, pp. 364-368.

[18] Vardeman, S. And Di-ou Ray. (1985). “Average Run Length for CUSUM schemes Where observations are Exponentially Distributed”, *Technometrics*, Vol. 27, No. 2, pp. 145-150.

[19] Venkatesulu.G., P. Mohammed Akhtar, B. Sainath and Narayana Murthy, B.R. (2017). “Truncated Gompertz Distribution and its Optimization of CASP- CUSUM Schemes”. *Journal of Research in Applied Mathematics*, Vol. 3, Issue 7, pp. 19-28.

G. Venkatesulu
Research Scholar.

P. Mohammed Akhtar
Professor, Dept. of Statistics,
Sri Krishanadevaraya University,
Ananthapuramu-515003.

B. Sainath
Assistant Professor (C),
Dept. of OR&SQC,
Rayalaseema University,
Kurnool-518002.

B.R. Narayana Murthy
Lecturer in Statistics Govt. (Autonomous) College,
Ananthapuramu-515001.
Email: palagiriakhtar@rediffmail.com