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One Vertex Union of Cycles and Quadrilateral Snake Graphs Attaching on Cycle Graphs are Cordial

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ABSTRACT

A cordial labeling as an application of communication network, in this paper we have given cordial labeling for the graph $C_p \bullet C_q^t, \forall p, q$ and t, i.e., attaching the one vertex union of cycles C_q on each vertex of cycle C_p, and also shown that the cordial labeling for the quadrilateral snake graph QS_n, $\forall\ n$ gluing with each vertex of graphs $C_p \bullet C_q^t$, i.e., $QS_n(C_p \bullet C_q^t), \forall n \geq 0.$ Finally the domination number of this graph is analyzed

Key words**:** Cordial labeling, Cycles, Quadrilateral snake graphs.

AMS Subjects classifications: 05C78

1. INTRODUCTION

Let the function be f from vertices of graph G to binary set/digit {0, 1} and for every edge consequently assign the label $| f(x) - f(y) |$. Call f a *cordial labeling* of G if the number of vertices label given 0 and the number of vertices label given 1 differ by at most 1 and correspondingly the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph with cordial labeling is called *cordial graph*.

The most important two types of classifications of graph labeling concept in graph theory are called graceful labeling and harmonious labeling. Graceful labeling was introduced independently by Rosa[5] in 1966 and Golomb[4] in 1972, while harmonious labeling was first evaluated by Graham and Sloane[3] in 1980. In [1], I. Cahit shown that the every tree is cordial and is cordial for all m and n and in[6], Sethuraman and Selvaraju, they are shown that the one edge union of shell graphs and one vertex union of complete Bipartite Graphs are Cordial Graphs. More information on graph labeling result can be referred in the latest version of dynamic survey written by Joseph A Gallian[2]. Let us denote the binary sets V_0 and V_1 respectively, defined as the number of all vertices (mapped with) allotted the label 0 and the number of vertices (mapped with) allotted the label 1 and let sets E_0 and E_1 define respectively, define the number of edges consequently getting the label/digit 0 and the number of edges consequently getting the label/digit 1.

In this paper, had an attempt to introduce new type of construction of graph that is different from usual graph operation like[2][4][7][8], one vertex or one edge union of graphs, Cartesian product, Normal product, General product and lexicographic product of graph, etc… Here we aimed at deriving new unique graph operation/ construction is called attaching of 't' number of cycle on any other cycle and further each vertex of the graph $C_p \bullet C_q^t, \forall p, q$ and t attached with Quadrilateral Snake graph QS_n, $\forall\ n$. We define QS_n, $\forall\ n$ as follows.

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The graph $QS_n, \ \forall \ n$ is called Quadrilateral Snake graph. It is defined as series connection of non-adjacent vertices of 'n' number of cycle C_4 and this graph $QS_n, \ \forall \ n$ vertex set V and edge set E have described below

$$V(QS_n) = \{c_k\}_{k=1}^{n+1} \cup \{u_i\}_{i=1}^{n} \cup \{v_j\}_{j=1}^{n}$$
$$E(QS_n) = \{c_k u_k\}_{k=1}^{n} \cup \{c_k v_k\}_{k=1}^{n} \cup \{u_k c_{k+1}\}_{k=1}^{n} \cup \{v_k c_{k+1}\}_{k=1}^{n} \quad (1)$$

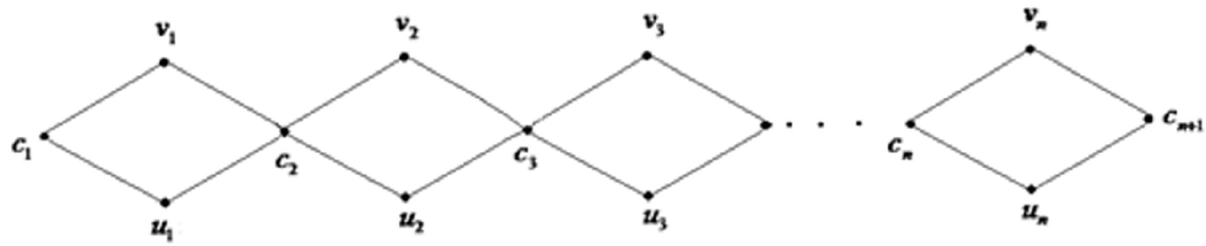
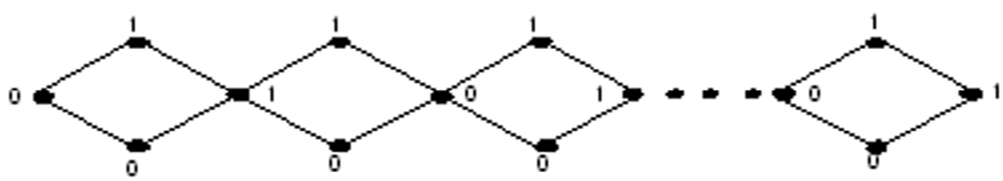


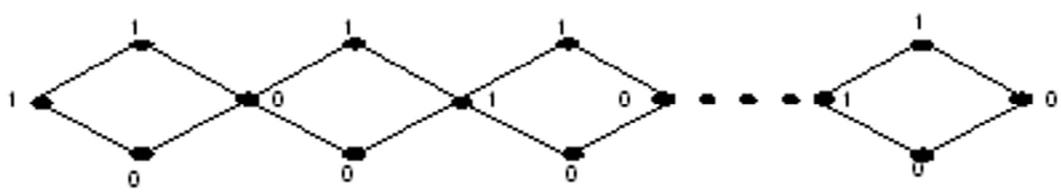
Diagram 1: Quadrilateral Snake Graph $QS_n, \ \forall \ n$

Result 1.1

For the conveniently to use Quadrilateral Snake Graph $QS_n, \ \forall \ n$, for attaching with other graph, there are two different possible types of cordial labeling of Quadrilateral Snake Graph $QS_n, \ \forall \ n$ as



Type-I



Type-II

In both the types, by definition of cordial labeling, $V_0 = V_1$ and $E_0 = E_1$.

Hence these graphs are called Cordial graph.

2. The cordial labeling of graph $G = C_p \bullet C_q^t, \ \forall \ p, \ q$ and t

The graph $G = C_p \bullet C_q^t, \ \forall \ p, \ q$ and t is a generated from cycle graph C_p with p number of vertices and common vertex union of 't' number of isomorphic graph of cycle C_q with q number of vertices is gluing at each vertex of C_p as shown in Figure 2.1. The following is the illustrative diagram of general form of graph G. For valuation of cordial labeling for graph G, the vertices are arranged in a vertex sequence in the following manner/pattern.

For vertices of cycle C_p,

$$V_{1,0}^{0},\ V_{2,0}^{0},\ \dots,\ V_{p,0}^{o} \tag{2}$$

For all the vertices of 'p' copies of cycle C_q^t (one vertex union of 't' isomorphic copies of cycle graph C_q)

$$v_{1,1}^{1}, v_{1,2}^{1}, \dots, v_{1,q-1}^{1}, v_{1,q-1}^{2}, v_{1,q-2}^{2}, \dots, v_{1,1}^{2}, v_{1,1}^{3}, \dots, v_{1,q}^{t}, v_{2,1}^{1}, v_{2,2}^{1}, v_{2,3}^{1}, \dots, v_{p,q}^{t} \tag{3}$$

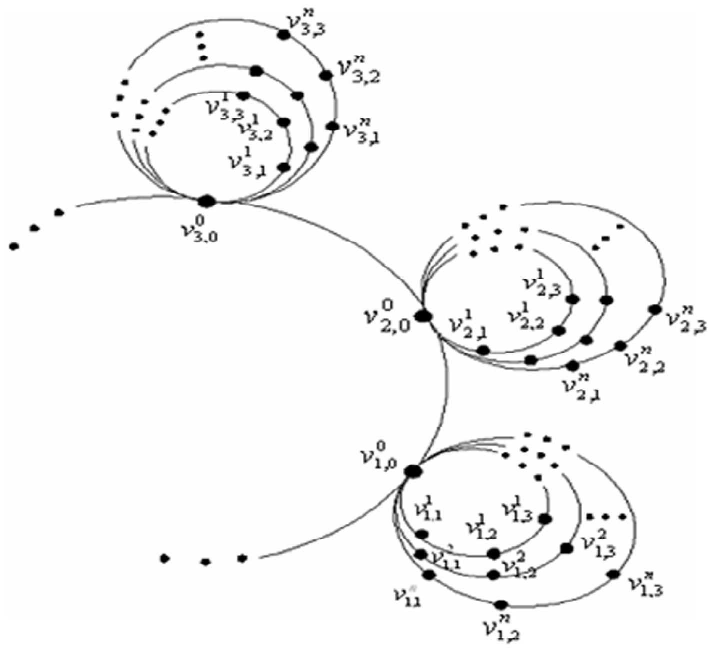


Figure 2: The Graph

Theorem: 2.1 The Graph $G = C_p \bullet C_q^t,\ \forall\ p,\ q$ and t is Cordial Graph

Proof: The graph $G = C_p \bullet C_q^t,\ \forall\ p,\ q$ and t be a graph and for convenience of cordial labeling, the proof of theorem divided into four types according to base cycle C_p of order type $4r$ or $2r + 1$, for $r = 1$ to n, form a one vertex union of isomorphic or non-isomorphic copies of cycle graph C_q of length $4r$, for $r = 1$ to n. For the Cordial Labeling of Graph, the vertices belong to base cycle follows a binary sequence 1100… (The labeling sequence starts with vertex $v_{n,0}^{0}$) and the vertices of one vertex union of cycles follow the binary sequence 0011… (The evaluation of labeling sequence starts with vertex $v_{1,1}^{1}$). The cordial labeling of Graph G is given the table 2.1 as follows and as four types with respect to order of base cycle

Type (i): If the base cycle graph C_p of order p is multiplication of 4 ($p = 4r$) and at each $p = 4r$ vertices the one vertex union of isomorphic 't' copies of cycle C_4, $\forall$ $r = 1$ or 2 or 3 or…. or n

The graph $G = C_{4r} \bullet C_{4r}^t,\ \forall\ r = 1\ to\ n$ has $V_1 = p(t(q - 1) + 1)$ vertices and has $E_1 = p(1 + tq)$ edges. In this case, the number of vertices allotted/mapped with label/digit 0 is equal to the number of vertices allotted/mapped with label 1 and number of edges correspondingly getting the label 0 is equal to number of edges correspondingly getting the label 1.

Type (ii): If the base cycle C_p of length is multiple of 4 ($p = 4r$) and one vertex union of non-isomorphic 't' copies of cycle C_{4r}, for $r = 1, 2, 3, \dots, n$.

Table 1

Types	E	V	V_0	V_1	E_0	E_1	$V_0 – V_1$	$E_0 – E_1$
Type (i)	E	$V = \begin{cases} (1100)^{\frac{p}{4}}, \text{for (II)} \\ (0011)^{\frac{pt(q-1)}{4}}, \text{for (III)} \end{cases}$	$V/2$	$V/2$	$E/2$	$E/2$	0	0
Type (ii)	E	$V = \begin{cases} (1100)^{\frac{p}{4}}, \text{for (II)} \\ (0011)^{\frac{pt(q-1)}{4}}, \text{for (III)} \end{cases}$	$V/2$	$V/2$	$E/2$	$E/2$	0	0
Type (iii)	E	$V = \begin{cases} (1100)^{r-1}110, \text{if p = 4r -1 for (II)} \\ (1100)^{r}1, \text{if p = 4r +1 for (II)} \\ (0011)^{\frac{pt(q-1)}{4}}, \text{for (III)} \end{cases}$	$\begin{cases} \frac{V}{2} \\ \left\lceil \frac{V}{2} \right\rceil \\ \left\lfloor \frac{V}{2} \right\rfloor \end{cases}$	$\begin{cases} \frac{V}{2} & \text{t = odd, p = 4r - 1} \\ \left\lfloor \frac{V}{2} \right\rfloor & \text{t = 4r - 2, p = 4r - 1} \\ \left\lceil \frac{V}{2} \right\rceil & \text{t = 4r, p = 4r - 1} \end{cases}$	$\left\lfloor \frac{E}{2} \right\rfloor$ $\left\lceil \frac{E}{2} \right\rceil$	$\left\lceil \frac{E}{2} \right\rceil$ t = odd, p = 4r - 1 $\left\lfloor \frac{E}{2} \right\rfloor$ t = odd, p = 4r + 1		0
1								
Type (iv)	E	$V = \begin{cases} (1100)^{r-1}110, \text{if p = 4r -1 for (II)} \\ (1100)^{r}1, \text{if p = 4r +1 for (II)} \\ (0011)^{\frac{pt(q-1)}{4}}, \text{for (III)} \end{cases}$	$\begin{cases} \frac{V}{2} \\ \left\lceil \frac{V}{2} \right\rceil \\ \left\lfloor \frac{V}{2} \right\rfloor \end{cases}$	$\begin{cases} \frac{V}{2} & \text{t = odd, p = 4r - 1} \\ \left\lfloor \frac{V}{2} \right\rfloor & \text{t = 4r - 1, p = 4r - 2} \\ \left\lceil \frac{V}{2} \right\rceil & \text{t = 4r, p = 4r - 1} \end{cases}$	$\left\lfloor \frac{E}{2} \right\rfloor$ $\left\lceil \frac{E}{2} \right\rceil$	$\left\lceil \frac{E}{2} \right\rceil$ p = 4r - 1 $\left\lfloor \frac{E}{2} \right\rfloor$ p = 4r + 1	0	1

The graph $G = C_{4r} \bullet C_{4r}^{t}$, $\forall\ r = 1\ \ to\ \ n$ has $V_2 = p\left(1 + t\left[\frac{(t+1)q}{2} - 1\right]\right)$ vertices and has $E_2 = p\left[1 + \frac{t(t+1)q}{2}\right]$ edges. In this case, number of vertices allotted (matched with) with label/digit 0 is equal to the number of vertices allotted (matched with) with label 1 and number of edges consequently/ correspondingly getting the label 0 is equal to the number of edges correspondingly getting the label 1.

Type (iii): If the base cycle C_p of length $2r + 1$, $r = 1$ to n and one vertex union of isomorphic' copies of cycle, C_{4r}, $r = 1$ or 2 or 3 or …. or n. The graph $G = C_{4r} \bullet C_{4r}^{t}$, $\forall\ r = 1\ to\ n$ has set $V_3 = p(t(q - 1)+1)$ vertices and has set $E_3 = p(1 + tq)$ edges. In this case, the number of vertices allotted/mapped with label 0 is equal to number of vertices allotted/mapped with label 1 and number of edges correspondingly getting label 0 is equal to the number of edges correspondingly getting label 1.

Type (iv): If the base cycle C_p of length $2r + 1$, $r = 1$ to n and one vertex union of non-isomorphic 't' copies of cycle, C_{4r}, $r = 1, 2, 3, …., n$.

The graph $G = C_{4r} \bullet C_{4r}^{t}$, $\forall\ r = 1\ to\ n$ has $V_4 = p(t(q - 1)+1)$ vertices and has $E_4 = p(1 + tq)$ edges. In this case, the number of vertices allotted/mapped with label 0 is equal to number of vertices allotted/ mapped with label 1 and the number of edges correspondingly getting label 0 is equal to number of edges correspondingly getting label 1.

Hence, it is clear that the Graph is Cordial Graph.

3. Cordial labeling of gluing of quadrilateral snake graph QS_n, $\forall\, n$ on each vertex of $C_p \bullet C_q^t$, $\forall\ p,\ q$ and t graph.

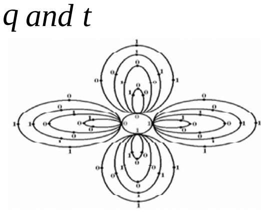
The graph QS_n, $\forall\, n$ gluing at each vertex of Graph are cordial graph because as mentioned in Result 1.1, cordial labeling of QS_n, $\forall\, n$ has the number of vertices with label 0 is equal to the number of vertices allotted with the label 1 and that the number of edges correspondingly getting with label 0 is equal to the number of edges correspondingly getting with label 1 in both the types and with the above table 2.1 the graph G satisfies the definition of Cordial Labeling and domination number of graph G is $\gamma(G) = \left\lceil \frac{p}{3} \right\rceil \left[\left\lceil \frac{q}{3} \right\rceil + (3t - 1)\left\lceil \frac{q-1}{3} \right\rceil \right]$.

4. CONCLUSION

In this paper proved that the graph G and also gluing of QS_n, $\forall\, n$ graph is Cordial Graph. The author put following open problem: Are there any different labeling will satisfy the Graph G and also gluing of quadrilateral snake graph QS_n, $\forall\, n$ with this each vertex of graph G.

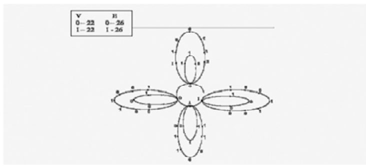
5. ILLUSTRATIVE EXAMPLE OF $C_p \bullet C_q^t$, $\forall p,\ q$ and t

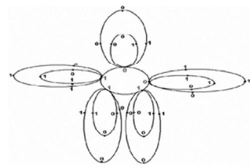
Type (i)	V	E
$p = 4$, $q = 4$, $t = 4$	0 – 26	0 – 34
	– 26	1 0 34

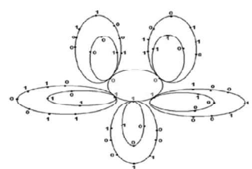


Type (ii)

$p = 4$, $q = 4$, $t = 2$ (non – isomorphic)



Type (iii)	V	E	
p = 5, q = 4, t = 2	0–18	0–23	
	1–17	1–22	

Type (iv)	V	E	
$p = 5$, $q = 4$, $t = 2$ (non – isomorphic)	0–28	0–33	
	1–27	1–32	

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