Design and Performance Comparison of Different Predictive Controllers for Magnetic Levitation System

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Abstract: This paper deals with design of model based predictive controllers for single axis magnetic levitation system and verifies the performance. The objective is to track the position of a ferromagnetic ball levitated with the aid of an electromagnetic force. Inherent nonlinearity and open loop unstable dynamics add challenges to this control problem. Three different controllers are designed: model predictive controller (MPC), approximate neural network MPC and adaptive neuro-fuzzy inference system (ANFIS) MPC. Controllers are designed in MATLAB environment and their performances are compared. MPC offers the advantage of minimum control effort while the other two controllers are found to require less computational time at the cost of comparatively higher control energy.

Keywords : Model predictive control; Adaptive neuro-fuzzy inference system; Magnetic levitation.

1. INTRODUCTION

Magnetic levitation is a quite attractive phenomenon where a ferromagnetic object is suspended by virtue of an electromagnetic force which counteracts gravitational pull. Magnetic levitation technology has been catching the attention due to its wide range of applications. Though it is extremely popular for its application in high speed transportation systems especially as maglev trains, but the list also includes wind turbines, magnetic elevator, magnetic bearing, space launch system, levitation of metal slabs during manufacturing, etc. [1]. Applications of magnetic levitation technology are extending to newer fields as it provides couples of advantages such as no friction, no abrasion, no desire for lubrication, long endurance, controllable support force, adjustable stiffness, environmental friendly, etc. It has the potential to change the world if accurate and precise control strategy is applied.

The system considered in the current analysis is a laboratory scale single axis magnetic levitation (maglev) system supplied by Feedback Instruments Limited, UK. The control objective is tracking the position of the ball suspended in a voltage controlled electromagnetic field. Maglev is a good candidate for control system designers as it has nonlinearity along with a very fast dynamics [2]. Open loop unstable dynamics of the system renders additional difficulty to the controller design. Different control strategies have been proposed by many researchers in recent years: PID, fuzzy control, back-stepping, feedback linearisation, etc. [3]-[10].

Model predictive control has had an exceptional history with early intimations in the academic literature coupled with an explosive growth due to its independent adoption by the process industries where it proved to be highly successful in comparison with alternative control methods [11]. Here a successful attempt has been made to design an MPC for magnetic levitation system which differs from slow process control problems due to its very fast dynamics. A linear MPC is developed by linearising the

process around an equilibrium point. An approximate neural network MPC and an adaptive neuro-fuzzy inference system (ANFIS) based controller are also designed. Performances of the aforesaid controllers are compared in terms of control effort as well as computational time.

Next section discusses the magnetic levitation system and formulation of the linear model used in the present work. Third section focuses on design procedure of different controllers. Fourth section highlights the results and performance comparison of the designed controllers and the last section draws the conclusion.

2. SYSTEM DESCRIPTION AND MATHEMATICAL MODELING

Magnetic levitation system considered in the current analysis and its phenomenological model are shown in Fig. 1. It includes a ferromagnetic steel ball which is suspended in a magnetic field controlled by voltage. Coil which is present at the top of the instrument acts as an electromagnetic actuator. An Infrared photo sensor is there which determines the position of the ball. Actuating force of the electromagnet can be controlled by regulating the current supplied to the circuit. If the electromagnetic force neutralises the weight of the ball, ball starts levitating. This can be achieved by designing a suitable controller. Magnetic levitation considered here is a single input single output (SISO) system where ball position is the output and voltage is the control signal [12].





Figure 1: Magnetic levitation system (Feedback 33-210) and its phenomenological model

Model linearisation

Assuming infrared sensor to be linear in the chosen operating range, yields a voltage Y that is related to distance Has

$$Y = \gamma H + \overline{Y} \tag{1}$$

where $\gamma > 0$ and $\overline{\mathbf{Y}}$ is a constant.

Current I fed to the coil holds a linear relation with input voltage U as

$$I = \rho U + \bar{I}, \rho > 0 \tag{2}$$

Here, \overline{I} is a constant that denotes the amount of current required to keepY = \overline{Y} . If deviations from the equilibrium point is denoted using lower case it follows from eq. (1) and (2) that

$$y = \gamma h \tag{3}$$

$$i = \rho u \tag{4}$$

The vertical motion of the ferromagnetic ball exhibits the following nonlinear relation

$$m\frac{d^2h}{dt^2} = mg - k\frac{I^2}{h^2}$$
(5)

where k > 0. Keeping current I at a fixed value, we get the following equilibrium point

$$\mathbf{H}_{0} = \mathbf{I}_{0} \sqrt{\frac{k}{mg}}.$$
 (6)

Linearisation of eq. (5) by first-order Taylor series expansion around the equilibrium operating point, $H = H_0$ yields

$$m\frac{d^{2}h}{dt^{2}} = \lambda h - \beta i$$

$$\lambda = \frac{2kI_{0}^{2}}{H_{0}^{3}} = \frac{2mg}{H_{0}},$$

$$\beta = \frac{2kI_{0}}{H_{0}^{2}} = \frac{2\sqrt{kmg}}{H_{0}}$$
(7) gives (7)

Substituting eq. (3) and (4) in eq. (7) gives,

$$\frac{d^2 y}{dt^2} = \eta y - A u \tag{8}$$

The transfer function is expressed as

$$G(s) = \frac{L(y)}{L(u)} = \frac{-A}{s^2 - \eta}$$
 (9)

where $\eta = \frac{2g}{H_0}$ and $A = \frac{2\rho\gamma}{H_0}\sqrt{\frac{kg}{m}}$. Since $\eta > 0$, one of the poles of G(s) lies on right hand side of

s – plane. Therefore eq. (9) reveals that system is open-loop unstable. Table 1 gives the parameters as well as their values used in the mathematical modeling.

Table 1						
Parameters	used	in	modeling [12]			

Parameters	Value	
Electromechanical constant, k	2.5 x 10 ⁻⁵	
Height to voltage multiplier, γ	143.48	
Height to voltage constant	-2.8	
Acceleration due to gravity, $g(m/s^2)$	9.8	
Current driver gain, p	1.045	
Mass of the ball, <i>m</i> (kg)	0.02	

3. CONTROLLER DESIGN

Design of model predictive control

Model predictive control has been developed to integrate the performance of the optimal control with the robustness of the feedback control. MPC refers to a control approach which makes explicit use of a process model to obtain a control sequence by optimising an objective function while accounting for constraints [13]-[15]. Only the first control signal of the calculated control sequence is applied at each instant. This procedure is repeated at each control interval and the new process measurements are used to update the optimisation problem. It is known as receding horizon principle.

The cost function J that reflects the control objective is

$$\operatorname{Min} \mathbf{J} = \sum_{i=1}^{N} ((r[k+i]) - \tilde{y}([k+i|k]))^2 + \mathbf{R} \sum_{i=1}^{M} (\Delta u[k-1+i])^2 \quad (10)$$

The first term tries to minimise the error between reference signal and predicted output while the second term is linked with minimisation of controller rate change. N is the prediction horizon, M is control horizon, Δu is control increment, and R is the weight on control input, 0 < R < 1. Smaller R causes increase in the speed of response allowing higher control input while larger *R* increases the robustness of the closed-loop system [15].

The transfer function model used for designing the controller is given by $G(s) = \frac{-1306}{s^2 - 2212}$

Design of approximate neural network MPC

First the closed loop system is excited by a reference voltage signal for identification. Magnitude of this signal varies randomly between operating range -1 V to +1 V and having minimum and maximum time interval 1 sec and 3 sec as shown in Fig. 2. The applied excitation signal is kept constant initially up to 20 sec for allowing the system to reach steady state. Error signal (the difference between desired output and actual system output), and change in error signal are noted. Control signal i.e. output of model predictive controller is also recorded.

These data are used to train a feedforward neural network. Ten thousand data are collected and out of which seven thousand data are used for training and fifteen hundred data are applied for validation and testing. Supervised learning is applied, where error and change in error constitute the inputs and controller output data act as target. Different networks are trained and finally a feedforward network having 50 neurons in the hidden layer is found to give best control performance. Levenberg-Marquardt training algorithm is applied for training.



Figure 2: Reference voltage signal used for closed loop identification (left) and corresponding output of maglev (right).

Design of ANFIS

Adaptive neuro-fuzzy inference system (ANFIS) realises fuzzy inference system through artificial neural network. It uses Takagi-Sugeno inference system of fuzzy. It integrates the benefits of both neural network and fuzzy logic in a single framework. Its inference system corresponds to a set of fuzzy 'IF-THEN' rules that have learning capability to approximate nonlinear functions [16], [17].

An adaptive neuro fuzzy controller is designed which can mimic a model predictive controller. The closed loop system is excited by a signal with random amplitude and frequency to extract data for all possible conditions. Error and change in error constitute the inputs and control signal data as output. Two dataset are collected having ten thousand samples each, one dataset is applied for training and other as checking data. Here, dataset are used to design a fuzzy controller, and the membership functions and parameters are tuned by training. The resulting ANFIS controller has two input variables having four gaussian membership functions each. Output of the controller has one variable having 16 linear membership functions. Sixteen rules are formed for the fuzzy inference system.

4. RESULTS AND DISCUSSIONS

Based on the linearised model of the system, a model predictive controller is designed with constraint on control input (-1.5V, 1.5V).

The tuning parameters used for the design are :

Prediction horizon : 20 Control horizon : 2 Weight on control signal : 0.1

Sampling time : 0.01 sec

Fig. 3 shows simulation result of MPC for tracking a sinusoidal reference signal and the control signal. Position of the ball is shown in meter and control signal in voltage. Fig. 4 shows the tracking result of pulse reference signal and respective control signal.





Figure 4: Ball position tracking of square wave by MPC (top) and Control signal used in voltage (bottom)

The approximate neural network model predictive controller is found to ensure stability as well as good tracking performance. Tracking performance is shown for sinusoidal and pulse input signals in Fig. 5.



Figure 5: Ball position tracking by Approximate Neural MPC for sinusoidal (top) and square references (bottom)

The result of ANFIS controller is shown in Fig. 6 for sinusoidal and square signals respectively. Performance is found to be satisfactory.





Figure 6: Ball position tracking by ANFIS controller for sinusoidal (top) and square signal (bottom) as reference

Table 2

Upto 100 sec	Criteria	Approximate Neural MPC	ANFIS
Sinusoidal input	IAE	1.65x10-2	4.203x10-2
	ISE	7.339x10-6	2.383x10-5
Square input	IAE	1.334x10-3	6.506x10-2
	ISE	3.3x10-8	3.234x10-4

Integral absolute error (IAE) and integral square error (ISE) for approximate neural MPC and ANFIS are calculated and listed in Table 2. Approximate neural network MPC is found to give better performance than ANFIS in terms of IAE and ISE. Performance of both the controllers can be further improved by choosing different network structures and different fuzzy membership functions.

Table 3Comparison of different controllers

Controller	CPU elapsed time (sec)	// u// ₂
MPC	60.20	10.264
Approximate neural MPC	22.88	22.07
ANFIS	24.99	20.06

CPU elapsed time taken by i3, 2.3 GHz processor is noted for different controllers which are shown in Table 3. The approximate neural MPC and ANFIS controllers are found to be taken very less time compared with MPC. So, we can conclude that computational burden of MPC is reduced by using neural MPC and ANFIS. $||u||_2$ is found to be minimum in case of MPC i.e. control energy requirement is minimum in case of MPC.

5. CONCLUSION

In this paper, ball position control of a magnetic levitation system is accomplished by designing three different controllers and performances of the designed controllers are compared. Computational burden of conventional MPC can be greatly reduced by using approximate neural MPC and ANFIS MPC controller, but compensated with an increase in control effort. Simulation results reveal that approximate neural MPC and ANFIS can be good alternatives of conventional MPC for control of maglev system. Improvement of performances of approximate neural MPC and ANFIS MPC controllers is the future direction of works.

6. **REFERENCES**

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