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Stability of Three Prey one Predator Model in Wireless Sensor Network

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Abstract: Wireless sensor network becomes of the hot area of research due to its applications. Sensor nodes having limited energy which is one of critical issue. So, how to use this energy to increase the lifetime of sensor network. One of the methods is that to use non operational mode that means sleep mode to reduce energy consumption. In this paper, discuss a mathematical model based on predator–prey model and analyze dynamics of the system as well as energy preservation in the passage of worms attacks in wireless sensor network (WSN). We discuss different equilibrium points and analyze the stability of the system. In last some numerical solution are given to explain the analyze model.

Keywords: Prey, Wireless sensor network, worms, Prey-Predator Model, stability.

1. INTRODUCTION

Wireless sensor network is a collection of sensor nodes which have memory, processing unit, battery, communication area. Sensor node is a resource constraint device such as having limited energy, memory, coverage area etc. Sensor nodes are deploying any where (1) to monitor and sense data. Sensor nodes collect the data and send to the base station from remote place. One of the greatest problems with sensor node is energy consumption when transmit and receive data; replacement of battery is most difficult, because nodes are deployed in unapproachable locations. So, nodes energy can be saved by exploiting the operational mode nodes. Sensor nodes are working in modes sleep and operational mode, when sensor is in sleep mode then nothing can be done by nodes, but when nodes in operational mode energy will be exhausted during collection and transmission of data. When nodes in operational mode, they become prone towards attack of malicious signals. With the expansion of internet use there are so many challenges appear in cyber world at data transmission time. To protect the system different types of mechanisms have been proposed and among these some are similar to the biological system. Researchers have studied various prey-predator models in computer network.

In (2), author discussed about active worm propagation model that use the concept of prey-predator biological model to detect and monitor the worms attack. In (3) two mathematical models were proposed to study the predator prey system inside a computer system. In (4) proposed a model that uses Lotka-Volterra equation to protect the internet from worm attack and minimize the numbers of predators to eradicate the worm threat. Castaneda et. al., (5) discussed four different techniques for detection and prevention on the basis of antiworm predation method. Delay concept used (6-10) for defence from active worm with different type of interaction. Mishra et. al., (3) proposed Stability analysis of a predator–prey model in wireless sensor network and find equilibrium points as well as saving of energy.

To analyze the attacking behavior of worms in WSN (11-12), to studied a various idea of prey–predator to derived a mathematical model. Ahead of moving with conceptualization of the model, it is significant to realize the objective of the system being modelled. A wireless sensor network is vulnerable against probable malicious signal attacks. It is also vital to be in view that, this objective requirements to be satisfied despite the fundamental energy and resource constraints on the nodes,which constitute the network. Need for effective working of a WSN some number of nodes required to perform function correctly. Because of cruel resource limitations, WSN faces so many problems like energy, memory and worm attack. Keeping the parameters of network, the model developed that should be feasible for WSN.

2. MODEL FORMULATION

In proposed model, we suppose three types of infected class: one is infectious category of 1 (Y) and second of infectious category of 2 (Z) and third category is of a infected class (L) in which third category of nodes do not have any opportunity of revival and they can infect other nodes quickly, when in active state. So, elongate the life time of sensor network this category of nodes sends to the sleep state, because they do not transmit of receive data as well malicious signals. When category 1 and category2 of nodes interact with susceptible class (X), both infects the susceptible class of nodes with different rates but they can be also recovered by different rate. When some of the nodes of class L come in active mode it transmit data as well as malicious signals to all types of nodes X, Y and Z. Class X, Y and Z are of prey category for predator L, so different interspecies interference coefficient rates between these. In this situation there is one predator and three pray. When L is in sleep state class Y and Z behaves like predator for X because these also of infected category. When all are in active state this is work as three pray and one predator. The number of susceptible nodes, infectious nodes of category 1, infectious nodes of category 2, infectious nodes of category 3 and recovered nodes at time t, are denoted by Let $X(t)$, $Y(t)$, $Z(t)$, $L(t)$ and $R(t)$ respectively.

Let $X(t) + Y(t) + Z(t) + L(t) + R(t) = N(t)$, for all t. Then the system can be described by the following set of non linear differential equation.

$$\left.\begin{aligned}
\frac{dX}{dt} &= rX\left(1-\frac{X}{k}\right) - \beta XY - \rho XL - \lambda_1 X - \omega XZ \\
\frac{dY}{dt} &= sY\left(1-\frac{Y}{l}\right) - \lambda YZ - \sigma YL - \lambda_2 Y + b\beta XY \\
\frac{dZ}{dt} &= pZ\left(1-\frac{Z}{n}\right) - \pi ZL + c\lambda YZ - \lambda_3 Z + c_1\omega XZ \\
\frac{dL}{dt} &= b_1\rho XL + b_2\sigma YL + b_3 ZL - \eta L \\
\frac{dR}{dt} &= \lambda_1 X + \lambda_2 Y + \lambda_3 Z
\end{aligned}\right\} \qquad (2.1)$$

Table 1
The Nomenclature used in the model

Variables/ Parameters	*Description*
X	Number of prey (susceptible) population at any time t
Y	Number of infected prey (infected category of 1) population at any time t
Z	Number of infected prey (infected category of 2) population at any time t
L	Number of predator population at any time t
R	Recovered Class
ρ	Interspecies interference to the prey class X and L
ω	Interspecies interference to the prey class X and Z
k	Carrying capacity of prey X
l	Carrying capacity of prey Y
n	Carrying capacity of prey Z
r	Growth rate of Prey (Susceptible) population X
s	Growth rate of Prey (infected category of 1) population Y
p	Growth rate of Prey (infected category of 2) population Z
λ	Interspecies interference to the prey (infected category of 1) class X and the prey (infected category of 2) Y
σ	Interspecies interference to the prey (susceptible) class X and the predator L
b	The conversion factor denoting the number of increase nodes in the class Y
c, c_1	The conversion factor denoting the number of increase nodes in the class Z
b_1, b_2, b_3	The conversion factor denoting the number of increase nodes in the predator class L
λ_1, λ_2, λ_3	Rate of removing the from the class X, Y, Z respectively
η	Death rate

3. EXISTENCE OF POSITIVE EQUILIBRIUM

(a) The equilibrium point $P_0(0, 0, 0, 0)$ always exist.

(b) The equilibrium point $P_1\left(\frac{k(r-\lambda_1)}{r}, 0, 0, 0\right)$ exist if $r > \lambda_1$

(c) The equilibrium point $P_2\left(0, \frac{l(s-\lambda_2)}{s}, 0, 0\right)$ exist if $s > \lambda_2$

(d) The equilibrium point $P_3\left(0, 0, \frac{m(p-\lambda_3)}{p}, 0\right)$ exist if $p > \lambda_3$

(e) The equilibrium point $P_4\left(\frac{k\{s(r-\lambda_1)-\beta l(s-\lambda_2)\}}{b\beta^2 lk+rs}, \frac{l\{b\beta k(r-\lambda_1)+r(s-\lambda_2)\}}{b\beta^2 lk+rs}, 0, 0\right)$ exist if $s(r-\lambda_1) - \beta l(s-\lambda_2) > 0$ and $b\beta k(r-\lambda_1) + r(s-\lambda_2) > 0$.

(f) The equilibrium point $P_5\left(0, \frac{l\{p(s-\lambda_2)-m\lambda(p-\lambda_3)\}}{m\lambda^2 lc+ps}, \frac{m\{c\lambda l(s-\lambda_2)+s(p-\lambda_3)\}}{m\lambda^2 lc+ps}, 0\right)$ exist if $p(s-\lambda_2) - m\lambda(p-\lambda_3) > 0$ and $c\lambda l(s-\lambda_2) + s(p-\lambda_3) > 0$.

(g) The equilibrium point $P_6\left(\dfrac{m\{c_1\omega k(r-\lambda_1)+r(p-\lambda_3)\}}{c_1\omega^2 mk+pr}, 0, \dfrac{k\{p(r-\lambda_1)-\omega m(p-\lambda_3)\}}{c_1\omega^2 mk+pr}, 0\right)$ exist if $c_1\omega k(r-\lambda_1)+r(p-\lambda_3)>0$ and $p(r-\lambda_1)-\omega m(p-\lambda_3)>0$.

4. STABILITY ANALYSIS

The Variational matrix of system (2.1) at P(X, Y, Z, L) is given by

$$J(P)=\begin{pmatrix} \left\{\begin{matrix} r-\dfrac{2Xr}{k}-\beta Y- \\ \rho L-\lambda_1-\omega Z \end{matrix}\right\} & -\beta X & -\omega X & -\rho X \\ b\beta Y & \left\{\begin{matrix} s-\dfrac{2Ys}{l}-\lambda Z- \\ \sigma L-\lambda_2+b\beta X \end{matrix}\right\} & -\lambda Y & -\sigma Y \\ c_1\omega Z & c\lambda Z & \left\{\begin{matrix} p-\dfrac{2Zp}{n}-\pi L+ \\ c\lambda Y-\lambda_3+c_1\omega X \end{matrix}\right\} & -\pi Z \\ b_1\rho L & b_2\sigma L & b_3\pi L & \left\{\begin{matrix} b_1\rho X+b_2\sigma Y \\ +b_3\pi Z-\eta \end{matrix}\right\} \end{pmatrix} \tag{4.1}$$

4.1. The Equilibrium Point $P_0(0, 0, 0, 0)$

The variational matrix at P_0 becomes

$$J(P_0)=\begin{pmatrix} r-\lambda_1 & 0 & 0 & 0 \\ 0 & s-\lambda_2 & 0 & 0 \\ 0 & 0 & p-\lambda_3 & 0 \\ 0 & 0 & 0 & -\eta \end{pmatrix} \tag{4.2}$$

The eigen values of above matrix are $w_1=(r-\lambda_1)$, $w_2=(s-\lambda_2)$, $w_3=(p-\lambda_3)$, $w_4=-\eta$, at p_0 therefore we have the following theorem:

Theorem 1: Equilibrium point P_0 is locally asymptotically stable if $\lambda_1<r$, $\lambda_2<s$, $\lambda_3<p$.

4.2 The equilibrium point $P_1\left(\dfrac{(r-\lambda_1)k}{r}, 0, 0, 0\right)$

At P_1, the variational matrix becomes

$$J(P_1)=\begin{pmatrix} r-\dfrac{2X^*r}{k}-\lambda_1 & -\beta X^* & -\omega X^* & -\rho X^* \\ 0 & s-\lambda_2+b\beta X^* & 0 & 0 \\ 0 & 0 & p-\lambda_3+c_1\omega X^* & 0 \\ 0 & 0 & 0 & b_1\rho X^*-\eta \end{pmatrix} \tag{4.3}$$

The eigen values of above matrix are $w_1 = -(r - \lambda_1)$, $w_2 = \dfrac{A_1}{r}$, $w_3 = \dfrac{A_2}{r}$, $w_4 = \dfrac{A_3}{r}$, at P_1, where, $A_1 = r(s - \lambda_2) + b\beta k(r - \lambda_1)$, $A_2 = r(p - \lambda_3) + c_1\omega k(r - \lambda_1)$, $A_3 = b_1\rho k(r - \lambda_1) - r\eta$, therefore we have the following theorem:

Theorem 2: Equilibrium point P_1 is locally asymptotically stable if $A_1 < 0$, $A_2 < 0$ and $A_3 < 0$.

4.3. The equilibrium point $P_2\left(0, \dfrac{(s-\lambda_2)l}{s}, 0, 0\right)$

At P_2, the variational matrix becomes

$$J(P_2) = \begin{pmatrix} r - \lambda_1 - \beta Y^* & 0 & 0 & 0 \\ p\beta Y^* & s - \lambda_2 - \dfrac{2Y^* s}{l} & -\lambda Y^* & -\sigma Y^* \\ 0 & 0 & p - \lambda_3 + c\lambda Y^* & 0 \\ 0 & 0 & 0 & b_2\sigma Y^* - \eta \end{pmatrix} \tag{4.4}$$

The eigen values of above matrix are $w_1 = -(s - \lambda_2)$, $w_2 = \dfrac{A_4}{s}$, $w_3 = \dfrac{A_5}{s}$, $w_4 = \dfrac{A_6}{s}$, at P_2, where $A_4 = s(r - \lambda_1) - \beta l(s - \lambda_2)$, $A_5 = s(p - \lambda_3) + cl\lambda(s - \lambda_2)$, $A_6 = b_2\sigma l(s - \lambda_2) - s\eta$, therefore we have the following theorem:

Theorem 3: Equilibrium point P_2 is locally asymptotically stable if $A_4 < 0$, $A_4 < 0$, and $A_6 < 0$.

4.4. The equilibrium point $P_3\left(0, 0, \dfrac{(p-\lambda_3)n}{p}, 0\right)$

At P_3, the variational matrix becomes

$$J(P_3) = \begin{pmatrix} r - \lambda_1 - \omega Z^* & 0 & 0 & 0 \\ 0 & s - \lambda Z^* - \lambda_2 & 0 & 0 \\ c_1\omega Z^* & c\lambda Z^* & p - \lambda_3 - \dfrac{2Z^* p}{n} & 0 \\ 0 & 0 & 0 & b_3\pi Z^* - \eta \end{pmatrix} \tag{4.5}$$

The eigen values of above matrix are $w_1 = -(p - \lambda_3)$, $w_2 = \dfrac{A_7}{p}$, $w_3 = \dfrac{A_8}{p}$, $w_4 = \dfrac{A_9}{p}$, at P_3, where $A_7 = p(r - \lambda_1) - n\eta(p - \lambda_3)$, $A_8 = p(s - \lambda_2) + \lambda n(p - \lambda_3)$, $A_9 = b_3\pi n(p - \lambda_3) - p\eta$, therefore we have the following theorem:

Theorem 4: Equilibrium point P_3 is locally asymptotically stable if $A_7 < 0$, $A_8 < 0$, and $A_9 < 0$

4.5. The equilibrium point $P_4\left(\dfrac{k\{s(r-\lambda_1) - \beta l(s-\lambda_2)\}}{b\beta^2 lk + rs}, \dfrac{l\{b\beta k(r-\lambda_1) + r(s-\lambda_2)\}}{b\beta^2 lk + rs}, 0, 0\right)$

At P_4, the variational matrix becomes

$$J(P_4)=\begin{pmatrix} r-\lambda_1-\dfrac{2X^*r}{k}-\beta Y^* & -\beta X^* & -\omega X^* & -\rho X^* \\ b\beta Y^* & s-\lambda_2-\dfrac{2Y^*s}{l}+b\beta X^* & -\lambda Y^* & -\sigma Y^* \\ 0 & 0 & p+c\lambda Y^*-\lambda_3+c_1\eta X^* & 0 \\ 0 & 0 & 0 & b_1\rho X^*+b_2\sigma Y^*-\eta \end{pmatrix} \quad (4.5)$$

The two eigen values of above matrix are the roots of the equation $a_0\lambda^2 + a_1\lambda + a_2 = 0$, where $a_1=\frac{1}{Q_3}\left[(r-\lambda_1)Q_6+(s-\lambda_2)Q_7\right]$, $a_2=\frac{1}{Q_3^2}\left[(r-\lambda_1)^2Q_8+(s-\lambda_2)^2Q_9+(r-\lambda_1)(s-\lambda_2)Q_{10}\right]$, and other two eigen value are $w_1=\frac{(r-\lambda_1)Q_1+(s-\lambda_2)Q_2-\eta Q_3}{Q_3}$, $w_2=\frac{(p-\lambda_3)Q_3+(r-\lambda_1)Q_4+(s-\lambda_2)Q_5}{Q_3}$, where, $Q_1=(b_1\rho ks+b_2\sigma lb\beta k)$, $Q_2=(b_2\sigma lr-b_1\rho l\beta k)$, $Q_3=(\beta^2kl+rs)$, $Q_4=(c_1\eta kl-bc\lambda l\beta k)$, $Q_5=(c\lambda lr-c_1\eta l\beta k)$, $Q_6=(rs+b\beta^2l(1-k)+bs\beta k)$, $Q_7=(-\beta lr+rs)$, $Q_8=((rs+b\beta^2l(1-k))bs\beta k)$, $Q_9=\beta lr^2s$ and $Q_{10}=((2rs+b\beta^2lk)(rs+b\beta^2l(1-k))+(rs+b\beta^2lk)^2-b\beta^2lksr$, clearly eigen value w_1, w_2, w_3 and w_4 will be negative if the following condition holds:

Theorem 5: Equilibrium point P_5 is locally asymptotically stable if $(r-\lambda_1)Q_6+(s-\lambda_2)Q_7>0$, $(r-\lambda_1)^2Q_8+(s-\lambda_2)^2Q_9+(r-\lambda_1)(s-\lambda_2)Q_{10}>0$, $(r-\lambda_1)Q_1+(s-\lambda_2)Q_2-\lambda Q_3<0$ and $(p-\lambda_3)Q_3+(r-\lambda_1)Q_4+(s-\lambda_2)Q_5<0$.

5. SIMULATION

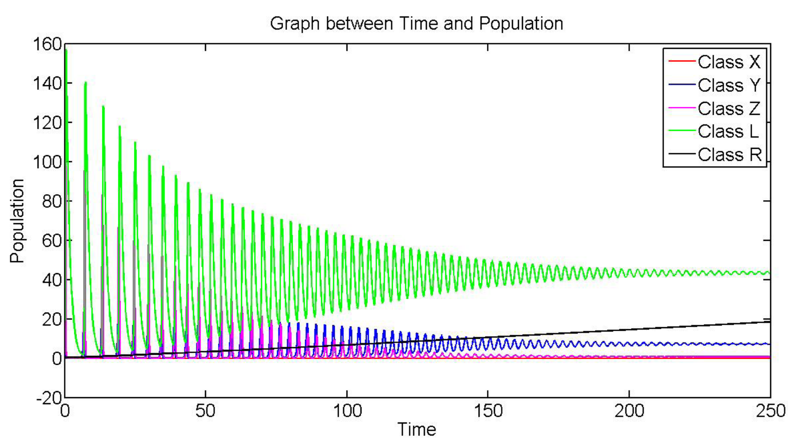

Figure 1:Plot between Time Vs variations in the growth rate Population

$r=6; k=500; \beta=.07; \rho=.25; \lambda_1=.01; \omega=.6; s=5.5; l=600; \lambda=.3; \sigma=.12; \lambda_2=.01; b=.6; p=5;$
$n=700; \pi=.16; c=.9; \lambda_3=.01; c_1=.5; b_1=.6; b_2=.8; b_3=.9; \eta=.8; [X, Y, Z, L, R]=[100, 5, 2, 1, 0].$

6. CONCLUSION

Motivated by the biological epidemic models, formulated a mathematical model, to considering predator–prey model for the attack of worms in WSN. Data and malicious signal are transmitted via neighbor nodes in the network. Sensor nodes operate in two modes sleep and active as per functional requirement to save the energy,

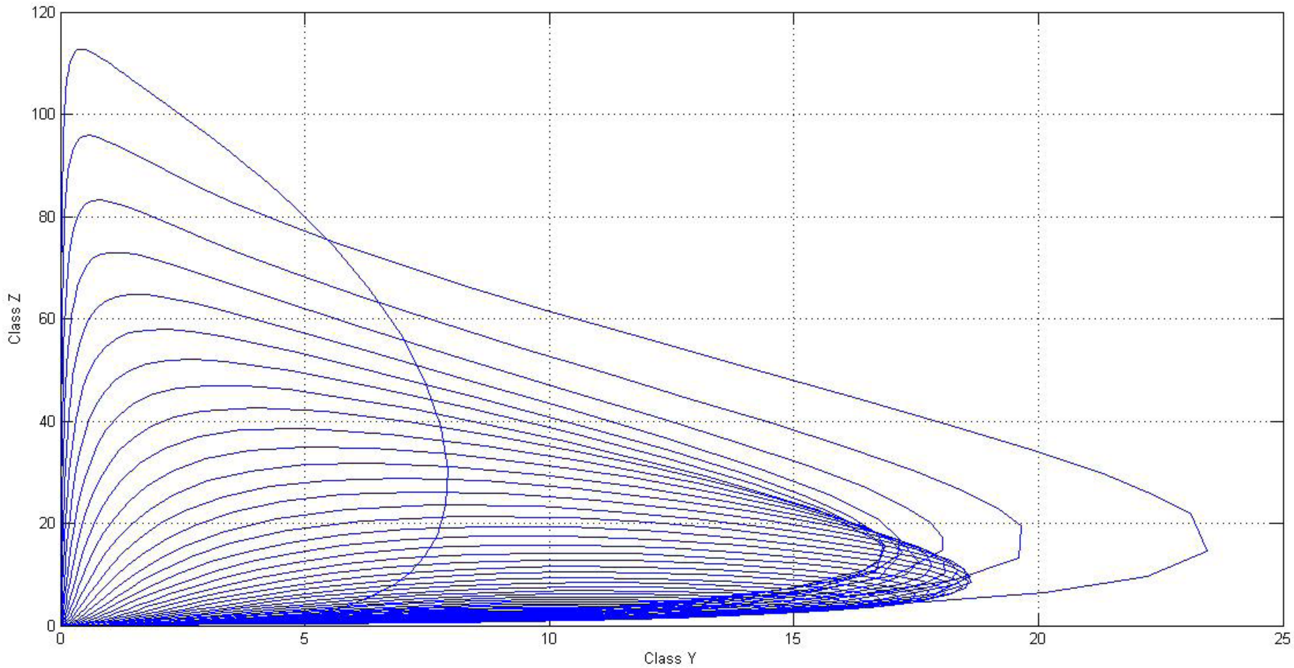

Figure 2: Plot between population class Y Vs population class Z

$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$
$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$

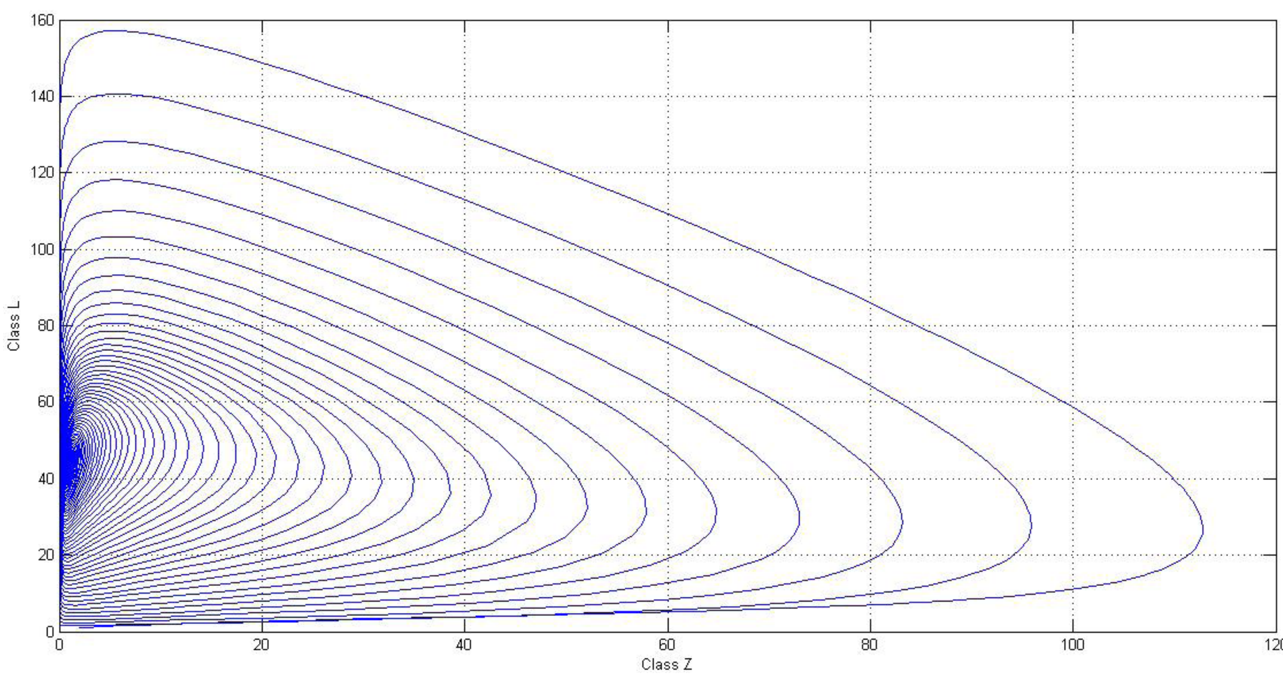

Figure 3: Plot between population class Z Vs. population class L

$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$
$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$

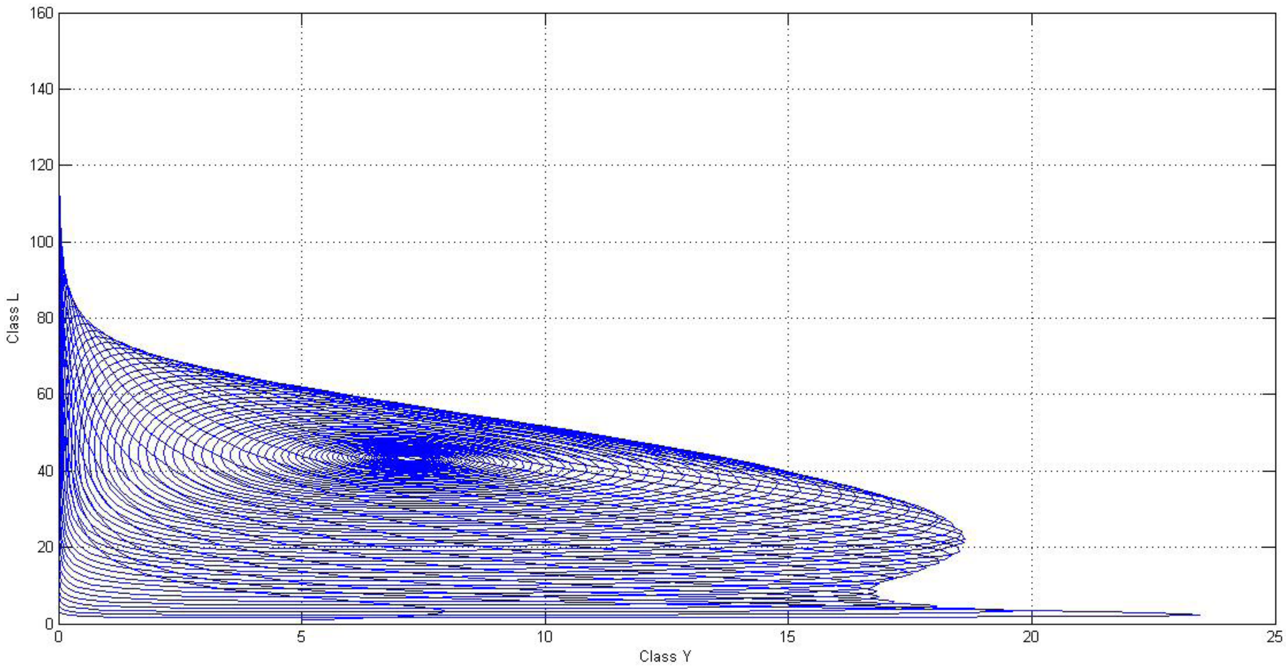

Figure 4: Plot between population class Y Vs population class L

$$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$$
$$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$$

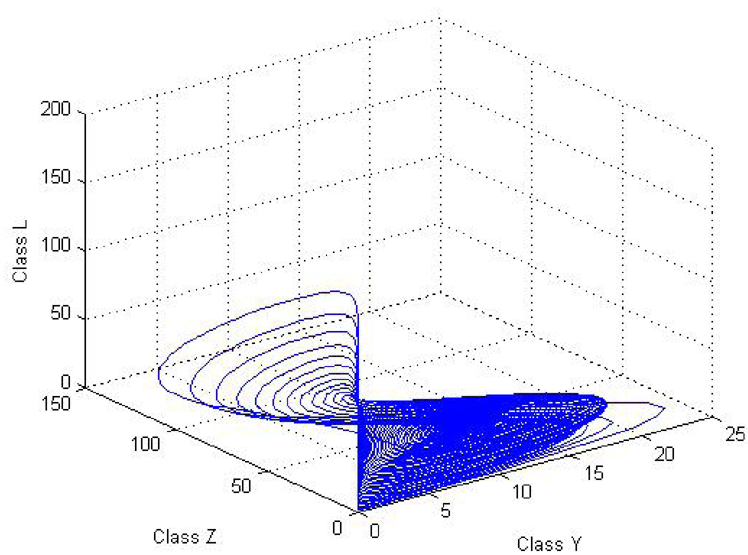

Figure 5: Phase plot of the system among population class Y, Z and L

$$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$$
$$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$$

that means low consumption in sleep mode and energy consumed in active mode.Taking the consideration of two operational modes, we calculated the various equilibrium points and their stability for the projected model. Applying ODE to find stability and equilibrium points of the model under user defined region. Simulation of the system by using MATLAB is conducted out to authenticate the model.Stability of remaining equilibrium points and persistence of model are discussed in future.

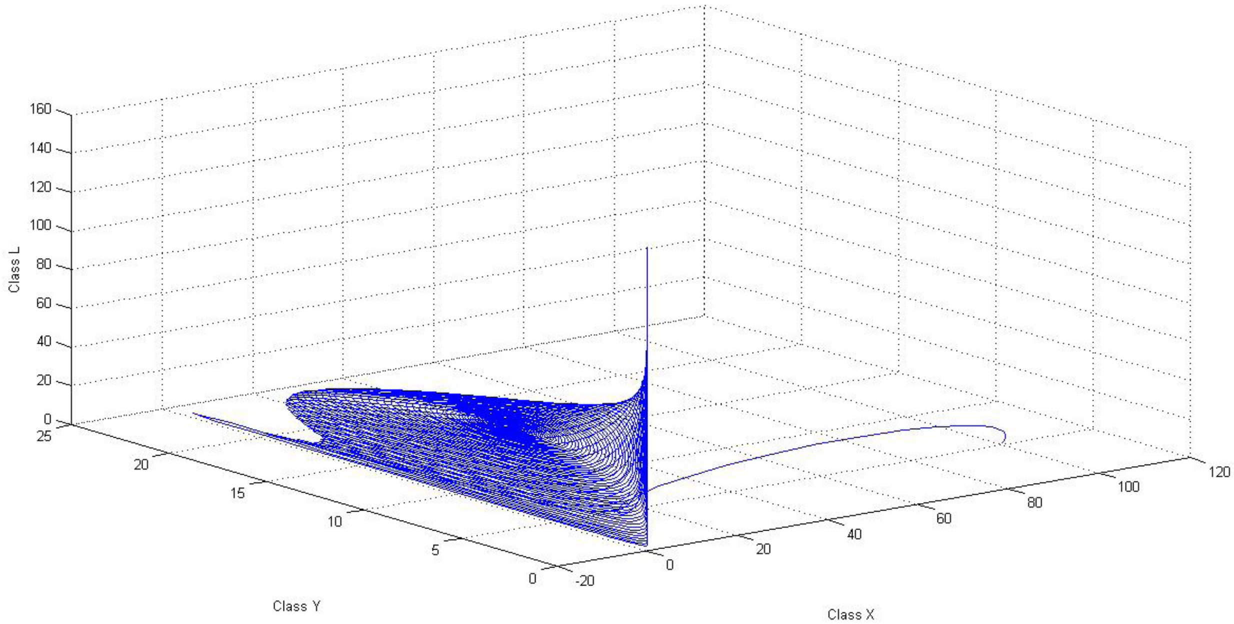

Figure 6: Phase plot of the system among population class X,Y and L

$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$
$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$

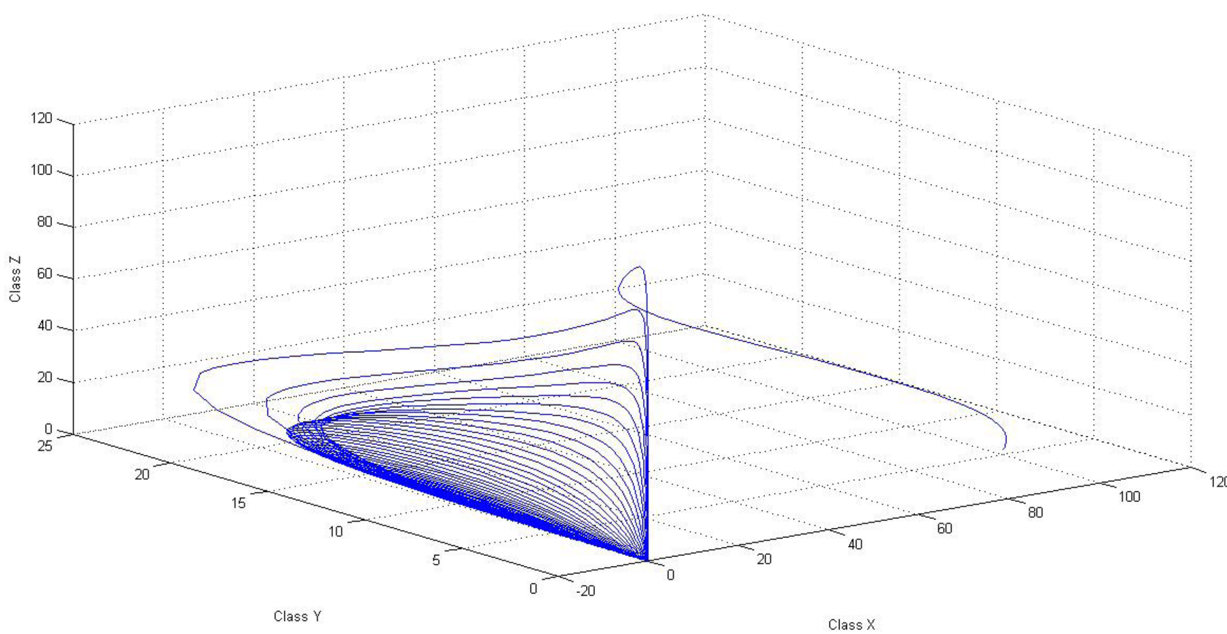

Figure 7: Phase plot of the system among population class X,Y and Z

$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$
$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$

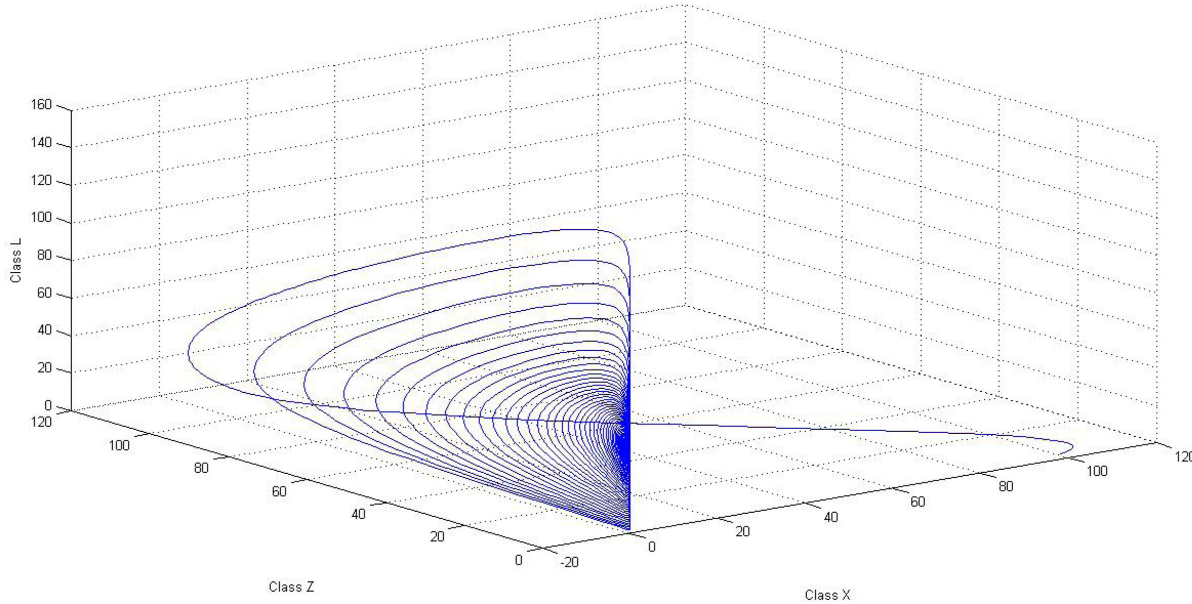

Figure 8: Phase plot of the system among population class X,Z and L

$$r = 6; k = 500; \beta = .07; \rho = .25; \lambda_1 = .01; \omega = .6; s = 5.5; l = 600; \lambda = .3; \sigma = .12; \lambda_2 = .01; b = .6; p = 5;$$
$$n = 700; \pi = .16; c = .9; \lambda_3 = .01; c_1 = .5; b_1 = .6; b_2 = .8; b_3 = .9; \eta = .8; [X, Y, Z, L, R] = [100, 5, 2, 1, 0].$$

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