

State Estimation of Power Systems Incorporating Facts Controllers

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ABSTRACT

This paper investigates a practical approach to incorporate Flexible AC Transmission Systems (FACTS) devices into a Weighted Least Squares (WLS) State Estimation (SE) algorithm. The FACTS devices included are Static Var Compensator (SVC) and Thyristor Controlled Series Compensator (TCSC). These devices are able to control the voltage, the active power and the reactive power flows based on predefined targets. The major procedure includes the modification of Jacobian matrix with the addition of new elements. MATLAB application is used for the development of state estimation algorithm and the simulation results are obtained by considering IEEE 14-bus system incorporating SVC and TCSC.

Keywords: State estimation, weighted least squares, FACTS devices.

I. INTRODUCTION

Power system is a complex interconnected system. The task of securely operating the system has become more challenging. Power system is controlled by system operators from area control centres. As the operating conditions vary during the daily operation, the system operator should ensure that the system is in a normal secure state. Accomplishing this goal requires identification of the operating state by continuous monitoring of the system conditions and determination of the necessary preventive action in case the state is found to be insecure. This security analysis of the power system can be done using Power System State Estimation. The first step in security analysis is to determine the current state of the system. This involves the acquisition of measurements from all parts of the system. Substations are equipped with Remote Terminal Units (RTU) which collect various types of measurements and transmits them to the control centre. Once the data are collected, they are processed to determine the current state of the system. Measurements received at the control centres will include measurements such as bus voltage magnitude, active power flow, reactive power flow real power injections, reactive power injections and line current flows. These raw data and measurements are processed by state estimator in order to filter the measurement noise and detect the gross errors. State estimator solution will provide reliable estimate of the system state based on available measurements and assumed system model. This solution obtained will be then passed to the Energy Management System (EMS) applications such as contingency analysis, automatic generation control, automatic load frequency control economic load dispatching, load forecasting, optimal power flow etc.

Transmission systems are undergoing continuous changes mainly from the strong increase in interconnected power transfers, opening of the market for delivery of cheaper energy and economic and ecological constraints which delay the building of new transmission facilities. Construction of new transmission lines are difficult due to growing power plant capacity and energy demand. Hence new innovative methods are adopted to make the existing transmission system more efficient. The need for more efficient power systems management, and the fast development of power electronics based on new and powerful semiconductor devices, have given rise to innovative technologies, such as FACTS controllers.

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The concept of power system state estimation has been intensively studied since the work of F.C. Schweppe during the late 70s [1-3]. Reviews of the state of the art in state estimation algorithms and comparative studies of numerically robust estimators for power networks can be obtained from [4-5]. Researches have been conducted in power flow analysis containing FACTS controllers which helps in understanding the changes that should be made in Jacobian matrix.[6-7]. Monitoring of the system state requires the integration of FACTS controllers' models into SE algorithms, taking into account their associated measurement set for the correct estimate of the equilibrium point of a power system containing these kinds of controllers [8-13].

II. WLS STATE ESTIMATION ALGORITHM

State estimation makes use of a set of redundant measurements in order to filter out measurement and telemetry errors and find optimal estimate. The measurement may include real and reactive power bus powers, real and reactive power flows, bus voltage magnitudes and line current magnitudes. Consider a case where there are n state variables and m measurements. The WLS estimator will minimize the following objective function [4]

$$J(x) = \sum_{i=1}^m (z_i - h_i(x))^2 / R_{ii} \quad (1)$$

$$x = [z - h(x)]^T R^{-1} [z - h(x)]$$

The steps involved in WLS state estimation algorithm are;

Step 1. Read the network data; measurements and their standard deviations; Form the Ybus matrix.

Step 2. Set iteration count $k = 0$; Initialize the state vector x^k , typically as a flat start.

Step 3. Knowing the measurement equations $h_i(x)$, Compute the measurement Jacobian matrix, $H(x^k)$

Step 4. Calculate the gain matrix, $G(x^k)$, using

$$G(x^k) = H^T(x^k) R^{-1} H(x^k) \quad (2)$$

Step 5. Calculate the right hand side vector t^k , given by

$$t^k = H^T(x^k) R^{-1} [z - h(x^k)] \quad (3)$$

Step 6. Decompose $G(x^k)$ and solve for,

$$G(x^k) \Delta x^k = H^T(x^k) R^{-1} [z - h(x^k)] \quad (4)$$

Update $x^{k+1} = x^k + \Delta x^k$,

Step 7. Test for convergence $\max. |\Delta x^k| \leq \epsilon$?

Step 8. If convergence not met, set $k = k+1$; go to step 3; else stop.

The structure of the Jacobian matrix will be as follows:

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \delta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \delta} & \frac{\partial Q_{flow}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix}$$

III. MODELING OF FACTS CONTROLLERS

(A) Modeling of SVC

SVC is a parallel combination of thyristor controlled reactor with a bank of capacitors. It's a shunt connected variable reactance, which either generates or absorbs reactive power in order to regulate the voltage magnitude at the point where it is connected in the AC network. As an important component for voltage control, it is usually installed at the receiving node of the transmission lines. Fig. 1 shows a SVC model with step down transformer.

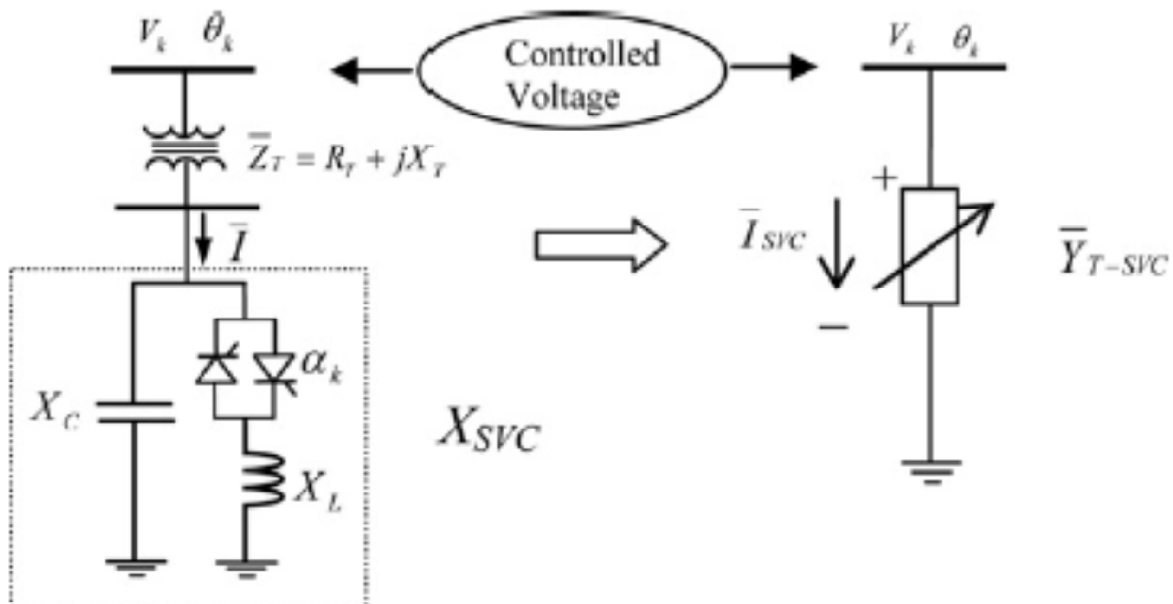


Figure 1: Combined SVC- transformer model

The total admittance of the combined SVC-transformer set as seen from the high-voltage side of the transformer is given by

$$Y_{T_SVC} = G_{T_SVC} + jB_{T_SVC} \quad (5)$$

$$X_{TCR} = \frac{\pi X_L}{2(\pi - \alpha_{SVC}) + \sin 2\alpha_{SVC}} \quad (6)$$

$$X_{EQ} = X_T + X_{SVC}$$

$$G_{T-SVC} = \frac{R_T}{R_T^2 + X_{EQ}^2} \quad \text{and} \quad B_{T-SVC} = \frac{X_{EQ}}{R_T^2 + X_{EQ}^2}$$

The active and reactive powers injected at node k by the single model are:

$$Q_k^{T-SVC} = -V_k^2 B_{T-SVC} \quad \text{and} \quad P_k^{T-SVC} = V_k^2 G_{T-SVC}$$

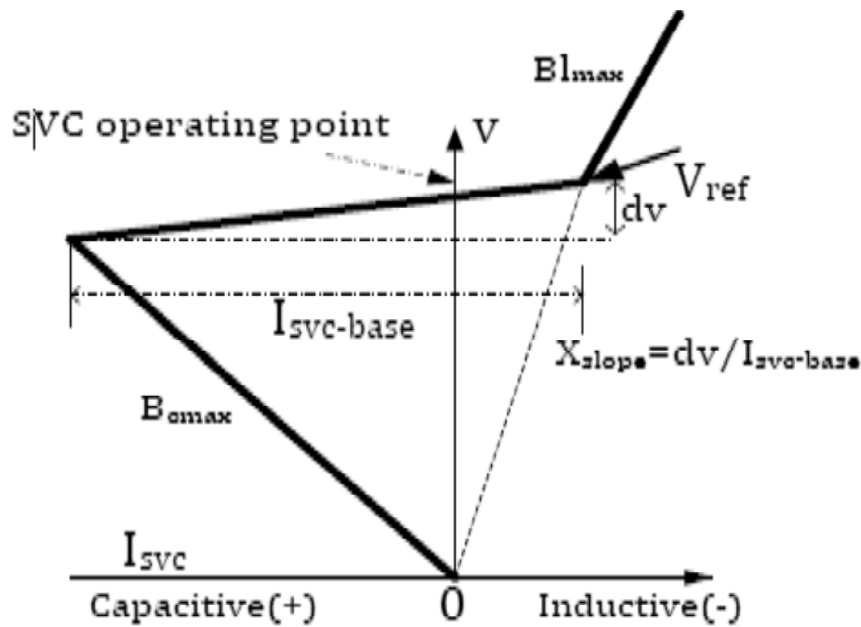


Figure 2: V-I Characteristics of SVC

The SVC can be operated in two different modes:

- (i) In voltage regulation mode wherein, the voltage is regulated with-in limits as explained below.
- (ii) In VAR control mode wherein the SVC susceptance is kept constant.

From V-I curve of SVC,

$$V = V_{ref} + X_S I$$

In regulation range $-B_{Cmax} < B < B_{Lmax}$

(B) Modeling of TCSC

TCSC device is a parallel combination of capacitor in parallel with a Thyristor controlled reactor (TCR). Fig 3 shows the general configuration of a TCSC module with a fundamental frequency TCSC equivalent reactance, as function of the TCSC firing angle, α_{TCSC} , is given by

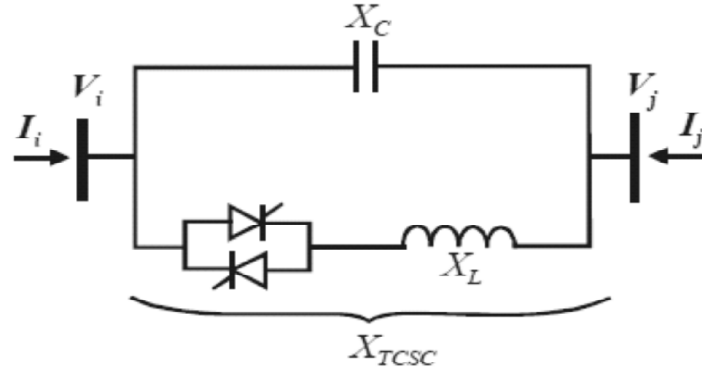


Figure 3: TCSC module

$$\begin{aligned}
 X_{TCSC}(\alpha) &= -\frac{1}{B_{TCSC}(\alpha)} \\
 &= -X_C + C_1(2(\pi - \alpha) + \sin(2(\pi - \alpha))) - \\
 &C_2 \cos^2(\pi - \alpha)(w \tan(w(\pi - \alpha)) - \tan(\pi - \alpha))
 \end{aligned} \quad (8)$$

where

$$C_1 = \frac{X_C + X_{LC}}{\pi} \quad \text{and} \quad C_2 = \frac{4X_{LC}^2}{X_L(\alpha)\pi}$$

The term X_{LC} corresponds to the parallel combination of the TCSC inductive and capacitive fixed reactance, $w = \frac{\omega_0}{\omega}$ where ω is the frequency of the system and $\omega_0 = \frac{1}{\sqrt{LC}}$

The active and reactive powers flowing through the controller from its terminal i to terminal j are given by

$$\begin{aligned}
 p_{ij} &= -V_i V_j B_{TCSC} \sin(\theta_i - \theta_j) \\
 q_{ij} &= -V_i^2 B_{TCSC} + V_i V_j B_{TCSC} \cos(\theta_i - \theta_j)
 \end{aligned} \quad (9)$$

III. MODIFICATION OF JACOBIAN MATRIX

(A) Modification of Jacobian Matrix with the Inclusion of SVC

The detailed formulation of state estimation and the changes made to Jacobian matrix with the inclusion of SVC can be understood by considering a 3-node power system network containing one SVC as shown in Fig.4.

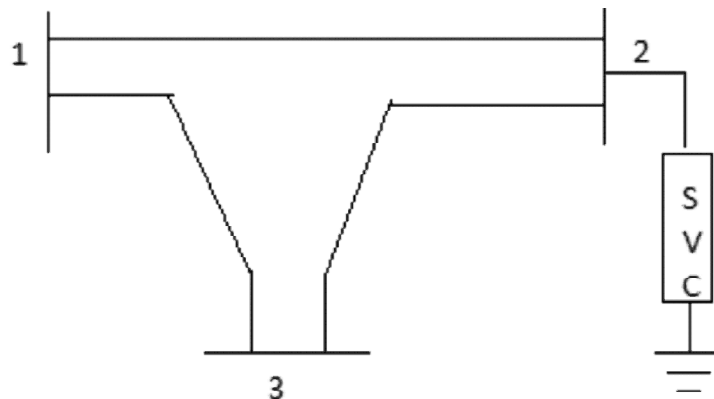


Figure 4: A three-node power system network with SVC

The state variables vector, the measurements vector and the non linear vector function relating the measurements and state variables are given by,

$$\hat{x} = [\theta_2 \theta_3 V_1 V_2 V_3 \alpha_{SVC}]'$$

$$z = [p_{12} p_{13} P_2 q_{12} q_{13} Q_2 V_1 V_2]$$

$$h = [p_{12} p_{13} (p_{12} + p_{24}) q_{12} q_{13} (q_{12} + q_{24}) V_1 V_2]$$

The Jacobian matrix of this system is given by

$$H = \begin{bmatrix} \frac{\partial p_{12}}{\partial \theta_2} & 0 & 0 & \frac{\partial p_{12}}{\partial V_1} & \frac{\partial p_{12}}{\partial V_2} & 0 & 0 \\ 0 & \frac{\partial p_{13}}{\partial \theta_3} & 0 & \frac{\partial p_{13}}{\partial V_1} & 0 & \frac{\partial p_{13}}{\partial V_3} & 0 \\ \frac{\partial P_2}{\partial \theta_2} & 0 & \frac{\partial P_2}{\partial \theta_4} & \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & 0 & \frac{\partial P_2}{\partial \alpha_{SVC}} \\ \frac{\partial q_{12}}{\partial \theta_2} & 0 & 0 & \frac{\partial q_{12}}{\partial V_1} & \frac{\partial q_{12}}{\partial V_2} & 0 & 0 \\ 0 & \frac{\partial q_{13}}{\partial \theta_3} & 0 & \frac{\partial q_{13}}{\partial V_1} & 0 & \frac{\partial q_{13}}{\partial V_3} & 0 \\ \frac{\partial Q_2}{\partial \theta_2} & 0 & \frac{\partial Q_2}{\partial \theta_4} & \frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_2} & 0 & \frac{\partial Q_2}{\partial \alpha_{SVC}} \\ 0 & 0 & 0 & \frac{\partial V_1}{\partial V_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial V_2}{\partial V_2} & 0 & 0 \end{bmatrix}$$

The additional state variable included is the firing angle of SVC given by α_{SVC} .

(B) Modification of Jacobian Matrix with the Inclusion of TCSC

The detailed formulation of state estimation and the changes made to Jacobian matrix with the inclusion of TCSC can be understood by considering a 4-node power system network containing one TCSC as shown in Fig. 5.

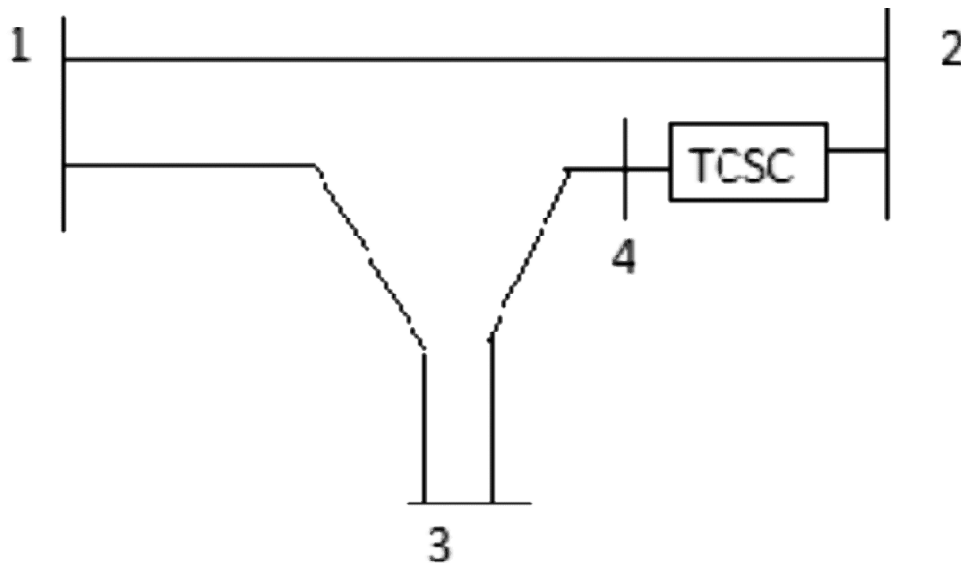


Figure 5: Four-node power system network with TCSC

The state variables vector, the measurements vector and the non linear vector function relating the measurements and state variables are given by

$$\begin{aligned}\hat{x} &= [\theta_2 \theta_3 \theta_4 V_1 V_2 V_3 V_4 \alpha_{TCSC}]^T \\ z &= [p_{12} p_{13} P_2 q_{12} q_{13} Q_2 V_1 V_2 p_{24} q_{24}] \\ h &= [p_{12} p_{13} (p_{21} + p_{24}) q_{12} q_{13} (q_{21} + q_{24}) V_1 V_2 p_{24} q_{24}]\end{aligned}$$

The Jacobian matrix of this system is given by

$$H = \begin{bmatrix} \frac{\partial p_{12}}{\partial \theta_2} & 0 & 0 & \frac{\partial p_{12}}{\partial V_1} & \frac{\partial p_{12}}{\partial V_2} & 0 & 0 & 0 \\ 0 & \frac{\partial p_{13}}{\partial \theta_3} & 0 & \frac{\partial p_{13}}{\partial V_1} & 0 & \frac{\partial p_{13}}{\partial V_3} & 0 & 0 \\ \frac{\partial P_2}{\partial \theta_2} & & \frac{\partial P_2}{\partial \theta_4} & \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & 0 & \frac{\partial P_2}{\partial V_4} & \frac{\partial P_2}{\partial \alpha} \\ \frac{\partial q_{12}}{\partial \theta_2} & 0 & 0 & \frac{\partial q_{12}}{\partial V_1} & \frac{\partial q_{12}}{\partial V_2} & 0 & 0 & 0 \\ 0 & \frac{\partial q_{13}}{\partial \theta_3} & 0 & \frac{\partial q_{13}}{\partial V_1} & 0 & \frac{\partial q_{13}}{\partial V_3} & 0 & 0 \\ \frac{\partial Q_2}{\partial \theta_2} & 0 & \frac{\partial Q_2}{\partial \theta_4} & \frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_2} & 0 & \frac{\partial Q_2}{\partial V_4} & \frac{\partial Q_2}{\partial \alpha} \\ 0 & 0 & 0 & \frac{\partial V_1}{\partial V_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial V_2}{\partial V_2} & 0 & 0 & 0 \\ \frac{\partial p_{24}}{\partial \theta_2} & 0 & \frac{\partial p_{24}}{\partial \theta_4} & 0 & \frac{\partial p_{24}}{\partial V_2} & 0 & \frac{\partial p_{24}}{\partial V_4} & \frac{\partial p_{24}}{\partial \alpha} \\ \frac{\partial q_{24}}{\partial \theta_2} & 0 & \frac{\partial q_{24}}{\partial \theta_4} & 0 & \frac{\partial q_{24}}{\partial V_2} & 0 & \frac{\partial q_{24}}{\partial V_4} & \frac{\partial q_{24}}{\partial \alpha}\end{bmatrix}$$

The additional state variable included is the firing angle of TCSC, given by α_{TCSC} . TCSC has two operating regions, inductive region and capacitive region. A resonant point exists in between these two regions which results in large variations in magnitude with respect to small variations in firing angle. This results in major perturbations in the Jacobian matrix. In this paper, the operation of TCSC in the capacitive region is considered as the goal is to increase the real power flow.

IV. SYSTEM CONSIDERED

State estimation study is carried out on IEEE 14 bus system with the addition of SVC and TCSC. Five sets of measurements are taken, viz. real power flows, reactive power flows, real power injections, reactive power injections and voltage magnitudes, as given in Tables 1-5.

V. RESULTS AND DISCUSSION

The results of state estimation after incorporating FACTS controllers SVC, and TCSC are presented in this section.

(A) State Estimation Results with SVC

Two SVC's are considered in this system. They are connected at bus 7 and 10. The SVC reactance parameters are $X_C = 1.07 \text{ pu}$, $X_L = 0.288 \text{ pu}$ and the transformer impedance $Z_T = j0.3 \text{ pu}$. Firing angle is varied in the range $90 \leq \alpha_{SVC} \leq 180$. The initial value of firing angle is taken as $\alpha_{SVC} = 130^\circ$. It is desired to increase the

Table 1
Real power flows

<i>Sl.No</i>	<i>From bus</i>	<i>To bus</i>	<i>Line no.</i>	<i>Measurement value</i>	<i>Standard deviation</i>
1	1	2	1	1.5708	0.001
2	2	3	3	0.7340	0.001
3	4	2	4	-0.5427	0.001
4	4	7	8	0.2707	0.001
5	4	9	9	0.1546	0.001
6	5	2	5	-0.4081	0.001
7	5	4	7	0.6006	0.004
8	5	6	10	0.4589	0.004
9	6	13	13	0.1834	0.004
10	7	9	15	0.2707	0.004
11	11	6	11	-0.0816	0.004
12	12	13	19	0.0188	0.006

Table 2
Reactive power flows

<i>Sl.No</i>	<i>From bus</i>	<i>To bus</i>	<i>Line no.</i>	<i>Measurement value</i>	<i>Standard deviation</i>
1	1	2	1	-0.1748	0.0064
2	2	3	3	0.0594	0.0064
3	4	2	4	0.0213	0.0064
4	4	7	8	-0.1540	0.0064
5	4	9	9	-0.0264	0.0064
6	5	2	5	0.0193	0.0064
7	5	4	7	-0.1006	0.0064
8	5	6	10	-0.2084	0.0064
9	6	13	13	0.0998	0.0064
10	7	9	15	0.1480	0.0064
11	11	6	11	-0.0864	0.0064
12	12	13	19	0.0141	0.0064

Table 3
Real power injections

<i>Sl. No</i>	<i>Bus No</i>	<i>Measurement value</i>	<i>Standard deviation</i>
1	2	0.3523	0.0041
2	3	0.0876	0.0041
3	7	0.0124	0.0041
4	8	0.2103	0.0041
5	10	-0.0580	0.0041
6	11	-0.0180	0.0041
7	12	-0.0160	0.0041
8	14	-0.0500	0.0041

Table 4
Reactive power injections

<i>Sl. No</i>	<i>Bus No</i>	<i>Measurement value</i>	<i>Standard deviation</i>
1	2	0.1830	0.004
2	3	-0.9420	0.004
3	7	0.0913	0.004
4	8	0.0560	0.004
5	10	-0.0900	0.004
6	11	-0.0350	0.004
7	12	-0.0610	0.004
8	14	-0.1490	0.004

Table 5
Voltage magnitude

<i>Sl.No.</i>	<i>Bus no.</i>	<i>Voltage magnitude</i>	<i>Standard deviation</i>
1	1	1.06	0.009

Table 6
State estimation results with and without SVC

	<i>Without SVC</i>		<i>With SVC</i>	
	<i>Voltage Magnitude</i>	<i>Phase Angle</i>	<i>Voltage Magnitude</i>	<i>Phase Angle</i>
1	1.0598	0	1.0603	0
2	1.0458	-5.0424	1.0540	-4.9696
3	1.0052	-13.0826	1.0608	-12.7576
4	1.0307	-11.1835	1.0356	-10.2132
5	1.0122	-9.5727	1.0398	-8.7249
6	1.0807	-15.2250	1.0790	-14.8033
7	1.0318	-11.5068	1.0400	-13.3048
8	1.0508	-11.4841	1.0551	-13.3049
9	0.9896	-13.3486	1.0367	-14.9396
10	0.9994	-14.0038	1.0400	-15.1937
11	1.0383	-14.7759	1.0624	-15.0933
12	1.0641	-16.0584	1.0624	-15.3960
13	1.0576	-16.0918	1.0591	-15.6592
14	1.0004	-15.6409	1.0308	-16.2943

volages at bus 7 and bus 10 from 1.0318 p.u. and 0.9994 p.u. to 1.04 p.u. The firing angle at which specified voltage is obtained at bus 7 is, $\alpha_{SVC} = 131.4725$ and the firing angle at which specified voltage is obtained at bus 10 is , $\alpha_{SVC} = 134.7521$. Estimated bus voltages without and with SVC are shown in Table 6.

(B) State Estimation Results With TCSC

The TCSC reactance parameters are $X_C = 9.375e-3$ pu and $X_L = 1.625e-3$ pu. The initial condition of the firing angle for TCSC is = 145° . TCSC is installed in line 3-4, to obtain the power flow in this line at a

desired value of 30 MW, from the initial value of 23.77 MW. The firing angle at which this desired power flow is obtained is, $\alpha_{TCSC} = 143.7427$.

Table 7
State estimation results with and without TCSC

Sl. No.	Line	Without TCSC		With TCSC	
		p_{ij}	p_{ji}	p_{ij}	p_{ji}
1	1-2	156.31	-152.05	156.64	-152.61
2	1-5	75.31	-72.56	75.60	-73.68
3	2-3	73.08	-70.77	73.86	-71.54
4	2-4	55.96	-54.30	55.60	-53.97
5	2-5	41.38	-40.48	41.55	-40.65
6	3-4	-23.39	23.77	-29.35	30.00
7	4-5	-61.04	61.56	-59.96	60.42
8	4-7	27.92	-27.92	27.12	-27.12
9	4-9	16.02	-16.02	15.35	-15.35
10	5-6	43.90	-43.90	45.59	-45.59
11	6-11	7.33	-7.27	7.58	-7.54
12	6-12	7.76	-7.69	8.07	-8.00
13	6-13	17.73	-17.51	18.32	-18.13
14	7-8	0.01	-0.01	0.02	-0.02
15	7-9	28.06	-28.06	27.20	-27.20
16	9-10	5.24	-5.23	5.03	-5.01
17	9-14	9.45	-9.33	9.10	-9.00
18	10-11	3.78	-3.76	3.88	-3.90
19	12-13	-1.61	1.66	-1.84	1.85
20	13-14	5.62	-5.57	5.99	-5.94

VII. CONCLUSION

A weighted least square algorithm for the state estimation of power systems containing FACTS devices SVC and TCSC has been developed. Required modification in the measurement Jacobian matrix for incorporating SVC and TCSC into the state estimator have been described in detail. MATLAB Coding was developed and tested with IEEE 14-bus system. It is seen that the desired level of voltage magnitudes are obtained at buses where SVCs are connected. In case of TCSC the desired level of real power flow is obtained in the line where TCSC is connected. Role of Unified Power Flow Controller (UPFC) on State Estimation is under investigation. State Estimation incorporating SVC, TCSC and UPFC on large power system will be the future work.

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