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AN M/M/1 RETRIAL QUEUE WITH TWO ORBITS

N. Paranjothi and G. Paulraj

Abstract: Consider an M/M/1 retrial queue. Assume that the server admits incoming customer into two orbit as follows: "any arriving customer finding the server busy enter in to the orbit -1 or otherwise occupies the sever for getting its first service. The server applies a set Ω of specifications and declares each customer as ' Satisfied Type (ST)' if the customer satisfies all the specifications of Ω; otherwise he declares him as 'Unsatisfied type (UST)' at the time of completing the customer's first service. For the present study such events of ST and UST are assumed to occur with probabilities 'p' and 'q' respectively , where p+q =1. Each ST leave the service area while each UST joins another group called 'orbit-2' of unsatisfied customers". From both the orbits each customers can apply for service after a random amount of time and can enter into service only when the server is free. The server doesnot apply the verification process to any one customers coming from orbit-2 to get its service i.e. when the customer receives its service second time but the server serves him up to his satisfaction without any verification. Steady state conditions, joint distribution of X_1 = number of busy servers and X_2=number of customers present in the system at a random epoch are derived. Some measures of X_2 are then obtained.

Key Words: Satisfied type customer, Unsatisfied Type Customer, Steady State Condition, and Joint Distribution.

1. Introduction

Consider an M/M/1 retrial queue. Assume that the server admits incoming customers into two orbits as follows: "any arriving customer finding the server busy enters into the orbit-1 or otherwise he occupies the server for getting its first service. The server applies a set Ω of specifications and declares each customers as ST if the customer satisfies all the specifications of Ω; otherwise he declares him as UST at the time of completing the customer's first service. For the present study such events of ST and UST are assumed to occur with probabilities 'p' and 'q' respectively, where p+q=1.

Each ST leaves the service area while each UST joins another group called 'obit-2' of unsatisfied customers". From both the orbit each customer can reapply for service after a random amount of time and can enter into service only when the server is free.

The server does not apply the verification process to any one customers coming from orbit-2 to get its service i.e. the server does not do the verification of specifications but serves him up to his full satisfactions when the customer receives its service second time. From the above setting it is evident that a customer can conduct the server at most two times.

Queues in which the above ST and UST customers are allowed may be used to model a number of real life problems. A few examples are the queues that conceptualize in front of service counters where verification of personal records such as Bio Data, Educational qualifications etc., of customers is checked out during the course of service duration of each customer.

Further the above situation may arise frequently in the stochastic modeling of number of telecommunication and computer systems. The most obvious example is provided by a person who desires to make a call to verify his records. If the data are insufficient then he is to go to orbit-2 to try again with full records at some time later; otherwise he gets served in his first attempt itself and leaves the service area.

Different variants of retrial queues can be found in Artalejo, Rajagopalan and Sivasamy[1] and Artalejo and Gomez- Corral[3]. For an exhaustive survey on retrial queues, see Falin and Templeton[5] and Artalejo[2]. Some basic details about queues are found in Gross and Harris[4].

In section 2, M/M/1 queue with above type of customers in two orbits is described. Section 3 provides the distribution of X_1 and X_2 under steady state conditions. Section 4 gives mean and variance of X_2 and a brief conclusion.

2. M/M/1 queue with two orbits

Consider an M/M/1 queue with Poisson arrivals, rate λ, and exponential service times with mean μ^{-1} for each customer who receive his first service and exponential service times with mean β^{-1} for each customer who receive his service for the second time. At the time of completing the first service of each customer,the server classifies him either as ST customer or as UST customer as stated above.

Thus the service time of all ST and UST customer of orbit-1 and the customers who are served for the first times are independent and identical distributed(i.i.d) and exponentially distributed random variable with parameter μp and μq respectively. On the other hand, the service times of all customers of orbit2 are exponentially distributed and i.i.d random variables with parameter β.

Each customer of the orbit is assumed to produce a Poisson process of repeated attempts with intensity ‘v’. Hence, if a repeated call finds the server’s state as free only it can enter in to the service and leaves the system after service, if it satisfies all requirements of the set Ω. Otherwise the state of the orbit does not change.

Even if there are some customers in the orbit who want to get service they can not occupy the server immediately as soon as he becomes free, because of their ignorance of the server's state.

The inter arrivals times between customers from external sources,inter occurrence times between repetitions from customer of the orbit and service times are mutually independent.

Let, at time 't', $X(t)=(X_1(t),X_2(t))$,

Where, $X_1(t)$=number of busy servers belongs to {0,1}

$X_2(t)$=number of customers present in the orbit(queue + service)

E= State Space ={(m,n): m=0,1 and n=0,1,2,…,∞}, and S={0,1,2,..,∞}

Let us further define the following:

$X_1 = X_1(t)$ and $X_2 = X_2(t)$ as $t\to\infty$

$P_{m,n} = P(X_1 = m, X_2 = n)$ for $(m,n) \in E$

$$\theta = \frac{\lambda}{\mu p + \beta} < 1$$

3. System under steady state

It can now be shown that the random process {X(t); t≥0} is a Markov process on the states space E which is the most important process associated with the above queueing system. Using the standard arguments the classical analysis for the present system is discussed below. The following equations are satisfied by $\{P_{mn}\}$ for (m,n) ∈E:

$$P_{mn} = P\left\{(X_1=m, X_2=n)\ \beta\right\} \text{ for } (m, n) \in E$$

and $$\theta=\frac{\lambda}{\mu p + \beta} < 1$$

4. The Steady State Analysis

It can now be shown that the random process {X(t); t ≥ 0 } is a Markov process on the state space 'E' which is the most important process associated with the above retrial queuing system. Classical analysis of the present system is discussed below. The following equations are satisfied by $\{P_{mn}\}$ for (m,n) ∈ E:

$$(\lambda + n\gamma\ )P_{00} = \mu p P_{1n} + \mu q P_{1n-1} + \beta P_{1n} \tag{1}$$

$$(\lambda+\mu+\beta)P_{1n} = \lambda P_{0n} + (n+1)\gamma P_{0n+1} + \lambda P_{1n-1}$$

Multiplying (1) by λ and (2) by μq and eliminating P_{1n-1},

we get the following equation:

$$(\mu+\beta)(\lambda+\mu q)P_{1n} = \lambda\mu q P_{00} + \mu q(n+1)\gamma P_{00+1} + \lambda(\lambda+n\gamma)P_{(}$$

from (1) and (2)

eliminating P_{1n}, we get the following equation:

$$(1+n\gamma)\{\lambda+\mu+\beta\}P_{0n} = \lambda(\mu p+\beta)P_{0n} + (\mu p+\beta)(n+1)\gamma P_{0,n+1} + \{(\mu+\beta)(\lambda+\mu q)\}P_{1,n-1}$$

Replacing n by (n-1) in (3) and using the result on $P_{1,n-1}$ in (4) we

$$(\lambda+n\gamma)(\lambda+\mu+\beta)P_{0n}$$

$$= \lambda(\mu p+\beta)P_{0n} + (\mu p+\beta)(n+1)\gamma P_{0,n+1} + \lambda\mu q P_{0,n-1} + \mu q n\gamma P_{0n} + \lambda(\lambda+(n-1)\gamma)P_{0,n-1}$$

$$\Rightarrow (\mu p+\beta)(n+1)\gamma P_{0,n+1} - \{\lambda(\lambda+n\gamma)+\lambda\mu q\}P_{0n}$$

$$= (\mu p+\beta)n\gamma P_{0n} - \{\lambda(\lambda+(n-1)\gamma)+\lambda\mu q\}P_{0,n-1}$$

$$\Rightarrow P_{0n} = \left[\frac{\lambda\{\lambda+\mu q+(n-1)\gamma\}}{(\mu p+\beta)n\gamma}\right]P_{0n-1} \qquad ;n\geq 1$$

$$= \frac{\theta^n}{n!\gamma^n}\prod_{i=0}^{n-1}(\lambda+\mu q+i\gamma)P_{00}$$

From (3) and (5), we deduce that

$$P_{1n} = \frac{\theta^{n+1}}{n!\gamma^n}\prod_{i=1}^{n}(\lambda+\mu q+i\gamma)P_{00}$$

The probability P_{00} can be found, with the help of the normalizin(

$$\text{Then}\quad \sum_{n=0}^{\infty} p_{0n} + \sum_{n=0}^{\infty} p_{1n} = 1$$

$$P_{00}^{-1} = \sum_{n=0}^{\infty}\frac{\theta^n}{n!\gamma^n}\prod_{i=0}^{n-1}(\lambda+\mu q+i\gamma) + \sum_{n=0}^{\infty}\frac{\theta^{n+1}}{n!\gamma^n}\prod_{i=1}^{n}(\lambda+\mu q+i\gamma)$$

using Binomial series

$$\sum_{n=0}^{\infty}\frac{\theta^n}{n!\gamma^n}\prod_{i=0}^{n-1}(\lambda+\mu q+i\gamma)=\sum_{n=0}^{\infty}\frac{\theta^n}{n!}\prod_{i=0}^{n-1}\left(\frac{\lambda+\mu q}{\gamma}+i\right)$$

$$=\sum_{n=0}^{\infty}\frac{(-\theta)^n}{n!}\prod_{i=0}^{n-1}\left(-\frac{\lambda+\mu q}{\gamma}-i\right)$$

$$=(1-\theta)^{-\left(-\frac{\lambda+\mu q}{\gamma}\right)}$$

$$\sum_{n=0}^{\infty}\frac{\theta^{n+1}}{n!\gamma^n}\prod_{i=1}^{n}(\lambda+\mu q+i\gamma)=\theta\sum_{n=0}^{\infty}\frac{(-\theta)^n}{n!}\prod_{j=0}^{n-1}\left(-\left(\frac{\lambda+\mu q}{\gamma}\right)-1-j\right)$$

$$=\theta(1-\theta)^{-\left(\frac{\lambda+\mu q}{\gamma}\right)-1}$$

$$P_{00}^{-1}=(1-\theta)^{-\left(\frac{\lambda+\mu q}{\gamma}\right)}+\theta(1-\theta)^{-\left(\frac{\lambda+\mu q}{\gamma}\right)-1}$$

$$=\quad(1-\theta)^{-\left(\frac{\lambda+\mu q}{\gamma}\right)-1}$$

5. Measures of Effectiveness

We can now get the various performance characteristics of the system in the steady state. In particular the mean and variance of the random variable X_2 are given by:

$$E(X_2)=\sum_{n=0}^{\infty}n\ P_{0n}+\sum_{n=0}^{\infty}n\ P_{1n}=\frac{\theta(\lambda+\mu q+\theta\gamma)}{(1-\theta)\gamma}$$

$$V(X_2)=\frac{\theta(\lambda+\mu q+\theta\gamma+\theta^2\gamma-\theta^3\gamma)}{(1-\theta)^2\gamma}$$

6. Conclusion

In this article we have analyzed an M/M/1 type of retrial queue which admits customers in two different orbits according to a set Ω of specification rules. This is an innovative induction to the literature, for this type of service mechanism in the context of retrial queues which have potential applications in vital areas. Hence it motivate the authors to pursue further to extend the analysis of M/G/1 and G/M/1 type of queues.

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N. Paranjothi and G. Paulraj
Department of Statistics
Annamalai University-608002