International Review of Pure and Applied Mathematics
Volume 14 • Number 2 • July-December 2018

OPTIMIZATION OF TYPE-C OC CURVES FOR TRUNCATED GENERALIZED EXPONENTIAL DISTRIBUTION

G. Venkatesulu

Abstract: Acceptance sampling plans was studied mainly to draw valid conclusions with regard to accept or reject the lots of finished products. . This Paper, it is proposed CASP-CUSUM Schemes for Optimizing Type-C OC curve and ARL, by assuming that the variable under consideration is distributed to Generalized Exponential distribution. Under this assumption, we determined Truncated Generalized Exponential distribution and its Type-C OC curve values and ARL values at different values of the shape parameter. Finally, we determined an optimal CASP-CUSUM scheme, such that P (A) is maximized. The obtained results were presented graphically.

Keywords: CASP-CUSUM Schemes, Type-C OC curve, Truncated Generalized Exponential Distribution.

I. INTRODUCTION

Product quality has become one of the most important issues that distinguish different commodities in a global business market. Two important techniques for ensuring quality are the statistical process control and statistical product control in the form of acceptance sampling.

Acceptance sampling is a quality control method used to accept or reject a lot after testing a random sample of a product. The purpose of acceptance sampling is to make a determination about lot of the product; accept the lot or reject it rather than to estimate the quality of the entire lot. Acceptance sampling is a very useful technique when a lot is so large in size or when testing is destructive. For a large lot, it is too time consuming and too costly to inspect every single of the product. Plus, checking every single product does not guarantee that the product will comply with required specification.

Life test refers to measurements of product life; product life can be measured in hours, miles, cycles or any other metric that applies to the period of successful operation of a particular product. Since time is a common measure of life, life data points are often called times-to-failure. There are different types of life products. Statistical distributions have been assumed by various authors (statisticians, mathematicians and engineers) to mathematically model or represent certain behavior of products. The probability density function (pdf) and cumulative distribution function (cdf) are mathematical functions that explain the distribution of life of an item.

Epstein[3] was first to discussed acceptance sampling based on truncated life tests for an exponential model. An extension of their work was carried out in Goode and Kao[4] by considering the Weibull model which includes the exponential distribution. Gupta also considered the gamma[5] and log-normal[6] distributions respectively.

Kakoty. S., Chakravarthy A.B.[9] determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable be first approximated as an exponential variable.

Vardeman.S, Di-ou Ray [14] was introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further the phenomena under study is the occurrence of rate of rare events and the inter arrival times for a homogenous poison process are identically independently distributed exponential random variables.

Lonnie. C. Vance[10] considers Average Run Length of cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are consider.

Sarma and Akhtar[1] studied Continuous acceptance sampling plans based on the truncated negative exponential distribution for Optimizing CASP-CUSUM schemes by solving the integral equation using Gauss-Chebyshev integration method with help of computer program. Finally, the obtained results were compared at different values of the parameters.

Narayana Murthy et.al[11] investigated CASP-CUSUM schemes based on the truncated Rayleigh distribution to determine ARL values for CASP-CUSUM schemes. The Optimum continuous acceptance sampling plans cumulative sums were obtained by evaluating integral equations using Lobatto integration method. Finally, obtained were compared with respect to different integration methods.

Sainath *et al.*[13] Considered Continuous acceptance sampling plan Cumulative sum to determine ARL values through truncated two parameters Burr distribution. To determine Optimum CASP-CUSUM values, a computer program is generated to solve the integral equations. By executing the computer program is generated to solve the integral equations. By executing the computer program, thus obtained ARL values for CASP-CUSUM schemes. Type-C OC curve values and ARL values are compared at different values of the parameters of the underlying probability distribution. They also determined an optimal CASP-CUSUM scheme at which the probability of acceptance is maximum.

Venkatesulu G. and Mohammed Akhtar P.[15] determined Truncated Lomax Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally critical comparisons his based on the obtained numerical results.

In the present paper, it is determined CASP-CUSUM Chart when the variable under study follows Truncated Generalized Exponential Distribution. Thus it is more worthwhile to study some interesting characteristics of this distribution.

GENERALIZED EXPONENTIAL DISTRIBUTION

The two- parameter generalized exponential as an extension of the exponential distribution has been introduced by Gupta and Kundu[7]. Moreover the two parameter generalized exponential is a particular case of the three-parameter Exponentiated Weibull distribution. At present generalized exponential distribution has received special attention in the probabilistic statistical literature and in various applications.

Definition: The non-negative random variable X is said to have a Generalized Exponential distribution if its P.D.F is given by

$$\mathbf{f(x; \alpha, \lambda) = \alpha\lambda\,(1 - e^{-\lambda x})^{\alpha-1}} \quad \text{Where } \alpha, \lambda \text{ and } x > 0 \tag{1.1}$$

The PDF and Hazard function of the generalized exponential distribution are very similar to those of Weibull and gamma distributions. Unlike exponential distribution, the generalized exponential distribution is capable of modeling various shapes of failure rate and hence various shapes of ageing criteria. Accordingly, the generalized exponential distribution is widely used in reliability and quality control.

The generalized exponential distribution has been used quite effectively to analyze lifetime data. In many cases it is observed that it provides a better than the Weibull, gamma, log-normal or generalized Rayleigh distributions.

TRUNCATED GENERALIZED EXPONENTIAL DISTRIBUTION

It is the ratio of probability density function of the Generalized Exponential distribution to their corresponding cumulative distribution function at the point B.

The random variable X is said to follow a Truncated Generalized Exponential Distribution as

$$f_B(x) = \frac{\alpha\lambda(1-e^{-\lambda x})^{\alpha-1}}{(1-e^{-\lambda B})^{\alpha}} \qquad \alpha > 0,\ \lambda > 0 \tag{1.2}$$

Where' **B'** is the upper truncated point of the Generalized Exponential Distribution.

2. DESCRIPTION OF THE PLAN AND TYPE- C OC CURVE

Beattie[2] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum (X_i - k_1) X_i 's (i = 1,2,3........)$ are distributed independently and k_1 is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

   When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

   The procedure, in brief, is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted $S_m = \sum (X_i - k) < h$; when $S_m < 0$, return cumulative to 0.
3. When $h < S_m < h + h'$ the product is rejected: when S_m crossed h, i.e., when $S_m > h + h'$ and continue rejecting product until $S_m > h + h'$ return cumulative to $h + h'$

The type-C, *OC* function, which is defined as the probability of acceptance of an item as a function of incoming quality, when the sampling rate is same in acceptance and rejection regions. Then the probability of acceptance *P* (*A*) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \tag{2.1}$$

Where *L* (0) = Average Run Length in acceptance zone and

L' (0) = Average Run Length in rejection zone.

Page E.S.[12] has introduced the formulae for *L* (0) and *L'* (0) as

$$L(0) = \frac{N(0)}{1 - P(0)} \tag{2.2}$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \tag{2.3}$$

Where *P* (0) = Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities

P (0), *N* (0), and *P*' (0), *N*' (0) as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z) dy \, , \tag{2.4}$$

$$N(z) = 1 + \int_0^h N(y) f(y + k_1 - z) dy \, , \tag{2.5}$$

$$P'(z) = \int_{k_1+z}^{B} f(y) dy + \int_0^h P'(y) f(-y + k_1 + z) dy \, . \tag{2.6}$$

$$N'(z) = 1 + \int_0^h N'(y) f(-y + k_1 + z) dy, \tag{2.7}$$

$$F(x) = 1 + \int_A^h f(x) dx :$$

$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y) dy$$

and *z* is the distance of the starting of the test in the normal chart from zero.

3. METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t)F(t)dt \tag{3.1}$$

Where

$$F(X) = P(z),$$
$$Q(X) = F(k - z),$$
$$R(X,t) = f(y + k - z)$$

Let the integral $I = \int_c^d f(x)dx$ be transformed to

$$I = \frac{d-c}{2}\int_c^d f(y)dy = \frac{d-c}{2}\sum a_i f(t_i) \tag{3.2}$$

$y = \dfrac{2x-(c-d)}{d-c}$ where a_i's and t_i's respectively the weight factor and abscissa for the Gauss-Chebyshev polynomial, given in Jain M. K. and *et al.*[8] using (3.1) and (3.2), (2.4) can be written as

$$F(X) = Q(X)\frac{d-c}{2}\sum a_i R(x,t_i)F(t_i) \tag{3.3}$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for $x = t_i$, i = 0 (1) n then obtain.

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i R(t_j,t_i)F(t_i) \quad j = 0(1)n \tag{3.4}$$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n, in$ (3.4), we get

$$F_0 = Q_0 + \frac{d-c}{2}[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \dots\dots\dots\dots a_n R(t_0,t_n)F_n)]$$

$$F_1 = Q_1 + \frac{d-c}{2}[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \dots\dots\dots\dots a_n R(t_1,t_n)F_n)]$$

………… ………… ……….. ………

………… …………. ……….. ……….

$$F_n = Q_n + \frac{d-c}{2}[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots\ldots\ldots a_n R(t_n,t_n)F_n)] \quad (3.5)$$

In the system of equations except for F_i, $i = 0,1,2\ldots\ldots\ldots\ldots n$ are known and hence can be solved for F_i, we solved the system of equations by the method of Iteration. For this, we write the system (3.5) as

$$[1 - Ta_0 R(t_0,t_0)]F_0 = Q_0 + T[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \ldots\ldots\ldots a_n R(t_0,t_n)F_n)]$$

$$[1 - Ta_1 R(t_1,t_1)]F_1 = Q_1 + T[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \ldots\ldots\ldots a_n R(t_1,t_n)F_n)]$$

……………. ……………. ………….. …………

……………. …………….. ..………… …………

$$[1 - Ta_n R(t_n,t_n)]F_n = Q_n + T[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots\ldots\ldots a_n R(t_n,t_n)F_n)] \quad (3.6)$$

Where $T = \frac{d-c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = \ldots = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = \ldots = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i $i = 0, 1, 2,\ldots,n$ which are just the refined values of F_i $i = 0, 1, 2,\ldots, n$. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P' (0), N (0), N' (0) can be obtained.

4. COMPUTATION OF ARL'S AND P (A)

We developed computer programs to solve the equations (2.4), (2.5), (2.6) and (2.7) and we got the following results given in the Tables (4.1) to (4.34).

Table 4.1
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2$, $\lambda = 0.2$, $k = 1$, $h = 0.01$, $h' = 0.01$

B	L(0)	L'(0)	P(A)
1.4	2.2152	1.0116723	0.6864857674
1.3	2.6642	1.0132182	0.7244721651
1.2	3.5693	1.0151222	0.7785694003
1.1	6.3012	1.0174992	0.8609720469
1.0	126878.0781	1.0205082	0.9999919534

Table 4.2
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2$, $\lambda = 0.2$, $k = 1$, $h = 0.02$, $h' = 0.02$

B	L(0)	L'(0)	P(A)
1.4	2.2037	1.0227205	0.6830135584
1.3	2.6485	1.0255955	0.7208551764
1.2	3.5452	1.0290754	0.7750288248
1.1	6.2516	1.0333109	0.8581576943
1.0	57576.9609	1.0384643	0.9999819398

Table 4.3
Values of ARL's AND TYPE-C OC CURVES when
α =2, λ = 0.2, k=1, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
1.4	2.1922	1.0331262	0.6796841621
1.3	2.6329	1.0371051	0.7174111009
1.2	3.5213	1.0418189	0.7716876864
1.1	6.2025	1.0473708	0.8555325270
1.0	30101.6855	1.0537667	0.9999650121

Table 4.4
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ =0.2, k=1, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
1.4	2.1808	1.0428734	0.6764991283
1.3	2.6175	1.0477244	0.7141425610
1.2	3.4977	1.0533198	0.7685507536
1.1	6.1538	1.0596325	0.8531031609
1.0	18001.4336	1.0663534	0.9999407530

Table 4.5
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.2, k=1, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
1.4	2.1695	1.0519488	0.6734596491
1.3	2.6022	1.0574349	0.7110517025
1.2	3.4742	1.0635546	0.7656211853
1.1	6.1056	1.0700680	0.8508744836
1.0	11838.4951	1.0762028	0.9999091029

Table 4.6
Values of ARL's AND TYPE-C OC CURVES when
α = 3 λ = 0.2, k=1, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
1.4	1.6954	1.0187808	0.6246507168
1.3	1.9904	1.0231498	0.6604819894
1.2	2.5928	1.0292374	0.7158436179
1.1	4.4296	1.0381075	0.8101400733
1.0	145350.0156	1.0518516	0.9999927878

Table 4.7
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.4, k=1, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
1.4	2.3978	1.0098860	0.7036448717
1.3	2.8864	1.0109615	0.7406064868
1.2	3.8702	1.0122619	0.7926736474
1.1	6.8363	1.0138466	0.8708500266
1.0	49815.2031	1.0157878	0.9999796152

Table 4.8
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ =0.4, k=1, h=0.02, h'=0.02

B	L(0)	L'(0)	P(A)
1.4	2.3512	1.0186875	0.6977104545
1.3	2.8246	1.0205176	0.7345915437
1.2	3.7773	1.0226462	0.7869468927
1.1	6.6492	1.0250932	0.8664250970
1.0	14741.5967	1.0278141	0.9999302626

Table 4.9
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.4, k=1, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
1.4	2.3060	1.0263815	0.6919999123
1.3	2.7647	1.0286372	0.7288306355
1.2	3.6877	1.0311114	0.7814911008
1.1	6.4693	1.0336843	0.8622294068
1.0	6727.1528	1.0360098	0.9998460412

Table 4.10
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ =0.4, k=1, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
1.4	2.2622	1.0329524	0.6865195632
1.3	2.7067	1.0353019	0.7233330607
1.2	3.6013	1.0376376	0.7763200402
1.1	6.2960	1.0396053	0.8582804799
1.0	3792.9751	1.0403894	0.9997257590

Table 4.11
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.4, k=1, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
1.4	2.2196	1.0383917	0.6812741756
1.3	2.6506	1.0405051	0.7181059122
1.2	3.5178	1.0422260	0.7714440227
1.1	6.1292	1.0428816	0.8545913696
1.0	2416.1895	1.0410476	0.9995692968

Table 4.12
Values of ARL's AND TYPE-C OC CURVES when
α = 3 λ =0.4, k=1, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
1.4	1.8728	1.0172942	0.6480036378
1.3	2.2154	1.0211449	0.6845000386
1.2	2.9133	1.0266367	0.7394278646
1.1	5.0363	1.0349103	0.8295373321
1.0	417242.5625	1.0483342	0.9999974966

Table 4.13
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ = 0.4, k=1, h=0.02, h'=0.02

B	L(0)	L'(0)	P(A)
1.5	1.7044	1.0326437	0.6227121353
1.4	1.9139	1.0398153	0.6479679346
1.3	2.2712	1.0500245	0.6838474274
1.2	2.9999	1.0653430	0.7379394770
1.1	5.2193	1.0899960	0.8272408843

Table 4.14
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ =0.4, k=1, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
1.5	1.7385	1.0547732	0.6223871112
1.4	1.9568	1.0680640	0.6469023824
1.3	2.3296	1.0876112	0.6817292571
1.2	3.0914	1.1182121	0.7343643904
1.1	5.4150	1.1704257	0.8222714067

Table 4.15
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ = 0.4, k=1, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
1.5	1.7739	1.0812402	0.6212976575
1.4	2.0014	1.1027361	0.6447528005
1.3	2.3909	1.1353328	0.6780281067
1.2	3.1881	1.1885582	0.7284295559
1.1	5.6247	1.2853571	0.8139878511

Table 4.16
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ =0.4, k=1, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
1.5	1.8106	1.1125287	0.6194052100
1.4	2.0479	1.1447746	0.6414398551
1.3	2.4551	1.1952360	0.6725664735
1.2	3.2905	1.2815468	0.7196968198
1.1	5.8499	1.4511949	0.8012364507

Table 4.17
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.2, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.4	3.9178	1.0234790	0.7928702235
2.3	4.9224	1.0250040	0.8276538849
2.2	6.9356	1.0267113	0.8710544109
2.1	12.9843	1.0286312	0.9265940189
2.0	203966.7188	1.0307987	0.9999949336

Table 4.18
Values of ARL's AND TYPE-C OC CURVES when
α = 3 λ =0.2, k=2, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
2.4	2.8339	1.0047146	0.7382586598
2.3	3.5101	1.0052185	0.7773740292
2.2	4.8697	1.0058101	0.8288118243
2.1	8.9641	1.0065109	0.8990519047
2.0	284679.5313	1.0073487	0.9999964833

Table 4.19
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ = 0.2, k=2, h=0.02, h'=0.02

B	L(0)	L'(0)	P(A)
2.4	2.8366	1.0095586	0.7375137806
2.3	3.5135	1.0105963	0.7766200304
2.2	4.8746	1.0118190	0.8281102180
2.1	8.9736	1.0132726	0.8985397220
2.0	289913.8750	1.0150189	0.9999964833

Table 4.20
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ =0.2, k=2, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
2.4	2.8393	1.0145341	0.7367470264
2.3	3.5170	1.0161362	0.7758415937
2.2	4.8796	1.0180300	0.8273831010
2.1	8.9832	1.0202907	0.8980064988
2.0	290240.5938	1.0230192	0.9999964833

Table 4.21
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ = 0.2, k=2, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
2.4	2.8420	1.0196431	0.7359581590
2.3	3.5204	1.0218410	0.7750383615
2.2	4.8846	1.0244476	0.8266300559
2.1	8.9928	1.0275710	0.8974516392
2.0	290567.9688	1.0313579	0.9999964237

Table 4.22
Values of ARL's AND TYPE-C OC CURVES when
α = 3, λ =0.2, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.4	2.8448	1.0248878	0.7351470590
2.3	3.5239	1.0277138	0.7742101550
2.2	4.8896	1.0310758	0.8258508444
2.1	9.0024	1.0351194	0.8968749046
2.0	290896.0000	1.0400441	0.9999964237

Table 4.23
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=3, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
3.4	6.0962	1.0029200	0.8587252498
3.3	7.8021	1.0030444	0.8860846758
3.2	11.2172	1.0031796	0.9179094434
3.1	21.4690	1.0033269	0.9553528428
3.0	1046444.3750	1.0034879	0.9999990463

Table 4.24
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=3, h=0.02, h'=0.02

B	L(0)	L'(0)	P(A)
3.4	6.0847	1.0058109	0.8581465483
3.3	7.7872	1.0060568	0.8855878115
3.2	11.1954	1.0063241	0.9175259471
3.1	21.4264	1.0066153	0.9551278949
3.0	982888.6875	1.0069335	0.9999989867

Table 4.25
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=3, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
3.4	6.0732	1.0086721	0.8575706482
3.3	7.7723	1.0090369	0.8850933909
3.2	11.1736	1.0094332	0.9171443582
3.1	21.3838	1.0098647	0.9549040198
3.0	877642.6250	1.0103358	0.9999988675

Table 4.26
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=3, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
3.4	6.0618	1.0115035	0.8569976091
3.3	7.7575	1.0119846	0.8846014142
3.2	11.1519	1.0125067	0.9167646766
3.1	21.3414	1.0130749	0.9546812177
3.0	756428.0000	1.0136946	0.9999986887

Table 4.27
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=3, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
3.4	6.0504	1.0143050	0.8564273715
3.3	7.7427	1.0148994	0.8841120601
3.2	11.1302	1.0155442	0.9163869023
3.1	21.2990	1.0162452	0.9544596076
3.0	664308.9375	1.0170096	0.9999984503

Table 4.28
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 3\ \lambda = 0.2$, k=3, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
3.4	4.2547	1.0021584	0.8093600273
3.3	5.3924	1.0022994	0.8432608247
3.2	7.6728	1.0024562	0.8844461441
3.1	14.5242	1.0026313	0.9354259968
3.0	730102.0000	1.0028278	0.9999986291

Table 4.29
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=4, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
4.4	8.7126	1.0020787	0.8968486190
4.3	11.2584	1.0021393	0.9182632565
4.2	16.3529	1.0022039	0.9422531724
4.1	31.6419	1.0022728	0.9692970514
4.0	***3349506.5000***	***1.0023466***	***0.9999997020***

Table 4.30
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=4, h=0.02, h'=0.02

B	L(0)	L'(0)	P(A)
4.4	8.6975	1.0041430	0.8964973688
4.3	11.2389	1.0042633	0.9179732800
4.2	16.3243	1.0043914	0.9420389533
4.1	31.5863	1.0045284	0.9691776037
4.0	2786321.2500	1.0046747	0.9999996424

Table 4.31
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=4, h=0.03, h'=0.03

B	L(0)	L'(0)	P(A)
4.4	8.6824	1.0061927	0.8961468935
4.3	11.2193	1.0063719	0.9176838994
4.2	16.2958	1.0065627	0.9418252110
4.1	31.5308	1.0067663	0.9690583944
4.0	2384057.2500	1.0069839	0.9999995828

Table 4.32
Values of ARL's AND TYPE-C OC CURVES when
$\alpha = 2, \lambda = 0.2$, k=4, h=0.04, h'=0.04

B	L(0)	L'(0)	P(A)
4.4	8.6674	1.0082279	0.8957972527
4.3	11.1998	1.0084649	0.9173952341
4.2	16.2674	1.0087174	0.9416120052
4.1	31.4754	1.0089868	0.9689393640
4.0	2082369.1250	1.0092745	0.9999995232

Table 4.33
Values of ARL's AND TYPE-C OC CURVES when
α = 2, λ = 0.2, k=4, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
4.4	8.6524	1.0102484	0.8954483867
4.3	11.1804	1.0105425	0.9171071649
4.2	16.2390	1.0108556	0.9413992763
4.1	31.4202	1.0111896	0.9688206911
4.0	1662958.2500	1.0115461	0.9999994040

Table 4.34
Values of ARL's AND TYPE-C OC CURVES when
α = 3 λ =0.2, k=4, h=0.01, h'=0.01

B	L(0)	L'(0)	P(A)
4.4	5.9990	1.0012902	0.8569635153
4.3	7.6987	1.0013474	0.8849036098
4.2	11.1020	1.0014094	0.9172624946
4.1	21.3198	1.0014766	0.9551334977
4.0	2099034.7500	1.0015496	0.9999995232

5. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters α, λ, *k*, *h* and *h'* are given at the top of each table, we determine optimum truncated point B at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 4.1 to 4.34 we made the following conclusions:

1. From the Table 4.1 to 4.34, it is observed that the values of P (A) are increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
2. From the Table 4.1 to 4.34, we observe that it can be maximized the truncated point B by increasing value of ***k***.
3. From the Table 4.1 to 4.34, it is observed that at the maximum level of probability of acceptance P (A) the truncated point '**B'** from 4.0 to 1.0 as the value of ***h*** changes from 0.01 to 0.05.
4. From the Table 4.1 to 4.34, it was observed that the truncated point '**B'** changes from 4.0 to 1 .0 and P (A) are as maximum **i.e. 0.9999997020.** Thus truncated point B and k are inversely related.
5. From Table 4.1 to 4.34 it is observed that the optimal truncated point changes from 4.4 to 4.0 as.
6. It is observed that the Table 5.1 values of Maximum Probabilities increased as the increased values of '**k**' at constant values of the parameter as shown below the Figure 5.1.

Table 5.1
α = 0.2, λ = 4, *h* = 0.01, *h'* = 0.01

k	*P(A)*
1	0.999991953
2	0.999997556
3	0.999999046
4	0.999999702

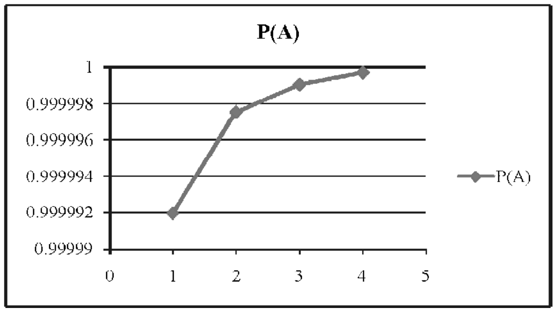

Figure 5.1

8 It is observed that the Table-5.2 values of Maximum Probabilities decreased as the increased values of h and h' at constant values of the parameter as shown below the Figure 5.2.

Table 5.2
α = 2, λ = 0.2, *k* = 4, *B* = 4.0

h and h'	*P(A)*
0.01	0.9999997020
0.02	0.9999996424
0.03	0.9999995828
0.04	0.9999995232
0.05	0.9999994040

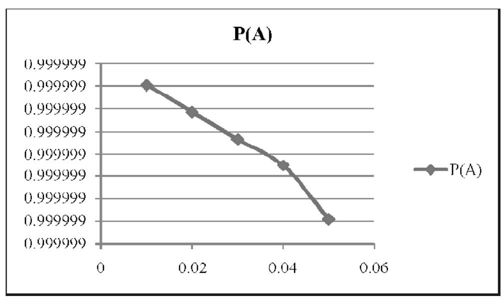

Figure 5.2

9. The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above table 4.1 to 4.34 are observed from the following Table.

Table 5.3
Consolidated Table from the Tables (4.1) to (4.34)

B	α	λ	*k*	*h*	*h'*	*L(0)*	*L'(0)*	*P(A)*
1.0	2	0.2	1	0.01	0.01	126878.0781	1.0205082	0.9999919534
2.0	2	0.2	2	0.02	0.02	362481.3438	1.0128206	0.9999971986
1.1	3	0.4	1	0.04	0.04	5.6247	1.2853571	0.8139878511
3.0	2	0.2	3	0.01	0.01	1046444.3750	1.0034879	0.9999990463
2.0	3	0.2	2	0.02	0.02	289913.8750	1.0150189	0.9999964833
1.0	2	0.2	1	0.04	0.04	18001.4336	1.0663534	0.9999407530
1.1	3	0.4	1	0.02	0.02	5.2193	1.0899960	0.8272408843

contd. table 5.3

B	α	λ	k	h	h'	$L(0)$	$L'(0)$	$P(A)$
4.0	3	0.2	4	0.04	0.04	2104700.5	1.0062803	0.9999995232
3.0	2	0.2	3	0.03	0.03	877642.6250	1.0103358	0.9999988675
2.0	2	0.2	2	0.03	0.03	307832.6250	1.0189831	0.9999966621
1.0	2	0.2	1	0.02	0.02	57576.9609	1.0384643	0.9999819398
1.0	3	0.4	1	0.01	0.01	417242.5625	1.0483342	0.9999974966
4.0	2	0.2	4	0.01	0.01	3349506.5	1.0023466	0.9999997020
3.0	3	0.2	3	0.04	0.04	732081.75	1.0115873	0.9999986291
1.0	3	0.2	1	0.03	0.03	187138.375	1.2198263	0.9999935031
2.0	3	0.2	2	0.05	0.05	290896.0	1.0400441	0.9999964237
1.1	3	0.4	1	0.03	0.03	5.4150	1.1704257	0.8222714067
4.0	2	0.2	4	0.04	0.04	2082639.1250	1.0092745	0.9999995232

By observing the Table-5.3, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL and P (A) reach their maximum i.e., ***3349506.5, 0.9999997020*** respectively, is

$$\begin{bmatrix} B = 4.0 \\ \alpha = 2 \\ \lambda = 0.2 \\ k = 4 \\ h = 0.01 \\ h' = 0.01 \end{bmatrix}$$

REFERENCES

[1] Akhtar, P. Md. and Sarma, K.L.A.P. (1993). "Optimization of Parameters in Continuous Acceptance Sampling Plan based on Truncated Negative Exponential Distribution", *Proceedings of the II International Symposium on Optimization and Statistics*, Dept.of Statistics and Operation Aligarh Muslim University, Aligarh, Held during Nov.1993, pp.105-111.

[2] Beattie, B.W. (1962). "A Continuous Acceptance Sampling procedure based upon a Cumulative Sums Chart for a number of defective". *Applied Statistics*, Vol. 11(No.2), pp. 137-147.

[3] Epstein, B. (1954), "Truncated life test in the exponential case", Annals of Mathematical Statistics, vol. 25, pp. 555 - 564.

[4] Goode, H.P. and Kao, J.H.K. (1961), "Sampling plans based on the Weibull distribution", Proceedings of the Seventh National Symposium on Reliability and Quality Control, Philadelphia, pp. 24 - 40.

[5] Gupta, S.S. (1962), "Life test plans for normal and log-normal distributions", Technometrics, vol. 4, pp. 151 - 160.

[6] Gupta, S. S. and Groll, P.A. (1961), "Gamma distribution in acceptance sampling based on life tests", *Journal of the American Statistical Association*, vol. 56, pp. 942 - 970.

[7] Gupta, R. D., and Kundu, D. (1999), "Generalized Exponential Distributions", *Australian and New Zealand Journal of Statistcs*, Vol. 41, pp. 173-188.

[8] Jain, M.K., Iyengar, S.R.K. and Jain, R.K. "Numerical Methods of Scientific and Engineering Computations", *Willy Eastern Ltd*., New Delhi.

[9] Kakoty, S and Chakravaborthy, A.B., (1990), "A Continuous Acceptance Sampling Plan for Truncated Normal distribution based on Cumulative Sums", *Journal of National Institution for Quality and Reliability*, Vol. 2 (No. 1), pp. 15-18.

[10] Lonnie, C. Vance. (1986). "Average Run Length of CUSUM Charts for Controlling Normal means". *Journal of Quality Technology*, Vol.18, pp.189-193.

[11] Narayana Muthy, B. R, Akhtar, P. Md and Venkataramudu, B. (2013) "Optimization of CASP-CUSUM Schemes based on Truncated Rayleigh Distribution". *International Journal of Engineering and Development*, Volum 6, Issue 2, pp. 37-44.

[12] Page, E.S., (1954), "Continuous Inspection Schemes", *Biometrika*, Vol. XLI, pp. 104-114.

[13] Sainath.B, P.Mohammed Akhtar, G.Venkatesulu, and Narayana Muthy, B. R, (2016), "CASP CUSUM Schemes based on Truncated Burr Distribution using Lobatto Integration Method". *IOSR Journal of Mathematics (IOSR-JM)*, Vol. 12, Issue 2, pp. 54-63.

[14] Vardeman, S. And Di-ou Ray. (1985), "Average Run Length for CUSUM schemes Where observations are Exponentially Distributed", *Technometrics*, vol. 27 (No. 2), pp. 145-150.

[15] Venkatesulu.G, P.Mohammed Akhtar, B.Sainath and Narayana Murthy, B.R. (2018), "Continuous Acceptance Sampling Plans for Truncated Lomax Distribution Based on CUSUM Schemes". *International Journal Mathematics Trends and Technology (IJMTT)* – Volume 55 Number 3 - March 2018.

G. Venkatesulu
Research scholar, Dept. of Statistics
Sri Krishanadevaraya University, Ananthapuramu-515003
E-mail: venkyvictory225@gmail.com