

# **Zitterbewegung and a formulation for quantum mechanics**

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***Abstract:** In this short note, we try to show that inside a vortex like region, like a black hole one can observe superluminality which yields some interesting results. Also, we consider the zitterbewegung fluctuations to obtain an interpretation of quantum mechanics.*

## **1. Introduction**

Contemporary quantum mechanics including quantum field theory is based upon the idea of point particles and points of spacetime. This we contend as an extreme idealization. The purport is the following:

As Dirac [1], Salecker and Wigner [2], Ginzburg [3] and others have pointed out, in the physical world, we can work with only intervals of spacetime. This again smudges the point particle. The age-old zitterbewegung is a manifestation of how imprecise our description of particles can be. As Fritz and Rohrlich [4, 5] have put it – the point particle is an oxymoron. This was also elaborated by Sidharth [6]. The work of Newton and Wigner also points in this direction.

As is well known, the phenomenon of zitterbewegung has been studied by many eminent authors like Schrodinger [7] and Dirac [1]. Zitterbewegung is a type of rapid fluctuation that is attributed to the interference between positive and negative energy states. Dirac pointed out that the rapidly oscillating terms that arise on account of zitterbewegung is averaged out when we take a measurement remembering that no measurement is instantaneous.

Interestingly, Hestenes [8, 9] too sought to explain quantum mechanics through the phenomenon of zitterbewegung. Besides, the authors of this paper have also studied the aforesaid phenomenon quite extensively in the past few years [10, 11, 12, 13, 14, 15]. However, in this paper we shall derive some interesting results that substantiate that the Compton scale and zitterbewegung can delineate several phenomena.

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## 2 Theory

Let us consider an object, such as a black hole or some particle, that is spiralling like a vortex. In that case, we know that the circulation is given by

$$\Gamma = \oint_C v dr = \frac{2\pi\hbar n}{m}$$

where,  $C$  denotes the contour of the vortex,  $n$  is the number of turns and  $m$  is the mass of the circulating particle. Now, using the relation derived by the author Sidharth [16] and using the theory of vortices [17]

$$v = \frac{D}{2\pi r}$$

and the diffusion constant being given as

$$D = \frac{\hbar}{m}$$

we obtain from the integral

$$\Gamma = \frac{\hbar}{2\pi m} \oint_l \frac{dr}{r} = \frac{2\pi\hbar n}{m} \quad (1)$$

Integrating this we derive the relation

$$r = e^{4\pi^2 n} \quad (2)$$

which shows that in case of vortices the radius of a particular vortex increases with the number of turns, which is expected. One may wonder how superluminality, forbidden by special relativity and causality, can arise? As we will see shortly that such ideas of superluminal velocities being forbidden etc., arise above the Compton scale or more precisely outside the zitterbewegung region belonging to the domain of usual physics. Now, suppose the velocity equals that of the velocity of light. Then, we have the circulation as

$$\Gamma = \oint_l c dr = \frac{2\pi\hbar n}{m}$$

This implies that

$$r = \frac{2\pi\hbar n}{mc}$$

Thus, we have finally

$$r = 2\pi n l_c \quad (3)$$

where,  $l_c = \frac{\hbar}{mc}$  is the Compton wavelength of the circulating particle or object. Essentially, we have quantized the radius. Now, it is known that the Langevin equation in the absence of external forces is given as [16]

$$m \frac{dv}{dt} = -\alpha v + F'(t)$$

where,  $\alpha$  is the coefficient of viscosity given by Stokes' law as

$$\alpha = 6\pi\eta a$$

Here,  $a$  is the radius of the sphere under consideration. So, when we have a cutoff time  $\tau$ , we get

$$\tau \approx \frac{ma^2}{mca} = \frac{a}{c}$$

Suppose, within the limit of the cut off time the radius of the sphere under consideration coincides with that of the Compton length, then

$$\tau \approx \frac{l_c}{c}$$

Thus, from (3) we have

$$r = 2\pi n c \tau$$

which finally yields

$$v_c = 2\pi n c \quad (4)$$

where,  $v_c = \frac{r}{\tau}$  can be defined as the velocity in the Compton scale. It is obvious from equation (4) that the velocity  $v_c$  is greater than the velocity of light. This means that in the Compton scale when a cut off time arises from considerations of stochastic behaviour and the Langevin equation one derives superluminal velocities. This is very interesting in the sense that in case of minimum spacetime intervals one can expect phenomena that are different from usual Physics that one observes above the Compton scale. It has also been pointed out by Joos [18] that from the Langevin like equation one can derive a cutoff time that can define a limit for the Compton scale and the phenomenon of zitterbewegung. Interestingly, when there is no cutoff time then we get [19]

$$r = D\sqrt{t}$$

where,  $D$  is the diffusion constant. It is known that from the last equation one can go to Nelson's derivation of the non-relativistic Schrodinger equation [16]. This

implies that the cutoff time provides a threshold limit beyond which Compton scale phenomena are not observed or essentially, such stochastic dynamics are averaged out. However, within the threshold limit we obtain results that are in stark contrast with known physics. This is a key feature of the Compton scale that has been discussed in several of our papers.

### 3 Consequence of a superluminal velocity

In the previous section we have obtained a novel result concerning the velocity of the circulating particle. We have seen that considering a vortex like motion of a particle in the Compton length one derives the velocity of the particle to be greater than the velocity of light.

Now, Sidharth had derived previously [20] that for a Kerr-Newman type charged black hole one would have an Aharonov-bohm type effect, on account of the vector potential  $\vec{A}$  which would give a shift for the magnetic field as

$$\Delta\delta_B = \frac{e}{\hbar} \oint \vec{A} \cdot d\vec{s}$$

and the shift due to the electric charge would be

$$\Delta\delta_E = -\frac{e}{\hbar} \oint A_0 \cdot dt$$

where,  $\vec{A} \approx \frac{1}{c} A_0$ . Using this relation it is easy to deduce that the magnetic effect is  $\sim \frac{v}{c}$  times the electric effect. Now, when  $v < c$  we have the electric effect to be stronger than the magnetic effect. But, from our derivation in the previous section it is obvious that the magnetic effect is stronger than the electric effect. This also may imply that there is an extra magnetic effect arising from the vortex like circulation of the particle. In case of a Kerr-Newman black hole the magnetic field is given by

$$B_r = \frac{2ea}{r^3} \cos \Theta$$

$$B_\Theta = \frac{ea \sin \Theta}{r^3} + O\left(\frac{1}{r^4}\right)$$

$$B_\phi = 0$$

Essentially, a short-ranged magnetic field that emerges due to the terms  $\sim \frac{1}{r^4}$  could result in the extra magnetic field that we mentioned earlier. It is interesting

to find that consideration of the Compton region brings about such interesting results and phenomena. Now, we would like to delve into another interesting aspect that stems from the consideration of Compton scale. The author Sidharth had derived the quantum mechanical spin from within the Compton scale [6]. We would see that from the vortex like consideration we obtain the same result.

#### 4 The quantum mechanical spin

Let us consider equation (3) once again. Multiplying both sides by the momentum of the particle we have

$$r \times p = 2\pi n l_c \times p$$

Now, if the De Broglie wavelength of the particle with vortex features is the Compton wavelength then we have

$$p = \frac{\hbar}{l_c}$$

Thus, we get

$$r \times p = 2\pi n \hbar \tag{5}$$

Now, we know that the spin of a particle is given as

$$S = \frac{\hbar}{2} \sqrt{n(n+2)}$$

where,  $n$  is some positive integer. Then, considering the following approximation

$$4\pi n \approx \sqrt{N(N+2)}$$

for some positive integer  $N$ , we have the spin for the circulating particle to be

$$r \times p = S \approx \frac{\hbar}{2} \sqrt{N(N+2)} \tag{6}$$

Therefore, as we can see, we have obtained the quantum mechanical spin from the consideration of the Compton scale and the vortex like feature of the particle. This could be looked upon as the foundation of quantum mechanics.

#### 5 Discussions

In terms of positive and negative energy solutions we have

$$\psi_p = e^{\frac{i}{\hbar}Et} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ or, } e^{\frac{i}{\hbar}Et} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (7)$$

$$\psi_n = e^{-\frac{i}{\hbar}Et} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ or, } e^{-\frac{i}{\hbar}Et} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

From this Sidharth had derived the following relation [6]

$$|\psi_p + \psi_n|^2 = |\psi_p|^2 + |\psi_n|^2 + (\psi_p\psi_n^* + \psi_n\psi_p^*) \quad (9)$$

It is interesting to note that the third term denotes that the positive and negative energy solutions interact with each other and this happens in the zitterbewegung region. So, basically, in this region classical physics gets molded into a quantum mechanical version on account of the fluctuations present. Outside this region, only the first two terms of the last equation exist and thereby we observe normal physical phenomena that includes the averaging out of the third term.

We would like to add that the positive and negative domains are represented by positive and negative energies insinuating forward and backward time flow. In general, it is not possible to have both these domains coexisting. However, it has been shown in great detail [21] that this is represented within the Compton scale by a twoWiener process leading to interesting results. It is worth mentioning that Newton and Wigner also had argued that there is a region outside which we get the usual physical phenomena.

Now, in accordance with Wilson's renormalization group and one of our previous papers we can consider localized wavepackets such that [22, 23, 24, 25]

$$\psi(x) = \psi_m(x) + \sum_n r_n \phi_n(x) \quad (10)$$

where the modified wavefunction is  $\psi_m(x)$  and each wavefunction  $\phi_n(x)$  fills a unit volume in the phase space. Here, the integration is performed upon the coefficients  $r_n$ . Thus, considering Ito's lemma [26, 27] as

$$d\psi(x_t) = \psi'(x_t)dx_t + \frac{1}{2}\psi''(x_t)\sigma_t^2 dt$$

we have

$$d\psi_m(x_t) = \{\psi'_m(x_t) + \sum_n r_n \phi'_n(x_t)\} dx_t + \frac{1}{2} \{\psi''_m(x_t) + \sum_n r_n \phi''_n(x_t)\} \sigma_t^2 dt - \sum_n d\{r_n \phi_n(x_t)\} \quad (11)$$

Interestingly, visualizing  $\phi(x)$  as a step function we were able to derive the following relation

$$\phi''(x_t) = \delta'(l) \frac{dl}{dx_t} + \delta(l) \frac{d^2 l}{dx_t^2} \quad (12)$$

Again, we know that in electromagnetism [28, 29], the gradient of the delta function represents a point magnetic dipole situated at the origin and that the function itself represents a point charge [30], owing to its distributional property. Thus, we can see that the electric charge emerges from the Compton scale when one considers the wavefunction  $\phi(x_t)$  to describe the phase space of unit volume. This phase space can essentially be looked upon as a cloud like space with fluctuations going on inside it. The properties of this cloud like space is entirely embedded in the wavefunction  $\phi(x_t)$ . From equation (12) we derive the electric charge that can be attributed to the collective fluctuations or zitterbewegung going on inside the phase space. When an observation is made upon this cloud like space the electron is created instantaneously.

We would also like to infer that these fluctuations are on account of the quantum vacuum that exists in the phase space. Considering the methodology of a quantized Klein-Gordon field in the vacuum state, one can calculate the probability that the configuration of this cloud like space is given by  $\phi(x_t)$ .

$$\rho_0[\phi(x_t)] = \exp\left[-\frac{1}{\hbar} \int \frac{1}{(2\pi)^3} \phi^*(k_t) \phi(k_t) \sqrt{|k|^2 + m^2} d^3 k_t\right] \quad (13)$$

where,  $\phi(x_t)$  is the Fourier transform of  $\phi(x_t)$  which essentially characterizes the electron. Incidentally, this implies that the electron is the Fourier transform of the cloud like structured phase space. Inasmuch, the cloud like space is connected to the point electron through a Fourier transform when the region is observed. Physically, one can argue that the very act of observation of the quantum vacuum leads to the detection of the electron and mathematically is represented by a Fourier transform given by equations of type (13).

Also, we had seen earlier that within the Compton scale or more precisely the zitterbewegung region we have “unphysical effects” like superluminality and a breakdown of causality [31, 32]. But, these are tweaked away once we return to the physical region. The whole point is that these ideas are based on notions of space time points which as noted by authors like Rohrlich [4, 5] is an oxymoron.

## 6 Conclusions

Thus, from various points of view we come to the conclusion that the zitterbewegung region is eliminated during observable physical processes. What we have argued is that classical physics represented by the first two terms of equation (9) where the positive and negative energy solution stand decoupled into separate domains. But, on the contrary, it is the interference of these two that gives rise to quantum mechanics.

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