IJMMS, Vol. 13, No. 1, (January-June 2017):85-98
ISSN: 0973-3329

COMMON FIXED POINT THEOREMS FOR SUB COMPATIBLE AND SUB SEQUENTIALLY CONTINUOUS MAPS IN INTUITIONISTIC FUZZY METRIC SPACE

Neena Vijaywargi*, Vijay Gupta** and Rajesh Shrivastava*

Abstract

Here we have extended the fixed point results for sub compatible and subsequential continuous mappings in intuitionistic fuzzy metric spaces. In this paper we have derived common fixed point for mappings under contractive conditions.

2000 Mathematics Subject Classification: *47H10, 54H25.*

Keywords: *Intuitionistic fuzzy metric space, common fixed point, sub compatible and sub sequentially continuous maps, coincidence point.*

1. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov [4] as a generalization of fuzzy sets which was introduced by L.A Zadeh [17] which laid the foundation of fuzzy mathematics. Fuzzy set theory has applications in mathematical modelling, engineering sciences, medical sciences communication etc. Park [9] in 2004, introduced a notion of intuitionistic fuzzy metric spaces, which is based on the idea of intuitionistic fuzzy sets and concept of fuzzy metric space given by George et al [5]. Kramosil and Michalek [8] introduced the notion of fuzzy metric space to the fuzzy situation. Using the idea of intuitionistic fuzzy sets, Alaca et al [2] defined the notion of IFM-space as Park [10] with the help of continuous *t*-norms and continuous *t*-conorms as a generalization of fuzzy metric

* Research Scholar , UIT, RGPV, Bhopal (M.P.), India. *Email: neenavijay09 @gmail.com*

** Head, Department of Mathematics, UIT, RGPV. Bhopal (M.P.), India. *Email: vkgupta_12873 @rediffmail.com*

*** Professor, Department of Mathematics, Govt. Science and commerce College Benazir Bhopal (M.P.), India. *Email: rajeshraju0101@rediffmail.com*

space. Turkoglu et al. [16] in 2006 studied the notion of compatible mappings in intuitionistic fuzzy metric space.

Some necessary and sufficient conditionswere obtained by Regan and Abbas [1] for the existence of common fixed point in fuzzy metric spaces .The idea of implicit function for proving a common fixed point theorem was introduced by Popa ([11]-[12]).

Jungck [7] in 1986 introduced the notion of compatible maps for a pair of self mappings. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the converse is not true.

In this paper, we introduce new concepts of subcompatibility and subsequential continuity using the coincidence point in intuitionistic fuzzy metric space.

2. PRELIMINARIES

Definition 2.1[13]: A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \epsilon [0,1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \epsilon [0,1]$

Definition 2.2[13]: A binary operation $\lozenge: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-norm if $\lozenge$ is satisfying the following conditions:

(i) $\lozenge$ is commutative and associative;

(ii) $\lozenge$ is continuous;

(iii) $a \lozenge 0 = a \; for \; all \; a\epsilon [0,1]$;

(iv) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \epsilon [0,1]$

Definition 2.3[2]: A 5 – tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t- norm , $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

(i) $M(x, y, t) + N(x, y, t) \leq 1$; for all $x, y \epsilon X$ and $t > 0$

(ii) $M(x, y, 0) = 0$; for all $x, y \epsilon X$;

(iii) $M(x, y, t) = 1 \; for \; all \; t > 0 \; if \; and \; only \; if \; x = y$;

(iv) $M(x, y, t) = M(y, x, t)$; for all $x, y \epsilon X$ and $t > 0$;

(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ $for\ all\ x,y,z \in X\ and\ s,t > 0$;

(vi) $M(x,y,.) : [0,\infty) \to [0,1]$ $is\ continuous$;

(vii) $lim_{t\to\infty} M(x,y,t) = 1$ $for\ all\ x,y\ in\ X\ and\ t > 0$;

(viii) $N(x,y,0) = 1$ for all $x,y \in X$;

(ix) $N(x,y,t) = 0$ $for\ all\ x,y \in X\ and\ t > 0\ if\ and\ only\ if\ x = y$;

(x) $N(x,y,t) = N(y,x,t)$ $for\ all\ x,y \in X\ and\ t > 0$;

(xi) $N(x,y,t) \lozenge N(y,z,s) \geq N(x,z,t+s) forall\ x,y,z \in X\ and\ s,t > 0$;

(xii) $N(x,y,.) : [0,\infty) \to [0,1]$ $is\ continuous\ for\ all\ x,y \in X$;

(xiii) $lim_{t\to\infty} N(x,y,t) = 0$ $for\ all\ x,y \in X\ and\ t > 0$;

Then (M,N) is called an intuitionistic fuzzy metric space. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between *x* and *y* with respect to *t*, respectively.

Remark 2.1[2]: Every fuzzy metric space $(X,M,*)$ *is* an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\lozenge)$ such that *t*-norm $*$ and *t*-conorm $\lozenge$ are associated as

$$x \lozenge y = 1 - \big((1-x)*(1-y)\big)\ for\ all\ x,y \in X$$

Example 2.1[9]: Let *(X,d)* be a metric space, define *t*-norm $a * b = Min\{a,b\}$ and *t*-conorm

$a \lozenge b = Max\{a,b\}$ and for all $x,y \in X$ and $t > 0$,

$$M_d(x,y,t) = \frac{t}{t+d(x,y)},\ N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then *(X, M, N , *, ◊)* is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric *(M,N)* induced by the metric d the standard intuitionistic fuzzy metric.

Remark 2.2[2]: In intuitionistic fuzzy metric space $(X,M,N,*,\lozenge)$, $M(x,y,.)$ is non decreasing and $N(x,y,.)$ is non- increasing for all $x,y \in X$.

Definition 2.4[2]: Let $(X,M,N,*,\lozenge)$ is an intuitionistic fuzzy metric space then

Sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$lim_{n\to\infty} M\big(x_{n+p},x_n,t\big) = 1,\ lim_{n\to\infty} N\big(x_{n+p},x_n,t\big) = 0$$

A Sequence $\{x_n\}$ in X is said to be Convergent to a point $x \in X$ if ,for all $t > 0$,

$$lim_{n\to\infty} M(x_n,x,t) = 1,\ lim_{n\to\infty} N(x_n,x,t) = 0\ .$$

Since $*$ and $\Diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of definition (2.3), respectively.

Definition 2.5[2]: An intuitionistic fuzzy metric space *(X,M,N, *, ◊)* is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 1[2]: let $X = \left\{\frac{1}{n} : n\epsilon N\right\} \cup \{0\}$ and let $*$ be the continuous t-norm and $\Diamond$ be the continuous t-conorm defined by $a * b = ab$ and $a \Diamond b = min\{1, a + b\}$ respectively, for all $a, b\epsilon[0,1]$. For each $t\epsilon(0, \infty)$ and $x, y\epsilon X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, t > 0 \\ 0, t = 0 \end{cases} \; and \; N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, t > 0 \\ 1, t = 0. \end{cases}$$

Clearly, $(X, M, N,*,\Diamond)$ is intuitionistic fuzzy metric space.

Definition 2.6[14]: Let A and B be mappings from an intuitionistic fuzzy metric space $(X, M, N,*,\Diamond)$ into itself. The maps A and B are said to be compatible if , for all $t > 0$,

$$lim_{n\to\infty} M\big(ABx_{n,} BAx_n, t\big) = 1 \, , lim_{n\to\infty} N\big(ABx_{n,} BAx_n, t\big) = 0 \, .$$

Whenever x_n is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = x \; for \; some \; x\epsilon X.$

Definition 2.7[16]: Two self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N,*,\Diamond)$ is said to be non-compatible if there exists at least one sequence $\{x_n\}$ such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = z \; for \; some \; z \; in \; X \; but \; neither$$

$$lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1 \, , and \; lim_{n\to\infty} N(ABx_n, BAx_n, t) \neq 0$$

Or the limit does not exists

Definition 2.8[5]: Let $(X, M, N,*,\Diamond)$ be an intuitionistic fuzzy metric space. Let A and B be self maps on X. Then a point x in X is called a coincidence point of A and B iff $Ax = Bx$. In this case, $w = Ax = Bx$ is called a point of coincidence of A and B.

In 1996, Jungck [6] introduced the notion of weakly compatible maps as follows.

Definition 2.9[7]: A pair of self mappings (A, B) of a intuitionistic fuzzy metric space $(X, M, N,*,\Diamond)$ is said to be weakly compatible if they commute at their coincidence points i.e $Ax = Bx$ for some x in X, then $ABx = BAx$.

It is easy to see that two compatible maps are weakly compatible but converse of this is not true.

Lemma(2.1): Let $(X, M, N,*,\lozenge)$ be an intuitionistic fuzzy metric space. Let A and B be self maps on X and let A and B have a unique point of coincidence, $w = Ax = Bx$, then w is the unique common fixed point of A and B.

Proof: Since A and B are owc, there exists a point x in X such that $Ax = Bx = w$ and $ABx = BAx$. Thus, $AAx = ABx = BAx$, which say that AAx is also a point of coincidence of A and B.

Since the point of coincidence $w = Ax$ is unique by hypothesis, $BAx = AAx = Ax$, and $w = Ax$ is a common fixed point of A and B.

Moreover, if z is any common fixed point of A and B, then $z = Az = Bz = w$ by the uniqueness of the point of coincidence.

Lemma(2.2)[16]: Let $(X, M, N,*,\lozenge)$ be an intuitionistic fuzzy metric space and for all x, y in X , $t > 0$and if there exists a number $h\epsilon(0,1)$

$$M(x, y, ht) \geq M(x, y, t) and\ N(x, y, ht) \leq N(x, y, t), then\ x = y$$

Definition2.10[15]: Two self mappings p and f of an intuitionistic fuzzy metric space $(X, M, N,*,\lozenge)$ are said to be occasionally weakly compatible(owc) iff there is a point x in X which is coincidence point of p and f at which p and f commute.

In this paper we weaken the above notion by introducing a new concept called subcompatibility for intuitionistic fuzzy metric space defined by H. Bouhadjera et. al., [6] in metric space as follows:

Definition (2.11): Self mappings p and f of a fuzzy metric space $(X, M,*)$are said to be subcompatible if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = z, , z \in X$ and satisfy $\lim_{n\to\infty} M(pfx_n, fpx_n, t) = 1$.

Two owc mappings are subcompatible, however the converse is not true in general. Here we introduce subsequential continuity in intuitionistic fuzzy metric space which weaken the concept of reciprocal continuity which was introduced by H. Bouhadjera et. al., [6] in metric space as follows.

Definition (2.12)[6]: Self mappings p and f of a fuzzy metric space $(X, M,*)$ are said to be subsequentially continuous if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = z,\ z \in X$ and satisfy $\lim_{n\to\infty} pfx_n = pz$ and $\lim_{n\to\infty} fpx_n = fz$

It is clear if p and f are continuous or reciprocally continuous then they are obviously subsequentially continuous. However, the converse is not true in general.

Definition (2.13): A function $\phi: [0, \infty) \to [0, \infty)$ is said to be a ϕ function if it satisfies the following conditions:

(i) $\phi\ (t) =\ 0$ if and only if $t = 0$;

(ii) $\phi\ (t)$ is strictly increasing and $\phi\ (t) \to \infty$ as $t \to \infty$;

(iii) ϕ is left continuous in $(0, \infty)$;

(iv) ϕ is continuous at 0.

3. IMPLICIT RELATIONS [3]

(a) Let $(\boldsymbol{\Phi})$ be the set of all real continuous functions $\phi: (R^+)^6 \to R^+$ satisfying the condition

$$\phi(u, u, v, v, u, u) \geq 0 \text{ imply } u \geq v, \text{ for all } u, v \in [0,1].$$

And

Let $(\boldsymbol{\Psi})$ be the set of all real continuous functions $\Psi: (R^+)^6 \to R^+$ satisfying the condition

$$\Psi(u, u, v, v, u, u) \leq 1 \text{ imply } u \leq v, \text{ for all } u, v \in [0,1].$$

(b) Let $(\boldsymbol{\Phi})$ be the set of real continuous functions $\phi: (R^+)^5 \to R^+$ satisfying the condition

$$\phi(u, u, v, u, u) \geq 0 \text{ imply } u \geq v, \text{ for all } u, v \in [0,1].$$

And

Let $(\boldsymbol{\Psi})$ be the set of real continuous functions $\Psi: (R^+)^5 \to R^+$ satisfying the condition

$$\Psi(u, u, v, u, u) \leq 1 \text{ imply } u \leq v, \text{ for all } u, v \in [0,1].$$

4. MAIN RESULTS

Theorem 4.1: Let p, q, f and g be four self maps of intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ defined by $t * t \geq 1$ for all $t \in [0,1]$. *If the* pairs (p, f) and (q, g) are subcompatible and subsequentially continuous, then

(a) p and f have a coincidence point.

(b) q and g have a coincidence point.

(c) For some $\phi \in \boldsymbol{\Phi}$ and $\Psi \in \boldsymbol{\Psi}$ for all $x, y \in X$ and every $t > 0$,

$$\phi\begin{Bmatrix} M(px, qy, t), M(fx, gy, t), M(fx, px, t), \\ M(gy, qy, t), M(fx, qy, t), M(gy, px, t) \end{Bmatrix} \geq 0$$

$$\Psi\begin{Bmatrix} N(px, qy, t), N(fx, gy, t), N(fx, px, t), \\ N(gy, qy, t), N(fx, qy, t), N(gy, px, t) \end{Bmatrix} \leq 1$$

Then p, q, f and g have a unique common fixed point.

Proof: Since the pairs (p, f) and (q, g) are subcompatible and subsequentially continuous, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z\epsilon X$ and

satisfy $lim_{n\to\infty} M(pfx_n, fpx_n, t) = M(pz, fz, t) = 1$

$lim_{n\to\infty} qy_n = lim_{n\to\infty} gy_n = z', z'\epsilon X$ *and*

satisfy $lim_{n\to\infty} M(qgy_n, gqy_n, t) = M(qz', gz', t) = 1$

$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z\epsilon X$ and

satisfy $lim_{n\to\infty} N(pfx_n, fpx_n, t) = N(pz, fz, t) = 0$

$lim_{n\to\infty} qy_n = lim_{n\to\infty} gy_n = z', z'\epsilon X$ and

satisfy $lim_{n\to\infty} N(qgy_n, gqy_n, t) = N(qz', gz', t) = 0$

Therefore, $pz = fz$ and $qz' = gz'$; that is, z is a coincidence point of p and f and $z'$is a coincidence point of q and g.

Now we prove $z = z'$

Put $x = x_n$ and $y = y_n$ in inequality (c), we get

$$\phi \begin{Bmatrix} M(px_n, qy_n, t), M(fx_n, gy_n, t), M(fx_n, px_n, t), \\ M(gy_n, qy_n, t), M(fx_n, qy_n, t), M(gy_n, px_n, t) \end{Bmatrix} \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\phi\{M(z, z', t), M(z, z', t), M(z, z, t), M(z', z', t), M(z, z', t)M(z', z, t)\} \geq 0$$

$$\phi\{M(z, z', t), M(z, z', t), 1,1, M(z, z', t)M(z', z, t)\} \geq 0$$

And

$$\Psi \begin{Bmatrix} N(px_n, qy_n, t), N(fx_n, gy_n, t), N(fx_n, px_n, t), \\ N(gy_n, qy_n, t), N(fx_n, qy_n, t), N(gy_n, px_n, t) \end{Bmatrix} \leq 1$$

Taking the limit as $n \to \infty$, we get

$$\Psi\{N(z, z', t), N(z, z', t), N(z, z, t), N(z', z', t), N(z, z', t), N(z', z, t)\} \leq 1$$

$$\Psi\{N(z, z', t), N(z, z', t), 0,0, N(z, z', t), N(z', z, t)\} \leq 1$$

In view of $\mathbf{3(a)}$ we get $z = z'$

Again, we claim that $pz = z$

Substitute $x = z$ and $y = y_n$ in inequality (c), we get

$$\phi \begin{Bmatrix} M(pz, qy_n, t), M(fz, gy_n, t), M(fz, pz, t), \\ M(gy_n, qy_n, t), M(fz, qy_n, t), M(gy_n, pz, t) \end{Bmatrix} \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\phi \begin{Bmatrix} M(pz, z', t), M(pz, z', t), M(pz, pz, t), \\ M(z', z', t), M(pz, z', t), M(z', pz, t) \end{Bmatrix} \geq 0$$

$$\phi\{M(pz, z', t), M(pz, z', t), 1,1, M(pz, z', t), M(z', pz, t)\} \geq 0$$

And

$$\Psi\begin{Bmatrix} N(pz,qy_n,t),N(fz,gy_n,t),N(fz,pz,t), \\ N(gy_n,qy_n,t),N(fz,qy_n,t),N(gy_n,pz,t) \end{Bmatrix} \le 1$$

Taking the limit as $n \to \infty$, we get

$$\Psi\begin{Bmatrix} N(pz,z',t),N(pz,z',t),N(pz,pz,t), \\ N(z',z',t),N(pz,z',t),N(z',pz,t) \end{Bmatrix} \le 1$$

$$\Psi\{N(pz,z',t),N(pz,z',t),0,0,N(pz,z',t),N(z',pz,t)\} \le 1$$

In view of $\mathbf{3}(\mathbf{a})$ we get $pz = z' = z$

Again we claim that $qz = z$

Substitute $x = z$ and $y = z$ in inequality (c) , we get

$$\phi\begin{Bmatrix} M(pz,qz,t),M(fz,gz,t),M(fz,pz,t), \\ M(gz,qz,t),M(fz,qz,t),M(gz,pz,t) \end{Bmatrix} \ge 0$$

$$\phi\{M(z,qz,t),M(z,qz,t),M(z,z,t),M(qz,qz,t),M(z,qz,t),M(qz,z,t)\} \ge 0$$

$$\phi\{M(z,qz,t),M(z,qz,t),1,1,M(z,qz,t),M(qz,z,t)\} \ge 0$$

And

$$\Psi\begin{Bmatrix} N(pz,qz,t),N(fz,gz,t),N(fz,pz,t), \\ N(gz,qz,t),N(fz,qz,t),N(gz,pz,t) \end{Bmatrix} \le 0$$

$$\Psi\{N(z,qz,t),N(z,qz,t),N(z,z,t),N(qz,qz,t),N(z,qz,t),N(qz,z,t)\} \le 1$$

$$\Psi\{N(z,qz,t),N(z,qz,t),0,0,N(z,qz,t),N(qz,z,t)\} \le 1$$

In view of$\mathbf{3}(\mathbf{a})$ we get $z = qz = gz$

Therefore, $z = pz = qz = fz = gz$, that is z is common fixed point of p, q, f and g.

Uniqueness: Let w be another common fixed point of p, q, f and g.

Then $pw = qw = fw = gw = w$

Put $x = z$ and $y = w$ in inequality (c), we get

$$\phi\begin{Bmatrix} M(pz,qw,t),M(fz,gw,t),M(fz,pz,t), \\ M(gw,qw,t),M(fz,qw,t),M(gw,pz,t) \end{Bmatrix} \ge 0$$

$$\phi\{M(z,w,t),M(z,w,t),M(z,z,t),M(w,w,t),M(z,w,t),M(w,z,t)\} \ge 0$$

$$\phi\{M(z,w,t),M(z,w,t),1,1,M(z,w,t),M(w,z,t)\} \ge 0$$

And

$$\Psi\begin{Bmatrix} N(pz,qw,t),N(fz,gw,t),N(fz,pz,t), \\ N(gw,qw,t),N(fz,qw,t),N(gw,pz,t) \end{Bmatrix} \le 1$$

$$\Psi\{N(z,w,t),N(z,w,t),N(z,z,t),N(w,w,t),N(z,w,t),N(w,z,t)\} \le 1$$

$$\Psi\{N(z,w,t),N(z,w,t),0,0,N(z,w,t),N(w,z,t)\} \le 1$$

In view of **3(a)** we get $z = w$

Therefore, uniqueness follows.

If we take $f = g$ in theorem 4.1, we get the following result:

Corollary 4.2: Let p, q, f be three self maps of intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ with continuous t norm $*$ defined by $t * t \geq 1$ for all $t \epsilon [0,1]$. If the pairs (p, f) *and* (q, f) are subcompatible and subsequentially continuous, then

(a) p and f have a coincidence point.

(b) q and f have a coincidence point.

(c) For some $\phi \epsilon \boldsymbol{\Phi}$ and $\Psi \epsilon \boldsymbol{\Psi}$ for all $x, y \epsilon X$ and every $t > 0$,

$$\phi \begin{Bmatrix} M(px, qy, t), M(fx, fy, t), M(fx, px, t), \\ M(fy, qy, t), M(fx, qy, t), M(fy, px, t) \end{Bmatrix} \geq 0$$

And

$$\Psi \begin{Bmatrix} N(px, qy, t), N(fx, fy, t), N(fx, px, t), \\ N(fy, qy, t), N(fx, qy, t), N(fy, px, t) \end{Bmatrix} \leq 1$$

Then p, q and f have a unique common fixed point.

Theorem 4.3: Let p, q, f and g be four self maps of intuitionistic fuzzy metric spac $(X, M, N, *, \lozenge)$ with continuous t-norm $*$ defined by $t * t \geq 1$ for all $t \epsilon [0,1]$. If the pairs (p, f) and (q, g) are subcompatible and subsequentially continuous, then

(a) p and f have a coincidence point.

(b) q and g have a coincidence point.

(c) For some $\phi \epsilon \boldsymbol{\Phi}$ and $\Psi \epsilon \boldsymbol{\Psi}$ for all $x, y \epsilon X$ and every $t > 0$,

$$\phi \begin{Bmatrix} M(px, qy, t), M(fx, gy, t), \dfrac{M(fx, px, t) + M(gy, qy, t)}{2}, \\ M(fx, qy, t), M(gy, px, t) \end{Bmatrix} \geq 0$$

And $$\Psi \begin{Bmatrix} N(px, qy, t), N(fx, gy, t), \frac{N(fx,px,t) + N(gy,qy,t)}{2}, \\ N(fx, qy, t), N(gy, px, t) \end{Bmatrix} \leq 1.$$

Then p, q and f and g have a unique common fixed point.

Proof: Since the pairs (p, f) and (q, g) are subcompatible and subsequentially continuous, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z \epsilon X \text{ and}$$

satisfy $$lim_{n\to\infty} M(pfx_n, fpx_n, t) = M(pz, fz, t) = 1.$$

$$lim_{n\to\infty} qy_n = lim_{n\to\infty} gy_n = z', z' \epsilon X \text{ and}$$

satisfy $lim_{n\to\infty} M(qgy_n, gqy_n, t) = M(qz', gz', t) = 1.$

$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z\epsilon X$ and

satisfy $lim_{n\to\infty} N(pfx_n, fpx_n, t) = N(pz, fz, t) = 0.$

$lim_{n\to\infty} qy_n = lim_{n\to\infty} gy_n = z', z'\epsilon X$ and

satisfy $lim_{n\to\infty} N(qgy_n, gqy_n, t) = N(qz', gz', t) = 0.$

Therefore, $pz = fz$ and $qz' = gz'$; that is, z is a coincidence point of p and f and $z'$is a coincidence point of q and g.

Now we prove $z = z'$

Put $x = x_n$ *and* $y = y_n$ in inequality (c), we get

$$\phi\left\{\begin{matrix} M(px_n, qy_n, t), M(fx_n, gy_n, t), \frac{M(fx_n, px_n, t)+M(gy_n, qy_n, t)}{2}, \\ M(fx_n, qy_n, t), M(gy_n, px_n, t) \end{matrix}\right\} \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\phi\left\{M(z, z', t), M(z, z', t), \frac{M(z, z, t) + M(z', z', t)}{2}, M(z, z', t), M(z', z, t)\right\} \geq 0$$

$$\phi\{M(z, z', t), M(z, z', t), 1, M(z, z', t), M(z', z, t)\} \geq 0$$

And

$$\Psi\left\{\begin{matrix} N(px_n, qy_n, t), N(fx_n, gy_n, t), \frac{N(fx_n, px_n, t) + N(gy_n, qy_n, t)}{2}, \\ N(fx_n, qy_n, t), N(gy_n, px_n, t) \end{matrix}\right\} \leq 1$$

Taking the limit as $n \to \infty$, we get

$$\Psi\left\{N(z, z', t), N(z, z', t), \frac{N(z, z, t) + N(z', z', t)}{2}, N(z, z', t), N(z', z, t)\right\} \leq 1$$

$$\Psi\{N(z, z', t), N(z, z', t), 0, N(z, z', t), N(z', z, t)\} \leq 1$$

In view of $\mathbf{3(b)}$ we get $z = z'$

Again we claim that $pz = z$.

Substitute $x = z$ and $y = y_n$ in inequality (c), we get

$$\phi\left\{\begin{matrix} M(pz, qy_n, t), M(fz, gy_n, t), \frac{M(fz, pz, t) + M(gy_n, qy_n, t)}{2}, \\ M(fz, qy_n, t), M(gy_n, pz, t) \end{matrix}\right\} \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\phi\begin{Bmatrix} M(pz,z',t), M(pz,z',t), \dfrac{M(pz,pz,t)+M(z',z',t)}{2}, \\ M(pz,z',t), M(z',pz,t) \end{Bmatrix} \geq 0$$

$$\phi\{M(pz,z',t), M(pz,z',t), 1, M(pz,z',t), M(pz,z',t)\} \geq 0$$

And

$$\Psi\begin{Bmatrix} N(pz,qy_n,t), N(fz,gy_n,t), \dfrac{N(fz,pz,t)+N(gy_n,qy_n,t)}{2}, \\ N(fz,qy_n,t), N(gy_n,pz,t) \end{Bmatrix} \leq 1$$

Taking the limit as $n \to \infty$, we get

$$\Psi\begin{Bmatrix} N(pz,z',t), N(pz,z',t), \dfrac{N(pz,pz,t)+N(z',z',t)}{2}, \\ N(pz,z',t), N(z',pz,t) \end{Bmatrix} \leq 1$$

$$\Psi\{N(pz,z',t), N(pz,z',t), 0, N(pz,z',t), N(pz,z',t)\} \leq 1$$

In view of **3**(**b**) we get $pz = z' = z$.

Again we claim that $qz = z$.

Substitute $x = z$ and $y = z$ inequality (c), we get

$$\phi\begin{Bmatrix} M(pz,qz,t), M(fz,gz,t), \dfrac{M(fz,pz,t)+M(gz,qz,t)}{2}, \\ M(fz,qz,t), M(gz,pz,t) \end{Bmatrix} \geq 0$$

$$\phi\begin{Bmatrix} M(z,qz,t), M(z,qz,t), \dfrac{M(z,z,t)+M(qz,qz,t)}{2}, \\ M(z,qz,t), M(qz,z,t) \end{Bmatrix} \geq 0$$

$$\phi\{M(z,qz,t), M(z,qz,t), 1, M(z,qz,t), M(qz,z,t)\} \geq 0$$

And

$$\Psi\begin{Bmatrix} N(pz,qz,t), N(fz,gz,t), \frac{N(fz,pz,t)+N(gz,qz,t)}{2}, \\ N(fz,qz,t), N(gz,pz,t) \end{Bmatrix} \leq 1$$

$$\Psi\left\{N(z,qz,t), N(z,qz,t), \frac{N(z,z,t)+N(qz,qz,t)}{2}, N(z,qz,t), N(qz,z,t)\right\} \leq 1$$

$$\Psi\{N(z,qz,t), N(z,qz,t), 0, N(z,qz,t), N(qz,z,t)\} \leq 1$$

In view of **3**(**b**) we get $z = qz = gz$

Therefore, $z = pz = qz = fz = gz$; that is z is common fixed point of p, q, f and g.

Uniqueness: Let w be another common fixed point of p, q, f and g.

Then $pw = qw = fw = gw = w$

Put $x = z$ and $y = w$ in inequality (c), we get

$$\phi\left\{\begin{matrix} M(pz,qw,t), M(fz,gw,t), \dfrac{M(fz,pz,t)+M(gw,qw,t)}{2}, \\ M(fz,qw,t), M(gw,pz,t) \end{matrix}\right\} \geq 0$$

$$\phi\left\{\begin{matrix} M(z,w,t), M(z,w,t), \dfrac{M(z,z,t)+M(w,w,t)}{2}, \\ M(z,w,t), M(w,z,t) \end{matrix}\right\} \geq 0$$

$$\phi\{M(z,w,t), M(z,w,t), 1, M(z,w,t), M(z,w,t)\} \geq 0$$

And

$$\Psi\left\{\begin{matrix} N(pz,qw,t), N(fz,gw,t), \dfrac{N(fz,pz,t)+N(gw,qw,t)}{2}, \\ N(fz,qw,t), N(gw,pz,t) \end{matrix}\right\} \leq 1$$

$$\Psi\left\{N(z,w,t), N(z,w,t), \frac{N(z,z,t)+N(w,w,t)}{2}, N(z,w,t), N(w,z,t)\right\} \leq 1$$

$$\Psi\{N(z,w,t), N(z,w,t), 0, N(z,w,t), N(z,w,t)\} \leq 1$$

In view of $\mathbf{3}(\mathbf{b})$ we get $z = w$. Therefore, uniqueness follows.

Theorem 4.4: Let p and f be two self maps of intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ with continuous t-norm $*$ defined by $t * t \geq 1$ for all $t \epsilon [0,1]$. If the pair (p, f) is subcompatible and subsequentially continuous, then

(a) p and f have a coincidence point.

(b) For some $\phi \in \boldsymbol{\Phi}$ and $\Psi \in \boldsymbol{\Psi}$ for all $x, y \in X$ and every $t > 0$,

$$\phi\left\{\begin{matrix} M(px,y,t), M(fx,y,t), M(fx,px,t), \\ M(y,y,t), M(fy,x,t), M(x,py,t) \end{matrix}\right\} \geq 0$$

$$\Psi\left\{\begin{matrix} N(px,y,t), N(fx,y,t), N(fx,px,t), \\ N(y,y,t), N(fy,x,t), N(x,py,t) \end{matrix}\right\} \leq 1.$$

Then p and f have a unique common fixed point.

Proof: Since the pair (p, f) is subcompatible and subsequentially continuous, then there exists a sequence $\{x_n\}$ in X such that

$$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z \epsilon X \text{ and}$$

satisfy $$lim_{n\to\infty} M(pfx_n, fpx_n, t) = M(pz, fz, t) = 1$$

$$lim_{n\to\infty} px_n = lim_{n\to\infty} fx_n = z, z \epsilon X \text{ and}$$

satisfy $lim_{n\to\infty} N(pfx_n, fpx_n, t) = N(pz, fz, t) = 0$

Therefore, $pz = fz$ that is, z is a coincidence point of p and f.

Again, we claim that $pz = z$

Substitute $x = x_n$ and $y = z$ in inequality (c), we get

$$\phi \begin{Bmatrix} M(px_n, z, t), M(fx_n, z, t), M(fx_n, px_n, t), \\ M(z, z, t), M(fz, x_n, t), M(x_n, pz, t) \end{Bmatrix} \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\phi \begin{Bmatrix} M(z, z, t), M(z, z, t), M(z, z, t), \\ M(z, z, t), M(pz, z, t), M(z, pz, t) \end{Bmatrix} \geq 0$$

And

$$\Psi \begin{Bmatrix} N(px_n, z, t), N(fx_n, z, t), N(fx_n, px_n, t), \\ N(z, z, t), N(fz, x_n, t), N(x_n, pz, t) \end{Bmatrix} \leq 1$$

Taking the limit as $n \to \infty$, we get

$$\Psi \begin{Bmatrix} M(z, z, t), M(z, z, t), M(z, z, t), \\ M(z, z, t), M(pz, z, t), M(z, pz, t) \end{Bmatrix} \leq 1$$

In view of $\mathbf{3(a)}$ we get $p\,z = z$

$\Rightarrow$ $$pz = fz = z$$

$\Rightarrow$ z is unique common fixed point of p and f.

References

[1] Abbas, M., Altun, I., and Gopal, D. 2009. Common fixedpoint theorems for non compatible mappings in fuzzy metric spaces, Bull. Of Mathematical Analysis and Applications ISSN, 1821-1291.

[2] Alaca C., Turkoglu D. and C. Yildiz C., "Fixed points in intuitionistic fuzzy metric spaces", Chaos, Solitons & Fractals, 29(5) (2006), 1073-1078.

[3] Asha Rani and Sanjay Kumar, Common Fixed point theorems in Fuzzy Metric Space using Implicit Relation, International journal of computer applications(0975-8887), volume 20-No.7, April2011.

[4] Atanassov. K , "Intuitionistic fuzzy sets", Fuzzy Sets and System, 20(1986), 87-96.

[5] George A. and Veeramani P., "On some results in fuzzy metric spaces, Fuzzy", sets and systems 64 (1994), 395-399.

[6] H. Bouhadjera and C. Godet-Thobie, "Common fixed point theorems for pairs of subcompatible maps" arXiv:0906.3159v1[math.FA](2009)

[7] Jungck G., Compatible mappings and common fixed points. International journal. Math. Sci. 9(1986), 771-779.

[8] Kramosil I. and Michalek J.: Fuzzy metric and Statistical metric spaces, Kybernetica, 11(1975) 326-334.

[9] Park J. H., "Intuitionistic fuzzy metric spaces", Chaos, Solitons & Fractals, 22(2004), 1039–1046.

[10] Park J.S and Kim S.Y.: Common fixed point and example in intuitionistic fuzzy metric space, j.k.I.I.S.18(2008), NO.4,524-529.

[11] Popa, V. A general coincidence theorem for compatible multivalued mappings satisfying an implicit relation, Demonsratio Math., 33,(2000) 159-164.

[12] Popa, V. Some fixed point theorems for compatible mappings satisfying on implicit relation, Demonsratio Math., 32 (1999), 157-163.

[13] Schweizer B. and Sklar A., "Statistical metric spaces", Pacific J. Math.10(1960), 313-33.

[14] Sharma Mami, "Fixed point theoremin Fuzzy metric soaces for non-compatible and weakly compatible maps" Journal of Applied and Fundamental Sciences"1(2015), 53-56.

[15] Thagafi Al- M.A. and Shahzad N: Generalized I-nonexpansive selfmaps and invariant Approximations, Acta. Math.Sinica, 24(5)(2008), 867-876.

[16] D. Turkoglu D., C. Alaca, and C.Yildiz: Compatible maps and compatible maps of type (α) and(β) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39(2006), 671-684.

[17] Zadeh, L. A., Fuzzy sets, Inform and Control 8(1965), 338-353.