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Labour versus Alternative Value Bases in Actual Joint Production Systems

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This paper extends the empirical investigation of the relationships between actual prices, labour values and commodity values to the case of joint production using data from the Supply and Use Tables of the Finnish (for the year 2004) and Japanese (for the year 2000) economies. Our findings show that (i) the systems under investigation do not have the usual properties of single-product systems; (ii) in the case of the Finnish economy, the exploration of the relationships between prices and additive labour values is without economic meaning; and (iii) in the case of the Japanese economy, there exist vectors of additive commodity values that are better approximations of actual prices than additive labour values. Thus, it is argued that the claim that actual economic systems can be adequately interpreted in terms of labour values is open to serious doubts.

INTRODUCTION

During the last decades, a significant number of empirical studies explore the relationships among labour values, *actual* prices of production and market prices.[1] The central conclusion of these studies is that labour values are quite close to production prices and market prices, while these results are *usually* interpreted as giving support to the labour theory of value as an analytical tool for the understanding of the laws of motion of actual economies. However, the aforesaid estimations can be criticized for not taking into account (i) joint production activities; and (ii) alternative value bases. Since in the real world, joint production activities are by no means rare (see Steedman, 1984; Faber *et al*., 1998, Kurz, 2006), the extension of the analysis to the joint production case seems to be necessary. On the other hand, it is well known that any 'basic' (*à la* Sraffa, 1960, §6) commodity can be considered as a 'value base' and, therefore, it is possible to determine the so-called 'commodity *i* values' (Gintis and Bowles, 1981; Roemer, 1986), *i.e.* the direct and indirect requirements of commodity *i* necessary

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to produce one unit of each commodity as gross output. Therefore, the issue that arises is that, strictly speaking, there is no theoretical reason to choose the labour theory of value as the most relevant amongst the alternative 'value theories'.

To the extent we know, there is only one study that explores the empirical relationships between labour values and prices in joint production systems (see Mariolis and Soklis, 2010). The main finding of this study is that the deviations of market prices from labour values and production prices are considerably greater than those estimated on the basis of single-product systems, while there were found cases where the exploration of the relationships between values and prices is without economic meaning. On the other hand, there are a few studies that extend the empirical investigation of the relationships between prices and values to the case of alternative value bases on the basis of single-product systems and data from the Symmetric Input-Output Tables (SIOT) of various countries.[2] Cockshott and Cottrell (1997), Tsoulfidis and Maniatis (2002) and Zachariah (2006) conclude that labour values are by far better approximations of prices than commodity values and, therefore, there is an empirical basis for preferring labour as value base, while the results reported in Soklis (2009, 2014) indicate that the empirical investigation of the relationships between prices and values cannot downplay alternative value bases. To the best of our knowledge, there is not any study that explores the empirical relationships between prices and commodity values in joint production systems. The purpose of this paper is to estimate the deviations of prices from labour values and commodity values associated with the Supply and Use Tables (SUT) of the Finnish (for the year 2004) and Japanese (for the year 2000) economies.[3]As is well-known, in the SUT (SIOT) there are (are no) industries that produce more than one commodity and (nor) commodities that are produced by more than one industry and, therefore, may be considered as the empirical *counterpart* of joint production (single-product) systems *à la* von Neumann (1945) and Sraffa (1960).[4]

The remainder of the paper is organized as follows. Section 2 presents the analytic framework. Section 3 provides the results of the empirical analysis. Finally, section 4 concludes the paper.

THE ANALYTIC FRAMEWORK

We assume a closed, square and linear system of joint production with circulating capital and homogeneous labour, which is not an input to the household sector. The net product is distributed to profits and wages that are paid at the beginning of the common production period and there are no savings out of this income.[5] The givens in our analysis are (i) the vector of market prices; (ii) the technical conditions of production, *i.e.* the triplet {**B**, **A**, **I**}, where **B** represents the $n \times n$ Make matrix, **A** the $n \times n$ Use matrix, and $\mathbf{I}^{\mathrm{T}}$ the $1 \times n$ vector of employment levels process by process ('*T*' is the sign for transpose); and (iii) the real wage rate, which is represented by the $n \times 1$ vector **b**. On the basis of these assumptions, the prices of production, **p**, are defined by the following relations

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} = (1 + r)\,(\mathbf{p}^{\mathrm{T}}\mathbf{A} + w\mathbf{I}^{\mathrm{T}}) \tag{1}$$

$$w = \mathbf{p}^{\mathrm{T}}\mathbf{b} \tag{2}$$

where r the uniform rate of profits and w the money wage rate. From relations (1) and (2) it follows that

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} = (1 + r)\mathbf{p}^{\mathrm{T}}\mathbf{D} \tag{3}$$

where $\mathbf{D}$ ($\equiv \mathbf{A} + \mathbf{b}\mathbf{I}^{\mathrm{T}}$) the 'augmented' Use matrix. Provided that **B** is non-singular, relation (3) entails that

$$(1 + r)^{-1}\,\mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}\mathbf{D}\mathbf{B}^{-1} \tag{4}$$

where $(1 + r)^{-1}$ is an eigenvalue of the matrix $\mathbf{D}\mathbf{B}^{-1}$ and $\mathbf{p}^{\mathrm{T}}$ is the corresponding left-hand side eigenvector. Nevertheless, *nothing* guarantees the existence of a (semi-) positive solution for (r, **p**).[6] However, in the case where $[\mathbf{B}-\mathbf{A}]^{-1} > 0$ ($[\mathbf{B} - \mathbf{A}]^{-1} \geq 0$) the system {**B**, **A**} is called 'all-engaging' ('all-productive') and it holds $\mathbf{p} > 0$ ($\mathbf{p} \geq 0$) for $0 \leq r \leq R$, where R is the only positive root of det $[\mathbf{B}-(1+r)\mathbf{A}]$ associated with a positive eigenvector.[7] Thus, in the case of 'all-engaging' ('all-productive') systems the *actual* production prices (i.e. the prices of production corresponding to the *actual* real wage rate) derived from equation (4) are uniquely determined and positive (semi-positive).

As is well known, in joint production systems is, by definition, impossible to determine the *embodied* labour content of each commodity and, therefore, it is impossible to determine the total (direct and indirect) labour necessary to produce each commodity of the system.[8] On the other hand, in joint production it is possible to determine the vector of the so-called '*additive* labour values',[9] $\mathbf{v} \equiv [v_j]$, where v_j represents the quantity of labour necessary to produce net product that consists of one unit of commodity j, and is defined by the following system

$$\mathbf{v}^{\mathrm{T}}\mathbf{B} \equiv \mathbf{v}^{\mathrm{T}}\,\mathbf{A} + \mathbf{l}^{\mathrm{T}} \tag{5}$$

Now, we define the extended $m \times m$ ($m = n+1$) input and output matrices as **C** and **V**, respectively, where[10]

$$\mathbf{C} \equiv \begin{pmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{l}^{T} & 0 \end{pmatrix} \tag{6}$$

and

$$\mathbf{V} \equiv \begin{pmatrix} \mathbf{B} & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

On the basis of these matrices we may define the so-called '*additive* commodity values' as follows[11]

$$\omega_i^{T}\mathbf{V} \equiv \omega_i^{T}\mathbf{C}_{(i)} + \mathbf{c}_i^{T} \tag{8}$$

where $\omega_i^{\mathrm{T}} \equiv [\omega_1^i, \omega_2^i, ..., \omega_m^i]$, ω_j^i denotes the additive commodity i value of commodity j, i.e. the total (direct and indirect) requirements of commodity i necessary to produce one unit of commodity j as net product, $\mathbf{C}_{(i)}$ denotes the matrix derived from **C** by replacing all the elements of its i-th row with zero, and $\mathbf{c}_i^{\mathrm{T}}$ denotes the i-th row of **C**. Thus, for $i = m$, relation (8) gives the vector of additive labour values (see relation (5)), with the m-th element of $\boldsymbol{\omega}_m$ representing the 'value of labour power'. However, nothing guarantees that the system (8) is consistent. Moreover, even if the system is consistent, it is possible to have more than one solution. In the case where the vector of additive commodity values is uniquely determined, then the solution of (8) is given by

$$\omega_i^{\mathrm{T}} = \mathbf{c}_i^{\mathrm{T}}[\mathbf{V} - \mathbf{C}_{(i)}]^{-1} \tag{9}$$

However, even in this case, nothing guarantees that the solution given by relation (9) is economically significant. Thus, in the case of joint production, is *a priori* unknown if the empirical investigation of the relationships between prices and values is economically meaningful.

Finally, it should be stressed that any 'complication' related to joint production, *i.e.* non-squareness, inconsistency or non-unique economically significant solution for (r, **p**) and/or ω_i can be adequately handled on the basis of general joint production models inspired by von Neumann (1945) and Sraffa (1960).[12]

EMPIRICAL INVESTIGATION

The application of the previous analysis to the SUT of the Finnish (for the year 2004) and Japanese (for the year 2000) economies gives the following results:[13]

(i) The matrices **B** and $[\mathbf{V} - \mathbf{C}_{(i)}]$ are non-singular. Consequently, **p** can be estimated from (4), and ω_i can be uniquely estimated from (9).

(ii) The matrices $[\mathbf{B}-\mathbf{A}]^{-1}$ contain negative elements. Consequently, the systems under consideration are not 'all-productive' and, therefore, they do not have the properties of a single-product system.

(iii) The system of production prices of the Finnish economy has 21 positive and 36 complex conjugate solutions for r, and only economically *insignificant* solutions for **p**. On the other hand, the system of production prices of the Japanese economy has a unique, positive solution for $(r, \mathbf{p})$, and $(1+r)^{-1}$ is the *dominant* eigenvalue of the matrix $\mathbf{DB}^{-1}$. Thus, it is found that the actual uniform rate of profits of the Japanese economy is almost 39%.[14]

(iv) The vector of additive labour values of the Finnish economy contains one negative element, which corresponds to the commodity 37 ('Secondary raw materials').[15] On the other hand, the vector of additive labour values of the Japanese economy is positive.[16]

(v) In the case of the Finnish economy, 29 from the 57 estimated vectors of additive commodity values are economically insignificant. More specifically, non economically significant are the vectors of commodity values that correspond to the following 'value bases' (by CPA code): 01, 02, 05, 10, 13, 14, 15, 16, 20, 23, 11, 24, 26, 27, 28, 29, 30, 31, 33, 37, 40, 41, 45, 52, 70, 72, 80, 85, 93, 95. The vectors associated with the commodities 01, 05, 13, 14, 15, 16, 20, 23, 24, 26, 27, 28, 29, 31, 33, 40, 41, 45, 52, 70, 72, 80, 85, 93 and 95 have one negative element that corresponds to the commodity 37. The vectors associated with the commodities 02, 30 and 37 have one negative element that corresponds to the commodities 10, 32 and 14, respectively. Finally, the vector associated with the commodity 10 has two negative elements that correspond to the commodities 23 11 and 37. Since we have already estimated that the vector of additive labour values is economically insignificant, it follows that, in the case of the Finnish economy, the comparison of the empirical relationships between prices-labour values and prices-commodity values is without economic meaning.

(vi) In the case of the Japanese economy, all the 24 estimated vectors of additive commodity values are positive. Therefore, in this case, it is economically meaningful to explore the empirical relationships between prices and values.

In order to assess the deviation of prices from values, we use a normalization bias-free measure of deviation that has been proposed by Steedman and Tomkins (1998) and is known as the '*d*-distance'.[17] The '*d*-distance' is defined as $d \equiv \sqrt{2(1-\cos\theta)}$, where θ is the Euclidean angle between the vectors $[\boldsymbol{\pi}_i^-]^{\mathrm{T}}(\hat{\omega}_i^-)^{-1}$ and **e**, $[\boldsymbol{\pi}_i^-]^{\mathrm{T}}$ is the vector derived from $\mathbf{p}^{\mathrm{T}} \equiv (\mathbf{p}^{\mathrm{T}}, w)$ if we extract its *i* th element, $\hat{\omega}_i^-$ a diagonal matrix formed from the elements of $\boldsymbol{\omega}_i$ if we extract its *i* th element and $[\boldsymbol{\pi}_i^-]^{\mathrm{T}}(\hat{\boldsymbol{\omega}}_i^-)^{-1}$ the ratio of prices to values.[18] Since the *theoretically* minimum value of cos θ equals $1/\sqrt{n}$, the theoretically maximum value of the '*d* distance', *D*, equals $\sqrt{2[1-(1/\sqrt{n})]}$. Thus, we may define the normalized '*d*-distance', as d/D (see also Mariolis and Soklis, 2010, p. 94).

In Table 1 and Figure 1 we report the deviations of prices from values for the Japanese economy. The first row of Table 1 refers to the deviations of prices from additive labour values, while the remaining rows report the deviations of prices from additive commodity values.[19] The last row refers to the average deviations of prices from values, *i.e.* the sum of the deviations divided by the total number of commodities that are used as value bases. Furthermore, in order to get a picture of the price-value deviations, in Figure 1 we display the deviations both of the prices of production from values and of market prices from values as well. The production (market) price-value deviations are displayed in the horizontal (vertical) axis, while the price-labour value deviations are taken as the origin of the axes. Thus, the points below (above) the horizontal axis indicate market price-commodity value deviations that are less (greater) than the market price-labour value deviations, while the points on the left (right) of the vertical axis indicate production price-commodity value

Table 1
Deviations of Prices from Additive Values; Japanese Economy, 2000

d/D(%) *'Value base'*	*Actual prices of production vs. values*	*Market prices vs. values*
Labour	16.3	44.2
Agriculture, forestry and fishing	23.8	49.4
Mining	27.8	56.3
Food products and beverages	20.8	46.5
Textiles	21.5	51.8
Pulp, paper and paper products	25.3	54.0
Chemicals	20.0	54.1
Petroleum and coal products	22.4	53.9
Non-metallic mineral products	23.6	37.8
Basic metals	31.6	50.4
Fabricated metal products	23.5	39.3
Machinery	23.5	48.9
Electrical machinery, equipment and supplies	23.3	49.3
Transport equipment	19.0	46.2
Precision instruments	20.3	47.3
Other manufacturing products	15.0	49.2
Construction	24.5	21.0
Electricity, gas and water supply	13.3	51.1
Wholesale and retail trade	18.5	45.2
Finance and insurance	18.3	36.3
Real estate	19.1	22.4
Transport and communications	17.3	48.0
Service activities	16.8	40.8
Government services	19.3	41.7
Private non-profit services to households	18.4	44.6
Average deviation of prices from values	20.9	45.2

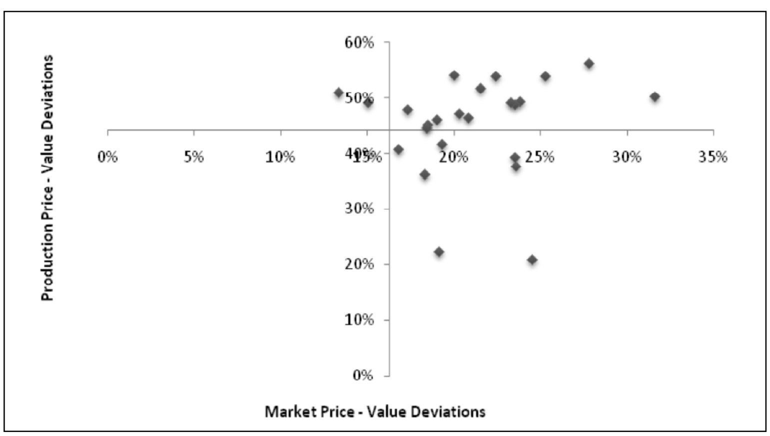

Figure 1: Deviations of Prices from Values; Japanese Economy, 2000

deviations that are less (greater) than the production price-labour value deviation.

From the Table 1 and the associated numerical results, we arrive at the following conclusions:

(i) The deviation of the vector of production (market) prices from the vector of labour values is almost 16.3% (44.2%).

(ii) The average deviation of production (market) prices from values is in the area of 20.9% (45.2%).

(iii) The deviation of production prices from the vectors of commodity values associated with the commodities 'Other manufacturing products' and 'Electricity, gas and water supply' is less than the corresponding production price-labour value deviation.[20]

(iv) The deviation of market prices from the vectors of commodity values associated with the commodities 'Non-metallic mineral products', 'Fabricated metal products', 'Construction', 'Finance and insurance', 'Real estate', 'Service activities' and 'Government services' is less than the corresponding market price-labour value deviation.[21]

(v) The smallest (largest) production price-value deviation is 13.3% (31.6%) and corresponds to the vector of commodity values associated with the commodity 'Electricity, gas and water supply' ('Basic metals').

(vi) The smallest (largest) market price-value deviation is 21.0% (56.3%) and corresponds to the vector of commodity values associated with the commodity 'Construction' ('Mining').

The next issue that comes up is whether the systems under consideration are 'r-all-engaging', i.e., characterized by $\mathbf{E}(r) \equiv [\mathbf{B} - (1+r)\mathbf{A}]^{-1} > \mathbf{0}$ for some $r > -1$.[22] As is well known, $\mathbf{E}(r) > \mathbf{0}$ is a sufficient condition for the existence of an interval of r, in which a joint production system retains all the essential properties of indecomposable single-product systems (see Schefold, 1971, p. 35; 1978b; Bidard, 1996).[23] The investigation can be based on the following theorem (Bidard, 1996, p. 328): Consider the eigensystems associated with the pair $\{\mathbf{B}, \mathbf{A}\}$, namely

$$\lambda \mathbf{B}\mathbf{x} = \mathbf{A}\mathbf{x} \tag{10}$$

$$\lambda \mathbf{y}^{\mathrm{T}}\mathbf{B} = \mathbf{y}^{\mathrm{T}}\mathbf{A} \tag{11}$$

The system {**B**, **A**} is '*r*-all-engaging' if there exist (λ, **x**, **y**) > 0, where **x** is determined up to a factor.[24]

The estimation of the characteristic values and vectors associated with the pairs {**B**, **A**} of the Finnish and Japanese economies gives the following result: the eigensystem of the Finnish economy has 20 positive (and simple) eigenvalues, while that of the Japanese economy has 7 positive (and simple) eigenvalues.[25] However, there are no positive left and right eigenvectors and, therefore, the considered systems are not *r*-all-engaging.

CONCLUDING REMARKS

This paper extended the empirical investigation of the relationships between prices and commodity values to the case of joint production, using data from the Supply and Use Tables of the Finnish (for the year 2004) and Japanese (for the year 2000) economies. The results showed that the systems under investigation are neither 'all-productive' nor '*r*-all-engaging' and, therefore, they do not have the properties of single-product systems. Moreover, in the case of the Japanese economy, it has been found that there exist vectors of additive commodity values that are better approximations of prices than additive labour values. This finding indicates that the empirical investigation of the relationships between values and actual prices should not *a priori* neglect alternative value bases. However, we do not consider that these results can provide support to an alternative value theory. On the contrary, by taking into account the finding of economically insignificant production prices and values in the case of the Finnish economy, these results cast doubt on the logic of the so-called 'empirical labour theory of value' (Stigler, 1958, p. 361). Future research efforts should use more disaggregated input-output data from various countries, concretize the model by including the presence of fixed capital and the degree of its utilization, depreciation, turnover times, taxes and subsidies, and explore the relationships between prices and hypothetical changes in income distribution.

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Notes

1. See Shaikh (1984, 1998), Petrović (1987), Ochoa (1989), Cockshott *et al.* (1995), Chilcote (1997), Tsoulfidis and Rieu (2006), T soulfidis and Mariolis (2007), Tsoulfidis (2008), *inter alia*.
2. See Cockshott and Cottrell (1997), Tsoulfidis and Maniatis (2002), Zachariah (2006), Soklis (2009, 2014).
3. See Appendix 1 for the available input-output data as well as the construction of relevant variables. It is worth noting that the system of the Finnish economy has also been investigated in Soklis (2011) for the estimation of wage-profit curves in joint production.
4. See, *e.g.*, Flaschel (1980, pp. 120-121) and Bidard and Erreygers (1998, pp. 434-436). It has to be noted, however, that some of the 'joint' products that appear in the SUT may result from statistical classification and, therefore, they do not correspond with the notion of joint production (see, *e.g.*, Semmler, 1984, pp. 168-169; United Nations, 1999, p. 77).
5. We hypothesize that wages are paid *ante factum* (for the general case, see Steedman, 1977, pp. 103-105) and that there are no savings out of this income in order to follow most of the empirical studies on this topic (see footnote 1).
6. See Filippini and Filippini (1982), Fujimoto and Krause (1988) and Hosoda (1993).
7. The concept of all-engaging (all-productive) systems, introduced by Schefold, is of significant importance since it corresponds with systems that retain all the essential properties of indecomposable (decomposable) single-product systems (see Schefold, 1971, pp. 34–5, 1978b; Kurz and Salvadori, 1995, pp. 238–40; Bidard, 1996).
8. As Sraffa (1960, p. 56) stresses: '[I]n the case of joint-products there is no obvious criterion for apportioning the labour among individual products, and indeed it seems doubtful whether it makes any sense to speak of a *separate* quantity of labour as having gone to produce one of a number of *jointly* produced commodities.'
9. For this concept, see Steedman (1975, 1976b, 1977, chs 12-13).
10. See, *e.g.*, Okishio (1963). The matrix **C** is also known as the 'complete' or 'full' matrix (Bródy 1970).
11. For this concept, see Mariolis (2003).
12. For a detailed exposition of the von Neumann/Sraffa-based analysis and the connection between the works of von Neumann and Sraffa, see Kurz and Salvadori (1995, ch. 8 and pp. 421–426, 2001) and Bidard (1997).
13. *Mathematica 7.0* is used in the calculations, while the precision in internal calculations is set to 16 digits. All the analytical results are available on request from the author.

14. The eigenvalues of systems (4) are reported in the Appendix 2.

15. As is well known, some labour values are negative iff a non-negative linear combination of some industries yields a greater net output per unit of labour employed than a non-negative linear combination of the remaining ones (see Filippini and Filippini, 1982, pp. 387-388).

16. The additive labour values of the Finnish and Japanese economies are reported in the Appendix 3, Tables A3.1 and A3.2, respectively. Notethatwereport the 'complete' *à la* Brò dy (1970) vectors, *i.e.* we include the additive value of the real wage bundle (or, equivalently, the value of labour power) as the last element of the vectors.

17. For a detailed discussion of the problem of measuring the deviation of prices from labour values, see *ibid*. Moreover, Mariolis and Soklis (2011) have shown that there exists an infinite number of numeraire-free measures (*à la* Steedman-Tomkins) of price-value deviation, whose ranking is *a priori* unknown, and the choice between them depends on either the theoretical viewpoint or the aim of the observer. Finally, it is worth noting that Mariolis and Tsoulfidis (2010) demonstrated that for realistic values of the relative rate of profit (*i.e.* not considerably greater that 40%), the Steedman-Tomkins distance and the traditional measures, such as the 'mean absolute deviation', the 'mean absolute weighted deviation' and the 'root-mean-square-percent-error', tend to be close to each other.

18. Note that for $i \neq m$ we get $[\boldsymbol{\pi}_i^-]^{\mathrm{T}} = (p_1, p_2, ..., p_{i-1}, p_{i+1}, ..., w)$, while for $i = m$ we get $[\boldsymbol{\pi}_i^-]^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}$. Furthermore, the '*d*- distance' between market prices and values is estimated on the basis of the Euclidean angle between the vectors $(\boldsymbol{\pi}_i^{\mathrm{M}})^{\mathrm{T}}(\hat{\boldsymbol{\omega}}_i^-)^{-1}$ and $\mathbf{e}$, where $(\boldsymbol{\pi}_i^{\mathrm{M}})^{\mathrm{T}} \equiv (p_1^{\mathrm{M}}, p_2^{\mathrm{M}}, ..., p_{i-1}^{\mathrm{M}}, p_{i+1}^{\mathrm{M}}, ..., p_m^{\mathrm{M}})$ denotes the vector of market prices. Since market prices are taken to be equal to 1 (see Appendix 1), it follows that for $i \neq m$ we get $(\boldsymbol{\pi}_i^{\mathrm{M}})^{\mathrm{T}} = (1,1,1,1,...,w_{\min}^{\mathrm{M}})$, while for $i = m$ we get $(\boldsymbol{\pi}_m^{\mathrm{M}})^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}$. I am grateful to Theodore Mariolis for an enlightening discussion on this point.

19. The price-commodity value deviations that are found to be less than the corresponding price-labour value deviations are indicated by bold characters.

20. The afore said vectors of commodity values are reported in Appendix 4, Tables A4.1-A4.2. The total requirements of a commodity necessary to produce net product that consists of one unit of it self are indicated by bold characters.

21. The aforesaid vectors of commodity values are reported in Appendix 4, Tables A4.3-A4.9.

22. See also Note 7.

23. It is important to note that this attribute of the considered systems is independent of the composition and the level of the real wage rate and, therefore, does not rely on our hypothesis that there are no savings out of

wages. Furthermore, since the matrices $[\mathbf{B}-\mathbf{A}]^{-1}$ contain negative elements, it follows that the systems under consideration can be '*r*-all-engaging' only for some $r > 0$ (*ibid*.).

24. In that case $\lambda^{-1} -1$ represents the maximum possible rate of growth (and profits), as defined by v. Neumann (1945), $\mathbf{y}^T$ the associated price vector, and x the associated intensity vector or, alternatively, the intensity vector of Sraffa's (1960, ch. 8) 'Standard system'.
25. The eigenvalues of the pairs {**B**, **A**} of the Finnish and Japanese economies are reported in Appendix 5, Tables A5.1 and A5.2, respectively.

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APPENDIX 1
A NOTE ON THE DATA

The SUT of the Finnish economy and the corresponding levels of sectoral employment are provided via the Eurostat website http://ec.europa.eu/eurostat. The SUT describe 59 products, which are classified according to CPA (Classification of Product by Activity) and 59 industries, which are classified according to NACE (General Industrial Classification of Economic Activities within the European Communities). The described products and their correspondence to CPA are reported in Table A1 below. However, all the elements associated with the product 12 (Uranium and thorium ores) and industry 12 (Mining of uranium and thorium ores) equal zero and, therefore, we remove them from our analysis. Furthermore, all the elements associated with the product 11 (Crude petroleum and natural gas) and industry 11 (Extraction of crude petroleum and natural gas) in the Make matrices (i.e. the part of the Supply Tables that describes domestic production) equal zero, and, therefore, we remove them from our analysis, while there are elements associated with the product 11 in the Use matrices (i.e. the part of the Use Tables that describes intermediate consumption) that are positive. In order to derive 'square' Make and Use matrices, we aggregate the product 11 with the 'primary product' (Coke, refined petroleum products and nuclear fuels) of industry 23 (Manufacture of coke, refined petroleum products and nuclear fuels). This choice is based on the fact that the product 11 is mainly used by the industry 23. Thus, we derive Make and Use matrices of dimensions 57×57.The SUT of the Japanese economy and the corresponding levels of sectoral employment are provided via the website of the Cabinet Office, http://www.cao.go.jp. The SUT describe 24 products and activities. Thus, the Make and Use matrices are of dimensions 24×24. The described products are reported in Table A2 below. It need hardly be said, that the SUT are not necessarily square (see, *e.g.*, United Nations (1999, p. 86, §4.41) and Eurostat (2008, p. 295, §11.1), while for the relevant theoretical discussion, see, *e.g.*, Steedman (1976a), Schefold(1978a) and Bidard (1986, 1997).

The SUT of the Finnish economy are measured in'basic prices', while those of the Japanese economy are measured in 'purchasers' prices'. It is important to note that we decided to use Finland's and Japan's SUT mainly because there were available Make and Use Tables at the same prices. Since, usually, Make Tables are constructed at basic prices, while Use Tables are constructed at purchasers' prices, most statistical offices do not provide these tables at the same prices. The market prices of all products are taken to be equal to one; that is to say, the physical unit of measurement of each product is that unit which is worth of a monetary unit (see, *e.g.*, Miller and Blair, 1985, p. 356).Wage differentials are used to homogenize the sectoral employment (see, *e.g.*, Sraffa, 1960, §10, and Kurz and Salvadori, 1995, pp. 322-325), *i.e.* the j-th element of the vector of employment levels process by process, **l**, is determined as follows: $l_j = L_j(w_j^M / w_{min}^M)$, where L_j and w_j^M are total employment and money wage rate, in terms of market prices, of the *j*-th sector, respectively, and w_{min}^M is the minimum sectoral money wage rate in terms of market prices. Alternatively, the homogenization of employment could be achieved, for example, through the economy's average wage; in fact, the empirical results are robust to alternative normalizations with respect to homogenization of labour inputs. Furthermore, by assuming that workers do not save and that their consumption has the same composition as the vector of private households consumption expenditure, **h**, directly available in the SUT, the vector of the real wage rate, **b**, is determined as follows: $\mathbf{b} = (w_{min}^M / \mathbf{eh})\mathbf{h}$, where $\mathbf{e} \equiv [1,1,...,1]$ represents the vector of market prices (see also, *e.g.*, Okishio and Nakatani, 1985, pp. 66-67). Finally, it should be noted that in the available SUT of the Finnish and Japanese economies we do not have data on the matrix of fixed capital coefficients and the non-competitive imports. As a result, our investigation is based on a model for a closed economy with circulating capital.

Table A1.1
Product classification; Finnish Economy

No	*CPA*	*Nomenclature*
1	01	Products of agriculture, hunting and related services
2	02	Products of forestry, logging and related services
3	05	Fish and other fishing products; services incidental of fishing
4	10	Coal and lignite; peat
5	11	Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying
6	12	Uranium and thorium ores
7	13	Metal ores
8	14	Other mining and quarrying products
9	15	Food products and beverages
10	16	Tobacco products
11	17	Textiles
12	18	Wearing apparel; furs
13	19	Leather and leather products
14	20	Wood and products of wood and cork (except furniture); articles of straw and plaiting materials
15	21	Pulp, paper and paper products
16	22	Printed matter and recorded media
17	23	Coke, refined petroleum products and nuclear fuels
18	24	Chemicals, chemical products and man-made fibres
19	25	Rubber and plastic products
20	26	Other non-metallic mineral products

contd. table

21	27	Basic metals
22	28	Fabricated metal products, except machinery and equipment
23	29	Machinery and equipment n.e.c.
24	30	Office machinery and computers
25	31	Electrical machinery and apparatus n.e.c.
26	32	Radio, television and communication equipment and apparatus
27	33	Medical, precision and optical instruments, watches and clocks
28	34	Motor vehicles, trailers and semi-trailers
29	35	Other transport equipment
30	36	Furniture; other manufactured goods n.e.c.
31	37	Secondary raw materials
32	40	Electrical energy, gas, steam and hot water
33	41	Collected and purified water, distribution services of water
34	45	Construction work
35	50	Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel
36	51	Wholesale trade and commission trade services, except of motor vehicles and motorcycles
37	52	Retail trade services, except of motor vehicles and motorcycles; repair services of personal and household goods
38	55	Hotel and restaurant services
39	60	Land transport; transport via pipeline services
40	61	Water transport services
41	62	Air transport services
42	63	Supporting and auxiliary transport services; travel agency services
43	64	Post and telecommunication services
44	65	Financial intermediation services, except insurance and pension funding services
45	66	Insurance and pension funding services, except compulsory social security services
46	67	Services auxiliary to financial intermediation
47	70	Real estate services
48	71	Renting services of machinery and equipment without operator and of personal and household goods
49	72	Computer and related services
50	73	Research and development services
51	74	Other business services
52	75	Public administration and defence services; compulsory social security services
53	80	Education services
54	85	Health and social work services
55	90	Sewage and refuse disposal services, sanitation and similar services
56	91	Membership organisation services n.e.c.
57	92	Recreational, cultural and sporting services
58	93	Other services
59	95	Private households with employed persons

Table A1.2
Product classification; Japanese Economy

No	*Nomenclature*
1	Agriculture, forestry and fishing
2	Mining
3	Foodproducts and beverages
4	Textiles
5	Pulp, paper and paper products
6	Chemicals
7	Petroleum and coal products
8	Non-metallic mineral products
9	Basic metals
10	Fabricated metal products
11	Machinery
12	Electrical machinery, equipment and supplies
13	Transport equipment
14	Precision instruments
15	Other manufacturing products
16	Construction
17	Electricity, gas and water supply
18	Wholesale and retail trade
19	Finance and insurance
20	Real estate
21	Transport and communications
22	Service activities
23	Government services
24	Private non-profit services to households

APPENDIX 2
EIGENVALUES OF THE SYSTEMS OF PRODUCTION PRICES OF THE FINNISH AND JAPANESE ECONOMIES

Table A2.1
Eigenvalues of DB^{-1}; Finland, 2004

1	0.761	21	0.102
2	0.678	22	0.076 ± 0.039 *i*
3	0.632	23	-0.003±0.083 *i*
4	0.463	24	0.075±0.004 *i*
5	0.386	25	-0.062±0.026 *i*
6	0.343±0.025 *i*	26	0.060
7	0.323	27	0.047±0.024 *i*
8	0.296	28	-0.048±0.015 *i*
9	0.287	29	0.037±0.034 *i*
10	0.272	30	0.032
11	0.242±0.047 *i*	31	0.030
12	0.227	32	-0.015±0.021 *i*
13	0.221	33	0.023
14	0.208	34	-0.007±0.013 *i*
15	0.194±0.028 *i*	35	0.014
16	0.183±0.013 *i*	36	-0.011±0.004 *i*
17	0.153±0.037 *i*	37	0.008±0.008 *i*
18	0.156	38	0.004
19	0.124±0.064 *i*	39	0.0002
20	0.063±0.081 *i*		

Table A2.2
Eigenvalues of DB^{-1}; Japan, 2000

1	0.721
2	0.449 ± 0.020
3	0.330 ± 0.006
4	0.278 ± 0.064
5	0.259
6	0.220
7	-0.210
8	0.191
9	0.128
10	0.127 ± 0.008
11	0.086
12	0.051 ± 0.025
13	-0.002 ± 0.029
14	0.010
15	0.007
16	-0.001 ± 0.007
17	-0.001

APPENDIX 3
ADDITIVE LABOUR VALUES (ALV) OF THE FINNISH AND JAPANESE ECONOMIES

Table A3.1
ALV; Finland, 2004

CPA	*ALV*	*CPA*	*ALV*
1	94.1	40	59.5
02	39.5	41	77.0
05	56.7	45	131.7
10	103.0	50	117.1
13	117.5	51	113.3
14	98.0	52	131.7
15	126.2	55	138.6
16	208.1	60	92.2
17	132.3	61	90.6
18	141.6	62	86.4
19	138.9	63	89.9
20	95.7	64	78.6
21	93.9	65	111.8
22	122.3	66	114.3
$23 \oplus 11$	45.6	67	114.3
24	98.8	70	49.0
25	117.7	71	91.2
26	116.6	72	147.2
27	108.9	73	183.0
28	133.1	74	146.0
29	141.8	75	160.6
30	145.9	80	178.4
31	129.4	85	173.4
32	48.4	90	98.7
33	135.1	91	161.3
34	146.1	92	135.7
35	153.2	93	92.4
36	131.1	95	230.6
37	-93.8	**Real wage**	**0.442**

Table A3.2
ALV; Japan, 2000

Nomenclature	*ALV*
Agriculture, forestry and fishing	0.054
Mining	0.104
Food products and beverages	0.075
Textiles	0.141
Pulp, paper and paper products	0.092
Chemicals	0.079
Petroleum and coal products	0.061
Non-metallic mineral products	0.104
Basic metals	0.083
Fabricated metal products	0.119
Machinery	0.117
Electrical machinery, equipment and supplies	0.105
Transport equipment	0.107
Precision instruments	0.121
Other manufacturing products	0.114
Construction	0.124
Electricity, gas and water supply	0.070
Wholesale and retail trade	0.108
Finance and insurance	0.085
Real estate	0.022
Transport and communications	0.104
Service activities	0.106
Government services	0.128
Private non-profit services to households	0.152
Real wage	0.416

APPENDIX 4
ADDITIVE COMMODITY VALUES (ACV) OF THE JAPANESE ECONOMY

Table A4.1
'Other manufacturing products values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.094
Mining	0.157
Food products and beverages	0.135
Textiles	0.198
Pulp, paper and paper products	0.216
Chemicals	0.148
Petroleum and coal products	0.091
Non-metallic mineral products	0.158
Basic metals	0.123
Fabricated metal products	0.161
Machinery	0.183
Electrical machinery, equipment and supplies	0.201
Transport equipment	0.218
Precision instruments	0.218
Other manufacturing products	**0.314**
Construction	0.222
Electricity, gas and water supply	0.105
Wholesale and retail trade	0.140
Finance and insurance	0.125
Real estate	0.030
Transport and communications	0.135
Service activities	0.164
Government services	0.175
Private non-profit services to households	0.222
Real wage	0.997

Table A4.2
'Electricity, gas and water supply values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.051
Mining	0.103
Food products and beverages	0.072
Textiles	0.136
Pulp,paper and paper products	0.141
Chemicals	0.127
Petroleum and coal products	0.068
Non-metallic mineral products	0.121
Basic metals	0.134
Fabricated metal products	0.116
Machinery	0.103
Electrical machinery, equipment and supplies	0.100
Transport equipment	0.104
Precision instruments	0.103
Other manufacturing products	0.108
Construction	0.096
Electricity, gas and water supply	**0.105**
Wholesale and retail trade	0.074
Finance and insurance	0.058
Real estate	0.018
Transport and communications	0.083
Service activities	0.092
Government services	0.107
Private non-profit services to households	0.106
Real wage	0.502

Table A4.3
'Non-metallic mineral products values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.011
Mining	0.014
Food products and beverages	0.018
Textiles	0.019
Pulp, paper and paper products	0.016
Chemicals	0.023
Petroleum and coal products	0.009
Non-metallic mineral products	**0.128**
Basic metals	0.030
Fabricated metal products	0.025
Machinery	0.028
Electrical machinery, equipment and supplies	0.035
Transport equipment	0.035
Precision instruments	0.043
Other manufacturing products	0.022
Construction	0.101
Electricity, gas and water supply	0.013
Wholesale and retail trade	0.013
Finance and insurance	0.010
Real estate	0.006
Transport and communications	0.013
Service activities	0.016
Government services	0.017
Private non-profit services to households	0.019
Real wage	0.091

Table A4.4
'Fabricated metal products values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.018
Mining	0.047
Food products and beverages	0.046
Textiles	0.033
Pulp, paper and paper products	0.026
Chemicals	0.037
Petroleum and coal products	0.027
Non-metallic mineral products	0.039
Basic metals	0.025
Fabricated metal products	**0.092**
Machinery	0.076
Electrical machinery, equipment and supplies	0.050
Transport equipment	0.048
Precision instruments	0.053
Other manufacturing products	0.041
Construction	0.014
Electricity, gas and water supply	0.024
Wholesale and retail trade	0.024
Finance and insurance	0.017
Real estate	0.010
Transport and communications	0.023
Service activities	0.027
Government services	0.030
Private non-profit services to households	0.031
Real wage	0.161

Table A4.5
'Construction values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.021
Mining	0.034
Food products and beverages	0.023
Textiles	0.039
Pulp, paper and paper products	0.038
Chemicals	0.035
Petroleum and coal products	0.022
Non-metallic mineral products	0.044
Basic metals	0.039
Fabricated metal products	0.042
Machinery	0.034
Electrical machinery, equipment and supplies	0.033
Transport equipment	0.032
Precision instruments	0.035
Other manufacturing products	0.034
Construction	**0.034**
Electricity, gas and water supply	0.067
Wholesale and retail trade	0.030
Finance and insurance	0.023
Real estate	0.048
Transport and communications	0.035
Service activities	0.031
Government services	0.043
Private non-profit services to households	0.051
Real wage	0.177

Table A4.6
'Finance and insurance values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.040
Mining	0.087
Food products and beverages	0.046
Textiles	0.086
Pulp, paper and paper products	0.058
Chemicals	0.054
Petroleum and coal products	0.055
Non-metallic mineral products	0.067
Basic metals	0.057
Fabricated metal products	0.068
Machinery	0.068
Electrical machinery, equipment and supplies	0.060
Transport equipment	0.064
Precision instruments	0.068
Other manufacturing products	0.066
Construction	0.071
Electricity, gas and water supply	0.050
Wholesale and retail trade	0.071
Finance and insurance	**0.069**
Real estate	0.018
Transport and communications	0.067
Service activities	0.061
Government services	0.067
Private non-profit services to households	0.083
Real wage	0.460

Table A4.7
'Real estate values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.102
Mining	0.205
Food products and beverages	0.142
Textiles	0.268
Pulp, paper and paper products	0.177
Chemicals	0.159
Petroleum and coal products	0.120
Non-metallic mineral products	0.201
Basic metals	0.163
Fabricated metal products	0.227
Machinery	0.223
Electrical machinery, equipment and supplies	0.202
Transport equipment	0.203
Precision instruments	0.231
Other manufacturing products	0.219
Construction	0.234
Electricity, gas and water supply	0.141
Wholesale and retail trade	0.227
Finance and insurance	0.173
Real estate	**0.047**
Transport and communications	0.211
Service activities	0.210
Government services	0.236
Private non-profit services to households	0.285
Real wage	1.800

Table A4.8
'Service activities values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.182
Mining	0.428
Food products and beverages	0.256
Textiles	0.416
Pulp, paper and paper products	0.318
Chemicals	0.333
Petroleum and coal products	0.250
Non-metallic mineral products	0.353
Basic metals	0.298
Fabricated metal products	0.371
Machinery	0.372
Electrical machinery, equipment and supplies	0.360
Transport equipment	0.343
Precision instruments	0.373
Other manufacturing products	0.371
Construction	0.392
Electricity, gas and water supply	0.282
Wholesale and retail trade	0.332
Finance and insurance	0.326
Real estate	0.082
Transport and communications	0.357
Service activities	**0.369**
Government services	0.365
Private non-profit services to households	0.420
Real wage	2.124

Table A4.9
'Government services values'

Nomenclature	*ACV*
Agriculture, forestry and fishing	0.009
Mining	0.018
Food products and beverages	0.013
Textiles	0.023
Pulp, paper and paper products	0.016
Chemicals	0.016
Petroleum and coal products	0.011
Non-metallic mineral products	0.018
Basic metals	0.015
Fabricated metal products	0.019
Machinery	0.019
Electrical machinery, equipment and supplies	0.017
Transport equipment	0.017
Precision instruments	0.019
Other manufacturing products	0.019
Construction	0.020
Electricity, gas and water supply	0.013
Wholesale and retail trade	0.018
Finance and insurance	0.014
Real estate	0.004
Transport and communications	0.024
Service activities	0.023
Government services	**0.021**
Private non-profit services to households	0.024
Real wage	0.135

APPENDIX 5
EIGENVALUES OF THE PAIRS {B, A} OF THE FINNISH AND JAPANESE ECONOMIES

Table A5.1
Eigenvalues of {B, A}; Finland, 2004

1	0.705	21	0.058 ± 0.073 *i*
2	0.652	22	0.080 ± 0.042 *i*
3	0.545	23	-0.003 ± 0.081 *i*
4	0.446	24	0.079
5	0.386	25	0.068
6	0.337 ± 0.032 *i*	26	-0.055 ± 0.021 *i*
7	0.324	27	0.049 ± 0.024 *i*
8	0.292	28	0.053
9	0.285	29	-0.052 ± 0.004 *i*
10	0.271	30	0.033 ± 0.034 *i*
11	0.235 ± 0.047 *i*	31	-0.009 ± 0.032 *i*
12	0.227	32	0.030 ± 0.005 *i*
13	0.218	33	0.023
14	0.208	34	0.018
15	0.191 ± 0.024 *i*	35	-0.012 ± 0.002 *i*
16	0.182 ± 0.019 *i*	36	0.007 ± 0.006 *i*
17	0.164	37	-0.004 ± 0.006 *i*
18	0.152 ± 0.040 *i*	38	0.004
19	0.119 ± 0.056 *i*	39	0
20	0.101		

Table A5.2
Eigenvalues of {B, A}; Japan, 2000

1	0.554
2	0.447 ± 0.014 *i*
3	0.331 ± 0.006 *i*
4	0.275
5	0.252 ± 0.051 *i*
6	0.222
7	-0.210
8	0.180
9	0.128
10	0.121 ± 0.011 *i*
11	0.085
12	0.050 ± 0.028 *i*
13	0.004 ± 0.015 *i*
14	-0.008
15	0.007
16	0.005 ± 0.003 *i*
17	-6.052×10^{-8}