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Characterization of Gamma Hemirings Interms of Intuitionistic Fuzzy *h*-Ideals Using Level Sets

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Abstract : In this paper, the notions of intuitionistic fuzzy *h*-ideal in Γ- hemiring are studied and some of the basic properties of this ideal are investigated. The level set of intuitionistic fuzzy *h*-ideal in Γ- hemiring is defined and described some of the characterizations. Finally a Cartesian product of intuitionistic fuzzy *h*-ideal in Γ- hemiring is introduced and its properties are discussed.

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1. INTRODUCTION

Ideals of hemirings have a significant role to play in the structure theory and they are instrumental in fulfilling scores of purposes. But the specific issue is that, they do not, in general, coincide with the usual ring ideals. Many results in rings apparently have no analogues in hemi rings using only ideals. Informal applications, hemirings have their utilitarian importance in automation and formal languages. It is universally only known that the set of regular languages does form the "*star, semirings*". The introduction of fuzzy sets by L.A.Zadeh[15] triggered an academic revolution and the fuzzy set theory hasbecome, over the years, the heart and soul of several applications in the royal domains of mathematics and other relevant fields. The idea of "Intuitionistic Fuzzy Set" was first published by K.T.Atanassov[1] as a generalization of the notion of fuzzy set. Jun and Lee [8] went a little further and applied the concept of fuzzy sets to the theory of Γ-rings. The notion of Γ-semiring was introduced by Rao[11]which, in course of time, gained momentum and included ternary semirings to provide algebraic home to non-positive cones of totally ordered rings. Henriksen[5] , Lizuka[6] and La Torre[9] dwelled deep in the study of *h*-ideals and *k*-ideals in hemirings to amend the gap between ring ideals and semiring ideals. These concepts have been extended to Γ-semiring by Rao[11], Dutta and Sardar[2].Jun et al [7] to study the ideals in hemirings in terms of fuzzy subsets. A characterization of an h-hemiregular hemiring in terms of a fuzzy h-ideal had been discussed in detail by Zhan et al [16]. Some salient properties of fuzzy *h*-ideals in Γ-hemirings had been studied by Sujit Kumar et al [13].The notion of intuitionistic fuzzy *h*-ideals in Γ-hemirings had been discussed Ezhilmaran et al [4] in the light of the previous findings**.** In this Paper some different characteristic properties of intuitionistic fuzzy *h*- ideals in Γ-hemiring by using level sets have been discussed and debated verbally.

2. PRELIMINARIES

Definition 2.1.

A hemiring (respectively semiring) is a nonemptyset S on which operations addition and multiplication have been defined such that (S, +) is a commutative monoid with identity 0, (S,.) is a semigroup (respectively monoid with identity 1_S) Multiplication distributes over addition from either side, $1_S \neq 0$ and $0_s = 0 = S_0$ for all $s \in$ S.

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Definition 2.2.

Let S and Γ be two additive commutative semigroupswithzero. Then S is called a Γ-hemiring if there exists a mapping $S \times \Gamma \times S \rightarrow S$ $((a, a, b)) \rightarrow aab)$ satisfying the following conditions :

(*i*) $(a + b)ac = aac + bac,$

(*ii*) $a\alpha(b + c) = aab + a\alpha c,$

(*iii*) $a(\alpha + \beta)b = a\alpha b + a\beta b,$

(*iv*) $a\alpha(b\beta c) = (a\alpha b)\beta c,$

(*v*) $0_s \alpha a = 0_s - a\alpha 0_s,$

(*vi*) $a_{0\Gamma} b = 0_\Gamma - b0_\Gamma a,$

for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. For simplification we write 0 instead of 0_S and 0_Γ.

Example 2.3.

Let S be the set of all $m \times n$ matrices over Z^-_0 (the set of allnon-positive integers) and Γ be the set of all $n \times m$ matrices over Z^-_0 then S forms a Γ-hemiring with usual addition and multiplication of matrices.

Definition 2.4.

A left ideal A of a Γ-hemi ring S is called a left *h*-ideal if for any $x, z \in S$ and $a, b \in A, x + a + z = b + z \Rightarrow x \in A$. A right *h*-ideal is defined analogously.

Definition 2.5.

Let μ be a non-empty fuzzy subset of a Γ-hemiring S (*i.e.*$\mu(x) \neq 0$ for some $x \in S$). Then μ is called a fuzzy left ideal (fuzzy right ideal) of S if

1. $\mu(x + y) \geq \min [m(x), m(y)]$ and
2. $\mu(x \gamma z) \geq (y)$ (respectively $\mu(x\gamma y) \geq \mu(x)$) for all $x, y \in S, \gamma \in \Gamma$.

A fuzzy ideal of a Γ-hemiring S is a non-empty fuzzy subset of S which is a fuzzy left ideal as well as fuzzy right ideal of S.Note that if μ is a fuzzy left or right ideal of a Γ-hemiring S, then $\mu(0) \geq \mu(x)$ for all $x \in S$.

Example 2.6.

Let S be the additive commutative semi group of all non positive integers and Γ be the additive commutative semigroup of all non-positive even integers. Then S is a Γ-hemiring if *a* and *b* denotes the usual multiplication of integers avb where a, $b \in S$ and $v \in \Gamma$. Let μ be a fuzzy subset of S, defined as follows

The fuzzy subset μ of S is a fuzzy ideal of S.

$$\mu(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0.7 & \text{if } x \text{ is even} \\ 0.1 & \text{if } x \text{ is odd} \end{cases}$$

The fuzzy subset μ of S is a fuzzy ideal of S.

Definition 2.7.

A fuzzy left ideal $A = <\mu_A, v_A>$ of a Γ-hemiring S is called a fuzzy left *h*-ideal if for all $x, a, b, z \in S, x + a + z = b + z$ implies $\mu_A(x) \geq \min\{\mu_A(a), \mu_A(b)\}$ and $v_A(x) \leq \max\{v_A(a), v_A(b)\}$

Definition 2.8.

Let μ be any fuzzy subset of X and let $t \in [0, 1]$. The set $\mu_t = [x \in X : \mu(x) \geq t]$ is called level subset μ ,clearly $\mu_t \subseteq \mu_s$ whenever $t > s$.

Definition 2.9 [16].

A fuzzy subset of a nonempty set X is defined as a function $\mu : X \to [0,1]$.

Definition 2.10 [1].

Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Notation

For the sake of simplicity, we shall use the symbol $A = <\mu_A, \nu_A>$ for the IFS $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$.

3. CHARACTERIZATION OF INTUITIONISTIC FUZZY H-IDEALS IN GAMMA HEMIRINGS

Definition 3.1.

Let A be an intuitionistic Fuzzy subset in Γ-hemiring S. For the intuitionistic Fuzzy pair $(t, s) \in [0, 1]$, the subset $A_{<t, s>} = \{x \in X \mid \mu_A(x) \ge t, \nu_A(x) \le s\}$ is called the level set of A.

Theorem 3.2.

A is an ideal of a Γ –hemiring S if and only if fuzzy $\overline{A} = <\mu_{\overline{A}}, \nu_{\overline{A}}>$ where

$$\mu_{\overline{A}}(x) = \begin{cases} 1 & x \in A, \\ 0 & \text{otherwise}, \end{cases}$$

$$\nu_{\overline{A}}(x) = \begin{cases} 0 & x \in A, \\ 1 & \text{otherwise}, \end{cases}$$

is an intuitionistic fuzzy left (resp. right) h- ideal of S.

Proof. (⇒): Let A be a left (resp. right) ideal of S.

Let $x, y \in S$ and $\alpha \in \Gamma$. $x, y \in A$ then $x + y \in A$ and $x\alpha y \in A$. Therefore.

$\mu_{\overline{A}}(x + y) = 1 \ge \{\mu_{\overline{A}}(x) \wedge \mu_{\overline{A}}(y)\}$ and $\mu_{\overline{A}}(x\alpha y) = 1 = \mu_{\overline{A}}(y)$(resp. $\mu_{\overline{A}}(x\alpha y) = \mu_{\overline{A}}(x) = 1$).

$\nu_{\overline{A}}(x + y) = 1 \le \{\nu_{\overline{A}}(x) \vee \mu_{\overline{A}}(y)\}$ and $\nu_{\overline{A}}(x\alpha y) = 0 = \nu_{\overline{A}}(y)$(resp. $\nu_{\overline{A}}(x\alpha y) = \nu_{\overline{A}}(x) = 0$).

If $x \notin A$ or $y \notin A$ then $\mu_{\overline{A}}(x) = 0$ or $\mu_{\overline{A}}(y) = 0$ $\nu_{\overline{A}}(x) = 1$ or $\nu_{\overline{A}}(y) = 1$

Thus we have

$\mu_{\overline{A}}(x + y) \ge \{\mu_{\overline{A}}(x) \wedge \mu_{\overline{A}}(y)\}$ and $\mu_{\overline{A}}(x\alpha y) \ge \mu_{\overline{A}}(y)$(resp. $\mu_{\overline{A}}(x\alpha y) \ge \mu_{\overline{A}}(x)$)

$\nu_{\overline{A}}(x + y) \le \{\nu_{\overline{A}}(x) \vee \nu_{\overline{A}}(y)\}$ and $\nu_{\overline{A}}(x\alpha y) \le \nu_{\overline{A}}(y)$(resp. $\nu_{\overline{A}}(x\alpha y) \le \nu_{\overline{A}}(x)$),

If $x + a + z = b + z$ for $x, a, b, z \in S$. Then

$$\mu_{\overline{A}}(x) = 1 \ge \{\mu_A(a) \wedge \mu_A(b)\},$$

$$\nu_{\overline{A}}(x) = 0 \le \{\nu_A(a) \vee \nu_A(b)\}$$

Hence $\overline{A}$ is an intuitionistic fuzzy left (resp. right) h- ideal of M.

(⇐): Let $\overline{A}$ is an intuitionistic fuzzy left (resp. right) ideal of M.

Let $x, y \in S$ and $\alpha \in \Gamma$. If $x, y \in A$ then

$$\mu_{\overline{A}}(x + y) \ge \{\mu_{\overline{A}}(x) \wedge \mu_{\overline{A}}(y)\} = 1$$

and $$\nu_{\overline{A}}(x + y) \le \{\nu_{\overline{A}}(x) \wedge \nu_{\overline{A}}(y)\} = 0$$

So, $x + y \in$ A.

Also $\mu_{\overline{A}}(x\alpha y) \geq \mu_{\overline{A}}(y) = 1$ (resp. $\mu_{\overline{A}}(x\alpha y) \geq \mu_{\overline{A}}(x) = 1$),

$v_{\overline{A}}(x\alpha y) \leq v_{\overline{A}}(y) = 0$ (resp. $v_{\overline{A}}(x\alpha y) \leq \mu_{\overline{A}}(x) = 0$)

So, $x\alpha y \in$ A

Hence $\overline{A}$ is a left (resp. right) ideal of S.

If $x + a + z = b + z$ for $x, a, b, z \in$ S. Then

$$\mu_{\overline{A}} \geq \{\mu_A(a) \wedge \mu_A(b)\} \geq 1 \text{ and}$$

$$v_{\overline{A}} \leq \{v_A(a) \vee v_A(b)\} \leq 0$$

Hence $\overline{A}$ is a left (resp. right) h- ideal of S.

Theorem 3.3.

Let A be an intuitionistic fuzzy left (resp. right) ideal of S and t $\in [0, 1]$ then

1. U(μ_A ; t) is either empty or an h-ideal of S.
2. L(v_A ; t) is either empty or an h- ideal of S.

Proof. Let $x, y \in$ U (μ_A ; t).

Then $$\mu_A(x + y) \geq \{\mu_A(x) \wedge \mu_A(y)\} \geq t.$$

Hence $x + y \in$ U (μ_A ; t).

Let $x \in$ S, $\alpha \in \Gamma$ and $y \in$ U(μ_A ; t).

Then $\mu_A(x\alpha y) \geq \mu_A(y) \geq t$ and so $x \in y$ (μ_A ; t).

Hence U(μ_A ; t) is an ideal of S.

If $x + a + z = b + z$ for $x, a, b, z \in$ U(μ_A ; t) then

$$\mu_A(x) \geq t\{\mu_A(a) \wedge \mu_A(b)\} \geq t \Rightarrow x + a + z = b + z \in \mathrm{U}(\mu_A ; t)$$

Hence U(μ_A ; t) is an h- ideal of S.

Let $x, y \in$ L(v_A ; t).

Then $$\mathrm{L}(v_A ; t) \leq \{v_A(x) \vee v_A(y)\} \leq t.$$

Hence $x + y \in$ L(v_A ; t).

Let $x \in$ S, $\alpha \in \Gamma$ and $y \in$ L(v_A ; t)

Then $v_A\{x\alpha y\} \leq v_A(y) \leq t$ and so $x\alpha y \in$ L(v_A ; t).

Hence L(v_A ; t) is an ideal of S.

If $x + a + z = b + z$ for $x, a, b, z \in$ L(v_A ; t) then

$$\mu_{\overline{A}}(x) \geq \{v_A(a) \wedge v_A(b)\} \leq t \Rightarrow x + a + z = b + z \in \mathrm{L}(v_A ; t)$$

Hence L(v_A ; t) is an h-ideal of S.

Theorem 3.4.

Let I be the left (resp. right) ideal of S. If the intuitionistic fuzzy set A = <μ_A, v_A> in S defined by

$$\mu_A(x) = \begin{cases} p & x \in \mathrm{I}, \\ s & \text{otherwise}, \end{cases}$$

$$v_A(x) = \begin{cases} u & x \in \mathrm{I}, \\ v & \text{otherwise}, \end{cases}$$

for all $x \in S$ and $\alpha \in \Gamma$, where $0 \le s < p$, $0 \le v < u$ and $p + u \le 1$, $s + v \le 1$, then A is an intuitionistic fuzzy let (resp. right) h-ideal of S and $U(\mu_A ; p) = 1 = L(v_A ; u)$.

Proof. Let $x, y \in S$ and $\alpha \in \Gamma$.

If at least one of x and y does not belong to I, then

$\mu_A(x+y) \ge s = \{\mu_A(x) \wedge \mu_A(y)\}$ and $v_A(x+y) \le v = \{v_A(x) \vee v_A(y)\}$.

If $y \in I$, $x \in S$ and $\alpha \in \Gamma$ then

$$\mu_A(x\alpha y) = p = \mu_A(y),\ v_A(x\alpha y) = v = v_A(y)$$

$$(\text{resp. } \mu_A(x\alpha y) = s = \mu_A(x),\ v_A(x\alpha y) = v = v_A(x)$$

If $y \notin I$ then

$$\mu_A(x\alpha y) = s = \mu_A(y),\ \upsilon_A(x\alpha y) = v = v_A(y)$$

$$(\text{resp. } \mu_A(x\alpha y) = s = \mu_A(x),\ \upsilon_A(x\alpha y) = v = v_A(x).$$

Therefore A is an intuitionistic fuzzy left (resp. right) ideal of S.

Let $x, a, b, z \in S$ be such that $x + a + z = b + z$ if , $b \in A$, we obtain

$$\mu_A(a) = \mu_A(b) = p,\ v_A(a) = v_A(b) = u \text{ and hence}$$

$$\mu_A(x) \ge \{\mu_A(a) \wedge \mu_A(b)\},\ v_A(x) \le \{v_A(a) \vee v_A(b)\}.$$

Therefore A is an intuitionistic fuzzy left (resp. right) h- ideal of S.

If either a or $b \notin A$.

Then $\{\mu_A(a) \wedge \mu_A(b)\} \le p \le \mu_A(x), \{v_A(a) \vee v_A(b)\} \ge u \le v_A(x)$.

Hence $A = <\mu_A, v_A>$ is an intuitionistic fuzzy left h-ideal of S.

Theorem 3.5.

An intuitionistic fuzzy set $A = <m_A, v_A>$ in a Γ-hemiring S is an intuitionistic fuzzy left (resp. right) h- ideal if and only if $A_{(t,s)} = \{x \in S | \mu_A(x) \ge t, v_A(x) \le s\}$ is a left (resp. right) h- ideal of S for $\mu_A(0)\ t, v_A(0) \le s$.

Proof : ($\Rightarrow$) Suppose that $A = <\mu_A, v_A>$ is an intuitionistic fuzzy left (resp. right) h- ideal of S and let $\mu_A(0) \ge t, v_A(0) \le s$.

Let $x, y \in A_{(t,s)}$ and $\alpha \in \Gamma$.

Then $\mu_A(x) \ge t, v_A(x) \le s$ and $\mu_A(y) \ge t, v_A(y) \le s$.

Hence

$\mu_A(x+y) \ge \{\mu_A(x) \wedge \mu_A(y)\} \ge t$ and $v_A(x+y) \le \{\mu_A(x) \vee v_A(y)\} \le s$

$\mu_A(x\alpha y) \ge \mu_A(y) \ge t$ and $v_A(x\alpha y) \le v_A(y) \le s$

(resp. $\mu_A(x\alpha y) \ge \mu_A(x) \ge t$ and $v_A(x\alpha y) \le v_A(x) \le x$)

Therefore $x + y \in A_{<t,s>}$ and $x\alpha y \in A_{<t,s>}$ for all $x, y \in A_{<t,s>}$ and $\alpha \in \Gamma$.

$\mu_A(x) \ge \{\mu_A(a) \wedge \mu_A(b)\} \ge t$ and $v_A(x) \le \{v_A(a) \vee v_A(b)\} \le s$.

Hence $\quad x + a + z = b + z \in A_{<t,s>}$

So $A_{<t,s>}$ is a left (resp. right) h-ideal of S.

($\Leftarrow$) Suppose that $A_{<t,s>}$ is a left (resp. right) h- ideal of S for $\mu_A(0) \ge t$ and $v_A(0) \le s$.

Let $x, y \in S$ such that, $\mu_A(x) = t_1, v_A(x) = s_1, \mu_A(y) = t_2$, and $v_A(y) = s_2$.

Then $x \in A_{<t,s>}$ and $y \in A_{<t_2,s_2>}$.

We may assume $t_2 \le t_1$ and $s_2 \ge s_1$ without loss of generality.

It follows that $A_{<t_2,s_2>} \subseteq A_{<t_1,s_1>}$ so that $x, y \in A_{<t_1,s_1>}$.

Since $A_{<t_1,s_1>}$ is an ideal of S, we have $x + y \in A_{<t_1,s_1>}$ and $x\alpha y \in A_{<t_1,s_1>}$ and for all $\alpha \in \Gamma$.

$$\mu_A(x+y) \geq t_1 \geq t_2 = \{\mu_A(x) \wedge \mu_A(y)\}$$
$$v_A(x+y) \leq s_1 \leq s_2 = \{v_A(x) \vee v_A(y)\}$$
$$\mu_A(x\alpha y) \geq t_1 \geq t_2 = \mu_A(y)$$
$$v_A(x\alpha y) \leq s_1 \leq s_2 = v_A(y)$$
$$\mu_A(x) \geq t_1 \geq t_2 = \{\mu_A(a) \wedge \mu_A(b)\}$$
$$v_A(x) \leq s_1 \geq s_2 = \{v_A(a) \vee v_A(b)\}.$$

Therefore A is an intuitionistic fuzzy left (resp. right) h-ideal of S.

Again suppose $A = \langle \mu_A, v_A \rangle$ is not an intuitionistic fuzzy left h-ideal

So there exits $x_0, z_0, a_0, b_0 \in S$ such that $x_0 + a_0 + z_0 = b_0 + z_0$ and

$$\mu_A(x_0) = \{\mu_A(a_0) \wedge \mu_A(b_0)\}$$

Taking
$$t_0 = \frac{1}{2}\{\mu_A(x_0) + \{\mu_A(a_0) \wedge \mu_A(b_0)\}\}$$

We have $\mu_A(x_0) < t_0 < \{\mu_A(a_0) \wedge \mu_A(b_0)\}$.

Clearly $\mu_A(a_0),\ \mu_A(b_0) \geq t_0$.

As $\mu_A(a_0) + v_A(a_0) \leq 1$, then $v_A(a_0) \leq 1 - \mu_A(a_0)$. So $v_A(a_0) \leq 1 - t_0$

Similarly $v_A(b_0) \leq 1 - t_0$.

Consider $A_{\langle t, s\rangle}$

which is clearly non empty is a left h- ideal of S and $a_0, b_0 \in A_{\langle t, s\rangle}$.

Therefore $x_0 \in A_{\langle t, s\rangle}$, so $\mu_A(x_0) \geq t_0$. which is a contradiction.

Theorem 3.6.

If the intuitionistic fuzzy set $A = \langle \mu_A, v_A \rangle$ in a Γ- hemiring S is an intuitionistic fuzzy left (resp. right) h-ideal if and only if $S_{\mu A} = \{x \in S| \mu_A(x) = \mu_A(0)\}$ and

$$Sv_A = \{x \in S| v_A(x) = v_A(0)\}$$ are left (resp. right) ideals of S.

Proof. Let $x, y \in S_{\mu A}$ and $\alpha \in \Gamma$.

Then $\mu_A(x) = \mu_A(0), \mu_A(y) = \mu_A(0)$.

Since A is an intuitionistic fuzzy left (resp. right)h- ideal of a Γ- hemiring S, we get

$$\mu_A(x+y) = \{\mu_A(x) \wedge \mu_A(x)\} = \mu_A(0).\text{ But } \mu_A(0) \geq \mu_A(x+y).$$

So, $x + y \in S_{\mu_A}$.

$$\mu_A(x\alpha y) \geq \mu_A(y) = \mu_A(0) \text{ (resp. } \mu_A(x\alpha y) \geq \mu_A(x) = \mu_A(0)).$$

Hence $x\alpha y \in S_{\mu_A}$.

If $x + a + z = b + z \Rightarrow \mu_A(x) \geq \{\mu_A(a) \wedge \mu_A(b)\} = \mu_A(0)$

But $\mu_A(0) \geq \mu_A(x)$

Therefore S_{μ_A} is a left (resp. right) h-ideal of S.

Similarly, let $x, y \in S_{\mu_A}$ and $\alpha \in \Gamma$. Then $v_A(x) = v_A(0), v_A(y) = v_A(0)$.

Since A is an intuitionistic fuzzy left (resp. right) h-ideal of a Γ-hemiring S.

$$v_A(x+y) = \{v_A(x) \vee v_A(y)\} = v_A(0).$$

But $(v_A)(0) \leq v_A(x+y)$. So $x + y \in Sv_A$.

$$v_A(x\alpha y) \leq v_A(y) = v_A(0) \text{ (resp. } v_A(x\alpha y)) \leq v_A(x) = v_A(0).$$

Hence $x\alpha y \in Sv_A$

If $\quad x + a + z = b + z \Rightarrow v_A(x) \leq \{v_A(a) \vee v_A(b)\} = v_A(0).$

But $\quad v_A(0) \leq v_A(x)$

Therefore S_{v_A} is a left (resp. right) h- ideal of S.

Definition 3.7.

Let A and B be an intuitionistic fuzzy subsets of X. The Cartesian product of A and B are defined by

$$(A \times B)(x, y) = \min\{\mu_A(x), \mu_A(y)\} \text{ and } (A \times B)(x, y)$$
$$= \max\{v_A(x), v_A(y)\} \text{ for all } x, y \in X.$$

Note 3.8. Strongest intuitionistic fuzzy relation on B is the Cartesian product of B with itself.

Theorem 3.9.

Let A and B be an intuitionistic fuzzy left h-ideals of a Γ-hemiring S.Then $A \times B$ is an intuitionistic fuzzy left *h*-ideal of the Γ-hemiring $S \times S$.

Proof. Let $(x_1, x_2), (y_1, y_2), \in S \times S$ and $v \in \Gamma$. Then

$$\begin{aligned}
(A \times B)((x_1, x_2) + (y_1, y_2)) &= (A \times B)(x_1 + y_1, x_2 + y_2) = \min\{\mu_A(x_1 + y_1), \mu_A(x_2 + y_2)\} \\
&\geq \min[\min\{\mu_A(x_1), \mu_A(y_1)\}, \min\{\mu_B(x_2), \mu_B(y_2)\}] \\
&= \min[\min\{\mu_A(x_1), \mu_B(y_2)\}, \min\{\mu_A(y_1), \mu_B(y_2)\}] \\
&= \min\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\} \\
(A \times B)((x_1, x_2) + (y_1, y_2)) &= (A \times B)(x_1 + y_1, x_2 + y_2) = \max\{v_A(x_1 + y_1), v_B(x_2 + y_2)\} \\
&\geq \max[\max\{v_A(x_1), v_A(y_1)\}, \max\{v_B(x_2), v_B(y_2)\}] \\
&= \max[\max\{v_A(x_1), v_B(x_2)\}, \max\{v_A(y_1), v_B(y_2)\}] \\
&= \max\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\} \\
(A \times B)((x_1, x_2)\, \gamma\, (y_1, y_2)) &= (A \times B)(x_1\, \gamma\, y_1, x_2\, \gamma\, y_2) = \min\{\mu_A(x_1\, \gamma\, y_1), \mu_B(x_2\, \gamma\, y_2)\} \\
&\geq \min\{\mu_A(y_1), \mu_B(y_2)\} = (A \times B)\,(y_1, y_2) \\
(A \times B)((x_1, x_2)\, \gamma\, (y_1, y_2)) &= (A \times B)(x_1\, \gamma\, y_1, x_2\, \gamma\, y_2) = \max\{v_A(x_1\, \gamma\, y_1), v_B(x_2\, \gamma\, y_2)\} \\
&\geq \max\{v_A(y_1), v_B(y_2)\} = (A \times B)\,(y_1, y_2)
\end{aligned}$$

Hence $A \times B$ is an intuitionistic fuzzy left ideal of $S \times S$.

Now, let $(a_1, a_2), (b_1, b_2), (x_1, x_2), (z_1, z_2) \in S \times S$ be such that

$$(x_1, x_2) + (a_1, a_2) + (z_1, z_2) = (b_1, b_2) + (z_1, z_2)$$

i.e., $\quad (x_1 + a_1 + z_1, x_2 + a_2 + z_2) = (b_1 + z_1,\ b_2 + z_2)$

Then $\quad x_1 + a_1 + z_1 = b_1 + z_1$ and $x_2 + a_2 + z_2 = b_2 + z_2$ so that

$$\begin{aligned}
(A \times B)(x_1, x_2) &\geq \min\{\mu_A(x_1), \mu_B(x_2)\} \geq \min\{\min\{\mu_A(a_1), \mu_A(b_1)\}, \{\min\{\mu_B(a_2), \mu_B(b_2)\}\} \\
&= \min\{\min\{\mu_A(a_1), \mu_A(b_1)\}, \min\{\mu_B(a_2), \mu_B(b_2)\}\} \\
&= \min\{(A \times B)(a_1, a_2), (A \times B)(b_1, b_2)\}
\end{aligned}$$

$$(A \times B)(x_1, x_2) \le \max\{v_A(x_1), v_B(x_2)\} \le \max\{\max\{v_A(a_1), v_A(b_1)\}, \max\{v_B(a_2), v_B(b_2)\}\}$$
$$= \max\{\max\{v_A(a_1), v_A(a_2)\}, \max\{v_A(b_1), v_B(b_2)\}\}$$
$$= \max\{(A \times B)(a_1, a_2), (A \times B)(b_1, b_2)\}$$

Consequently, $A \times B$ is an intuitionistic fuzzy left h-ideal of $S \times S$. Like hemi rings

(Example 4.10 of [8]) the converse of Theorem 5.3 is not true. To conclude the paper, we obtain the following characterization of an intuitionistic fuzzy left h-ideal of a Ã-hemiring.

Theorem 3.10.

Let B be an intuitionistic fuzzy set in a Γ-hemiring S and A_B be the strongest an intuitionistic fuzzy relation on S. Then B is an intuitionistic fuzzy left h-ideal of S if and only if A_B is an intuitionistic fuzzy left h-ideal of $S \times S$.

Proof. Assume that B is an intuitionistic fuzzy left h-ideal of S.

Let $(x_1, x_2), (y_1, y_2) \in S \times S$ and $\gamma \in \Gamma$

Let $(x_1, x_2), (y_1, y_2) \in S \times S$ and $\gamma \in \Gamma$. Then

$$(A_B)((x_1, x_2) + (y_1, y_2) = (A_B)(x_1 + y_1, x_2 + y_2) = \min\{\mu_B(x_1 + y_1), \mu_B(x_2 + y_2)\}$$
$$\ge \min\{\min\{\mu_B(x_1), \mu_B(y_1)\}, \min\{\mu_B(x_2), \mu_B(y_2)\}]$$
$$= \min\{\min\{\mu_B(x_1), \mu_B(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}]$$
$$= \min\{\mu_B(x_1, x_2), \mu_B(y_1, y_2)\} \text{ and}$$
$$(A_B)((x_1, x_2)\,\gamma\,(y_1, y_2)) = (A_B)(x_1\,\gamma\,y_1, x_2\,\gamma\,y_2) = \min\{\mu_B(x_1\,\gamma\,y_1), \mu_B(x_2\,\gamma\,y_2)\}$$
$$= \min\{\min\{\mu_B(y_1), \mu_B(y_2)\} = (A_B)(y_1, y_2)$$
$$(A_B)((x_1, x_2) + (y_1, y_2)) = (A_B)(x_1 + y_1, x_2 + y_2) = \max\{v_B(x_1 + y_1), v_B(x_2 + y_2)\}$$
$$\le \max\{\max\{v_B(x_1), v_B(y_1)\}, \max\{v_B(x_2), v_B(y_2)\}]$$
$$= \max\{\max\{v_B(x_1), v_B(x_2)\}, \max\{v_B(y_1), v_B(y_2)\}]$$
$$= \max\{v_B(x_1, x_2), v_B(y_1, y_2)\}.$$
$$(A_B)((x_1, x_2)\,\gamma\,(y_1, y_2)) = (A_B)(x_1\,\gamma\,y_1, x_2\,\gamma\,y_2) = \max\{v_B(x_1\,\gamma\,y_1), v_B(x_2\,\gamma\,y_2)\}$$
$$= \max\{\max\{v_B(y_1), v_B(y_2)\} = (A_B)(y_1, y_2)$$

Hence A_B is an intuitionistic fuzzy left ideal of $S \times S$.

Let $(a_1, a_2), (b_1, b_2), (x_1, x_2), (z_1, z_2) \in S \times S$ be such that

$$(x_1, x_2) + (a_1, a_2) + (z_1, z_2) = (b_1, b_2) + (z_1, z_2)$$

i.e.,
$$(x_1 + a_1 + z_1, x_2 + a_2 + z_2) = (b_1 + z_1, b_2 + z_2)$$

Then $x_1 + a_1 + z_1 = b_1 + z_1$ and $x_2 + a_2 + z_2 = b_2 + z_2$ so that

$$(A_B)(x_1, x_2) = \min\{\mu_B(x_1), \mu_B(x_2)\} \ge \min\{\min\{\mu_B(a_1), \mu_B(b_1)\}, \min\{\mu_B(a_2), \mu_B(b_2)\}]$$
$$= \min\{\min\{\mu_B(a_1), \mu_B(a_2)\}, \min\{\mu_B(b_1), \mu_B(b_2)\}]$$
$$= \min\{\mu_B(a_1, a_2), \mu_B(b_1, b_2)\}].$$
$$(A_B)(x_1, x_2) = \max\{v_B(x_1), v_B(x_2)\} \le \max\{\max\{v_B(a_1), v_B(b_1)\}, \max\{v_B(a_2), v_B(b_2)\}]$$
$$= \max\{\max\{v_B(a_1), v_B(a_2)\}, \max\{v_B(b_1), v_B(b_2)\}]$$
$$= \min\{v_B(a_1, a_2), v_B(b_1, b_2)\}].$$

There fore A_B is an intuitionistic fuzzy left h-ideal of S × S.

Conversely, suppose that A_B is an intuitionistic fuzzy left *h*-ideal of

Let $x_1, x_2, y_1, y_2 \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}\min\{\mu_B(x_1+y_1), \mu_B(x_2+y_2)\} &= \mu_B((x_1+y_1,\ x_2+y_2)) \\ &= \mu_B((x_1, x_2)+(y_1, y_2)) \\ &\geq \min\{\mu_B(x_1, x_2), \mu_B(y_1, y_2)\} \\ &= \min\{\min\{\mu_B(x_1), \mu_B(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}\}\end{aligned}$$

$$\begin{aligned}\max\{v_B(x_1+y_1), v_B(x_2+y_2)\} &= v_B((x_1+y_1,\ x_2+y_2)) \\ &= v_B((x_1, x_2)+(y_1, y_2)) \\ &\leq \max\{v_B(x_1, x_2), v_B(y_1, y_2)\} \\ &= \max\{\max\{v_B(x_1), v_B(x_2)\}, \max\{v_B(y_1), v_B(y_2)\}\}.\end{aligned}$$

Now, putting $x_1 = x, x_2 = 0, y_1 = y$ and $y_2 = 0$, in this inequality and noting that

$$\mu_B(0) \geq \mu_B(x), v_B(0) \leq v_B(x), \text{ for all } x \in S.$$

We obtain $\mu_B(x+y) \leq \min\{\mu_B(x), \mu_B(y)\}$ & $v_B(x+y) \leq \max\{v_B(x), v_B(y)\}$.

Next, we have

$$\begin{aligned}\min\{\mu_B(x_1\gamma y_1), \mu_B(x_2\gamma y_2)\} &= \mu_B(x_1\gamma y_1,\ x_2\gamma y_2) \\ &= \mu_B((x_1, x_2)\gamma(y_1, y_2)) \geq \mu_B(y_1, y_2) = \min\{\mu_B(y_1), \mu_B(y_2)\}\end{aligned}$$

$$\begin{aligned}\max\{v_B(x_1\gamma y_1), v_B(x_2\gamma y_2)\} &= v_B(x_1\gamma y_1,\ x_2\gamma y_2) \\ &= v_B((x_1, x_2)\gamma(y_1, y_2)) \leq v_B(y_1, y_2) = \max\{v_B(y_1), v_B(y_2)\}.\end{aligned}$$

Taking $x_1 = x, x_2 = 0, y_1 = y$ and $y_2 = 0 x_1 = x, y_1 = y$ and $y_2 = 0$, we obtain

$$\mu_B(x\gamma y) \geq \mu_B(y)\} \ \& \ v_B(x\gamma y) \leq v_B(y).$$

Hence B is an intuitionistic fuzzy left ideal of S.

Let $a, b, x, z \in S$ be such that $x + a + z = b + z$. Then

$$(x, 0) + (a, 0) + (z, 0) = (b, 0) + (z, 0)$$

Since A_B is a fuzzy left *h*-ideal of S × S, we have

$$\begin{aligned}\mu_B(x) &= \min\{\mu_B(x), \mu_B(0)\} = \{\mu_B(x, 0) \geq \min\{\mu_B(a, 0), \mu_B(b, 0)\} \\ &= \min\{\min\{\mu_B(a), \mu_B(0)\}, \min\{\mu_B(b), \mu_B(0)\}\}\} \\ &= \min\{\mu_B(a), \mu_B(b)\} \text{ and}\end{aligned}$$

$$\begin{aligned}v_B(x) &= \max\{v_B(x), v_B(0)\} = v_B(x, 0) \leq \max\{v_B(a, 0), v_B(b, 0)\} \\ &= \max\{\max\{v_B(a), v_B(0)\}, \max\{v_B(b), v_B(0)\}\} \\ &= \max\{v_B(a), v_B(b)\}.\end{aligned}$$

Consequently, B is an intuitionistic fuzzy left h-ideal of S.

4. REFERENCES

1. **Atanassov, K T**., 1986. Intuitionistic Fuzzy Sets and Systems, 20: 87-96.
2. **Dutta, T K., S K Sardar**., **2002.** On Matrix Ã-Semirings. Far East Journal of Mathematics and Science, 117-31.
3. **Dutta, T K., S K Sardar., 2002.** On the Operator Semiring of a Ã-Semiring. Southeast Asian bulletin of Mathematics, Springer-Verlag**,** 6: 203-213.
4. **Ezhilmaran, D., V Krishnamoorthy.**, 2014. Characterization Of Intuitionistic Fuzzy h-ideal Ã- hemirings. Global journal of pure and applied mathematics, 10: 101-112
5. **Henriksen, M.,** 1958. Ideals in semirings with commutative addition. American. Mathematical Society. Notices, 6.
6. **Lizuka, K.**, 1959 **.**On the Jacobson radical of Semiring. Tohoku Mathematical journal,**2:** 409-421.
7. **Jun, Y B., M A Ozt¨urk and S Z Song.**, 2004. On Fuzzy h-ideals in hemiring. Information Sciences, 162: 211-226.
8. **Jun, Y B., C Y Lee.,** 1992. Fuzzy Ã-rings. Pusan Kyongnam Mathematical Journal, 2: 163-170.
9. **LaTorre, D R.**, 1965. On h-ideals and k-ideals in hemirings. Publications Mathematicea, 12: 219-226.
10. **Zhan, Ma J.,** 2010. Fuzzy h- ideals in h-hemiregular and h-simple Ã-hemirings, Neural Computing and Application, 19: 477-485.
11. **Rao, M M K.**, 1995. Ã-Semirings–1, Mathematics, 19: 49-54.
12. **Rosenfeld, A.**, **1971 .** Fuzzy groups. Southeast Asian Bulletin of Journal of mathematics analysis and application**,** 35: 512-517.
13. **Sardar, S K., D Mandal., 2009.** Fuzzy h-ideal in Ã-hemiring. International journal of Pure and applied mathematics**,** 3: 439-450.
14. **Xing-Yun Xie.,** 2001.Fuzzy ideal Extensions of semi groups. Soochow Journal of Mathematics, 27: 125-138.
15. **Zadeh, L A.,** 1965.Fuzzy Sets. Information and Control, 8:338-353.
16. **Zhan, J., W A Dudek**.,2007 .Fuzzy h-ideals of hemirings. Information sciences, 177: 876-886.