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The Quarternionic Description of Quantum Mechanics

Abstract: *It was pointed out by Ezra Newman in the sixties that an imaginary shift of the coordinate in purely Classical equations leads to the purely Quantum Mechanical gyromagnetic ratio g = 2. Newman puzzled about it for decades and finally could not explain this enigmatic finding. The author, Dr. B G Sidharth, Director, B M Birla Science Centre, Hyderabad, has been working on this for a few decades and has concluded the following: 1. The explanation lies at very small scales where the square of the Compton scale is retained and 2. When a complex coordinate is generalized to three dimensions, as Sachs had pointed out we end up with a four dimensional space, which moreover has a Minkowski invariant thrown in. On a further analysis the author noted that in this quarternionic description the spacetime is rather different to the simple Minkowski spacetime. To put it pictorially the former resembles the curly spiral binding while the latter is more like the smooth paper The author also concluded that this was the reason why despite a century of efforts Einstein's gravitation could not be reconciled with Particle Physics. Moving on we consider the second order representation of the quarternions in terms of the 2 × 2 Pauli matrices. This time the line element will be given by* $\sigma_{(i)} x^i$. *We get again an invariant but unlike in the 4 × 4 matrix consideration, this time there is no invariance under the reflection symmetry. We consider the different situations like neutrinos, noncommutative geometry and two dimensional surfaces like Graphene where this latter case applies.*

It was pointed out by Ezra Newman in the sixties that an imaginary shift of the coordinate in purely Classical equations leads to the purely Quantum Mechanical gyromagnetic ratio g = 2.

The Kerr-Newman metric can be written as:

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - asin^2\Theta d\phi]^2 + \frac{sin^2\Theta}{\rho^2}[(r^2 + a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\Theta^2 \quad (1)$$

where, $\Delta = r^2 - 2Mr + a^2 + Q^2, \rho^2 = r^2 + a^2cos^2\Theta$

Even for distances much smaller than the Compton wavelength 'a', as above, this goes over to,

$$-dt^2 + a^2 sin^2\Theta d\phi^2 + cos^2\Theta dr^2 + a^2 cos^2\Theta d\Theta^2$$

where $\theta \neq \pi/2.$.

Newman noticed in the sixties that an imaginary shift of the coordinate in the above purely Classical equations leads to the purely Quantum Mechanical gyromagnetic ratio g = 2.

Newman puzzled about it for decades and finally could not explain this enigmatic finding.

The author on the other hand has been working on this for a few decades and has concluded the following:

1. The explanation lies at very small scales where the square of the Compton scale is retained and not neglected as a contemporary theory.
2. When a complex coordinate is generalized to three dimensions, as Mendel Sachs had pointed out we end up with a four dimensional space, which moreover has a Minkowski invariant thrown in.

While the usual Minkowski four vector transforms as the basis of the four dimensional representation of the Poincare group, the two dimensional representation of the same group, given by the right hand side in terms of Pauli matrices, obeys the quaternionic algebra of the second rank spinors (Cf. ref.[1,2,3] for details).

To put it briefly, the quarternion number field obeys the group property and this leads to a number system of quadruplets as a minimum extension. In fact one representation of the two dimensional form of the quarternion basis elements is the set of Pauli matrices. Thus a quarternion may be expressed in the form

$$Q = i\sigma_\mu x^\mu$$

$$\sigma_0 x^4 - i\sigma_1 x^1 - i\sigma_2 x^2 - i\sigma_3 x^3$$

$$\sigma_0 x^4 - i\vec{\sigma}.\vec{r}$$

This can also be written as

$$Q = -i\begin{pmatrix} ix^4 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & ix^4 - ix^3 \end{pmatrix}$$

As can be seen from the above, there is a one to one correspondence between a Minkowski four-vector and Q. The invariant is now given by $\bar{Q}\,Q$, where $\bar{Q}\,Q$ is the complex conjugate of .

However, as is well known, there is a lack of spacetime reflection symmetry in this latter formulation. If we require reflection symmetry also, we have to consider the four dimensional representation,

$$(I,\vec{\sigma}) = \left[\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}\right] = \Gamma^{\mu}$$

(Cf.also.ref. [4] for a detailed discussion). The motivation for such a reflection symmetry is that usual laws of physics, like electromagnetism do indeed show the symmetry.

On a further analysis the author noted that in this 2×2 quarternionic description the spacetime is rather different to the simple Minkowski spacetime. To put it pictorially the former resembles the curly spiral binding while the latter is more like the smooth paper. The author also concluded that this was the reason why despite a century of efforts Einstein's gravitation could not be reconciled with Particle Physics, as indeed Wolfgang Pauli had pointed out decades ago.

Let us now consider the second order representation of the quarternions and this is through the 2×2 Pauli matrices. This time the line element will be given by $\sigma_{(i)}\, x^i$. We get again an invariant but unlike in the 4×4 matrix consideration, this time there is no invariance under the reflection symmetry. In a sense this is a new and non Minkowskian relativity. There are many different situations where this lack of reflection symmetry applies. For example the neutrinos do not have this symmetry. Similarly noncommutative geometry and two dimensional surfaces like Graphene also display this lack of reflection symmetry. So this quarternionic description in terms of 2×2 Pauli matrices has applications beyond Einstein's Minkowski geometry.

It may be pointed out that the author has shown (Cf.ref.[5]) that only left handed particles can decay and this is borne out by the results of the B factory decays.

References

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