Communications on Stochastic Analysis
Vol. 12, No.4 (2018) 507-511

MARTINGALE STOPPING RULE TO ANALYZE POPULATION CONTROL THROUGH STERILIZATION PROGRAM

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ABSTRACT: An effort has been made to develop a method to estimate the stopping time with respect to sterilization programmes especially for women in pursuit of achieving the population control using Martingale stopping rule. A birth and death process has been used for analysing the expected stopping time for different values of birth and death rates with respect to different reference time period.

Introduction

Controlling the population growth in India has been a concern for the Indian Government after the independence. In fact, India was the first nation to introduce family planning program in 1952 with mixed success as described by Ledbetter (1984). Sterilization gradually got acceptance among the people and peaked during 1970's as one of the programme in India for controlling the population growth. Ramnathan et. al. (1995) and Mathew et. al. (2009) elaborate analysis on the impact of forced sterilization on male population particularly during mid-seventies the family planning proceeds towards family happiness and female sterilization became the only acceptable option. In various surveys, it was found that a large number of male-population regretted after going through sterilization due to lack of awareness of side effects. As per IIPS (2007) about five percent sterilized females regretted later because of various socio-economic reasons including loss of kids. The study of IIPS (2017) using NFHS data showed that 37% of married women rely on sterilization program for their family planning. Interestingly, sterilization regret female population increased from 4.4% (NFHS, 2005-06) to 6.9% (NFHS, 2015-16). Mishra et. al (2004) and Ram et.al. (2004) have studied the impact of sterilization by pooling different rounds of NFHS data and performed regression analysis. Biswas and Pachal (1987) assumed differing vital rates and developed a multistate Markov chain model to analyse sterilization policy in India. In the present study, an effort has been made to employ birth and death process. Because of stochastic nature of response, the Martingale approach

Received 2018-1-2; Communicated by Shishir Kumar Jha
Keywords: Birth and death process, Markov chain, Martingale stopping rule, Sterilization.

may be used for developing methods to solve certain problems associated with sterilization policy for checking the unmanageable growth of population.

Methodology

Notations and assumptions

Let us assume that the focus is to sterilize mothers who have a certain number of surviving children as per guidelines of the state for implementing the population control policy. Further, suppose that mothers constitute a homogeneous group with respect to age.

Suppose, our study constitutes of the population of mothers who have m_0 number of surviving children (m_0 is the number of children at which mother becomes eligible for sterilization). Any increase in this number makes mother compulsorily eligible for sterilization.

⇒ Mothers whose number of off spring moves from $m_0 \rightarrow m_0 + 1$ are to be encouraged for sterilization in the time interval (0, τ) and mothers whose number of off spring moves from $m_0 \rightarrow m_0 - 1$ are not encouraged for sterilization in the time interval (0, τ) (where, τ is assumed to be large).

Let T is the time when a decision is taken for sterilization. Therefore, T is the stopping time.

Development of the model

Let us consider the birth and death process, where S(t) denotes the number of children surviving at time t with birth and death parameters b_t and d_t respectively.

Let S_0 and S_2 are two states of mothers representing 'state of no need of sterilization' and 'state of need of sterilization' respectively. Let, S_1 is the state when mothers become eligible for sterilization.

⇒ S_0 is the state when m_0 is reduced to $m_0 - 1$ (mother is not encouraged to go for sterilization) and S_2 is the state when m_0 is increased to $m_0 + 1$ (mother is encouraged to go for sterilization).

Let $\alpha_0(\text{t}) = \text{P}[\text{S(j)} = m_0 - 1 \mid \text{S(0)} = m_0 \text{ and S(j)} = m_0 \text{ for all j} < \text{t}] = \alpha_0$ (say),

$$\beta\,(\tau) = \text{P}[\text{S(j)} = m_0 \mid \text{S(0)} = m_0 \text{ for all j} \in [0, \tau]]$$

$$\Rightarrow 1 - \beta\,(\tau) = \text{P}[\text{S(j)} = m_0 - 1,\, m_0 + 1 \mid \text{S(0)} = m_0 \text{ for all j} \in [0, \tau]]$$

Further, let p_n(t) denotes the probability of n children at time t (n=0,1,2,3,…..)

i.e. $p_n(\text{t}) = \text{P}[\text{S(t)} = \text{n}]$; and

p_n(t + δt) = p_n(t)[1- (b_tδt + d_tδt) + o(δt)] + p_{n-1}(t) b_{t-1} δt + p_{n+1}(t) b_{t+1} δt + o(δt);
where, n = 0,1,2,3,4,……; such that p_{n-1}(t) = 0 for n = 0.

Let us define $h(j) = 1+\frac{d_1}{b_1}+\frac{d_1 d_2}{b_1 b_2}+ \ldots + \frac{d_1 d_2 \ldots d_{j-2} d_{j-1}}{b_1 b_2 \ldots b_{j-2} b_{j-1}}$; j = 1, 2, 3, ….. (1)

Therefore, for the σ- field ⨍[h(S(u);0≤u≤t], h(S(t)) is a Martingale [vide: Karlin and Taylor (1975)].
Further, the stopping time

T = min [S(t) = $m_0 - 1$, S(t) = m_0 and S(t) = m_0 + 1| S(0) = m_0]. (2)

⇒ E[h(S(T))] = E [h(S(0))]

= E [h(m_0)], using optimal sampling theorem of Martingales [Biswas (2004)]

= h(m_0) [m_0 is fixed as per assumption]

= $(1 - \alpha_0)$ [β h(m_0) + (1 -β)h(m_0 + 1)] + α_0 [h($m_0 - 1$)] [vide: Biswas and Pachal (1990)] (3)

$$\Rightarrow \alpha_0 = \frac{\beta\ h(m_0)+\left(1-\beta\right)h(m_0+1)-h(m_0)}{\beta\ h(m_0)+\left(1-\beta\right)h(m_0+1)-h(m_0-1)} \quad (4)$$

Therefore, α_0 is the fraction of the mother population, which will not be sterilized.

$$\Rightarrow (1-\alpha_0) = \left[\frac{h(m_0)-h(m_0-1)}{\beta\ h(m_0)+\left(1-\beta\right)h(m_0+1)-h(m_0-1)}\right] \quad (5)$$

Therefore, $(1 - \alpha_0)$ is the fraction of the mother population being sterilized.

Where, β = β (t) = $e^{-(b_{m_0} - d_{m_0})\tau}$ such that (0, τ) is the period of sterilization program. (6)

Assuming that sterilization is not made mandatory for mothers having less than two surviving children.

⇒ m_0 = 2

⇒ h ($m_0 - 1$) = 1, (7)

h (m_0) = 1+ $\frac{d_1}{b_1}$ and (8)

h ($m_0 + 1$) = 1+ $\frac{d_1}{b_1}$ + $\frac{d_1 d_2}{b_1 b_2}$ [using equation (1)] (9)

Suppose, f[S(t)] is a function of S(t).

⇒ X(T) = [f[S(T)] − T] is a Martingale. (10)

⇒ E[X(T)] = E[X(0)] = E[f[S(T)] − T], using Martingale stopping rule [vide: Biswas (2004)]

⇒ X(0) = f[S(0)] = f(m_0)

⇒ h(m_0) = E[f(S(T)] − E(T)

$\Rightarrow$ E(T) = E[f(S(T)] − h(m_0) (11)

Following Biswas and Pachal (1987), f(j) [j = 1, 2, ….] and E (T) is given by

$$f(j+1) = \frac{1}{b_1} + \frac{1}{b_2}\left(1+\frac{d_2}{b_1}\right) + \ldots\ldots \quad \text{for } j \geq 1 \text{ with } f(0) = f(1) = 0 \tag{12}$$

For m_0 = 2, f(m_0 − 1) = 0, f(m_0) = $\frac{1}{b_1}$ and f(m_0 +1) = $\frac{1}{b_1} + \frac{1}{b_2}\left(1+\frac{d_2}{b_1}\right)$ (13)

E[f(S(T)] =

$$\frac{\left[h(m_0-1)+\beta\, h(m_0+1)\right]\left(1-\beta\right)f(m_0+1)+\left[h(m_0-1)-\beta\, h(m_0+1)\right]\beta\, f(m_0)+[h(m_0)-h(m_0-1)]f(m_0)}{[h(m_0-1)-\left(1-\beta\right)h(m_0+1)-\beta\, h(m_0)]} \tag{14}$$

Substituting (14) in (11) and using (7), (8), (9), (12) and (13), we get;

E(T) =

$$\frac{b_2 d_1(d_2+b_1 b_2)-b_1 b_2\left[\left(1-\beta\right)\left\{b_1\left(1+\beta\right)+d_1\beta\right\}+b_2\left(1+2\beta-\beta^2\right)\right]-d_1\left(1-\beta\right)[b_1 b_2\left(\beta-b_2\right)+\beta\, b_2(b_2+d_2)]}{b_2 d_1+\left(1-\beta\right)d_1 d_2} \tag{15}$$

Numerical illustration

Suppose that the starting number of surviving children for sterilizing the female population is 2. i.e. m_0=2. For different values of τ, expected stopping time of the sterilization program has been tabulated in **table1** under hypothetical values of b_i and d_i (i = 1, 2) using equation (15).

For τ = 1, 2, 3, 4 and 5; the corresponding values of β are 1.10517, 1.2214, 1.3499, 1.4918 and 1.6487 respectively [using equation (6)].

TABLE 1(Expected stopping time)

τ	β	E(T)			
		b_i=d_i=.1 (i=1, 2)	b_i=d_i=.2 (i=1, 2)	b_1=d_1= b_2=.1, d_2=.2	b_1=d_1= d_2=.1, b_2=.2
1	1.10517	0.21134	0.44058	0.11453	-
2	1.2214	0.34136	0.69829	0.35370	0.10151
3	1.3499	0.50711	1.02723	1.12290	0.20773
4	1.4918	0.71701	1.44418	31.62500	0.32876
5	1.6487	0.98191	1.97085	-	0.73670

Conclusion

Under the assumption that mothers who have two surviving children are taken as starting point of transition for implementation of sterilization program, the findings in the table 1 indicate that stopping time increases as time observation period (0, τ) of sterilization program increases. When death parameter d_2 is more than the birth parameter b_2 (keeping $b_1$and $d_1$unchanged) then the stopping time sharply increases with increase in time observation period (0, τ). When death parameter d_2 is less than the birth parameter b_2 (keeping b_1 and d_1 unchanged) then the stopping time feebly increases with increase in time observation period (0, τ). From the table 1, it may be concluded that increase in reference time period results in increase in expected stopping time irrespective of the values of birth and death rates. The analysis has been done with hypothetical birth rate and death rate data. Using primary data with respect to b_i and d_i may result in interesting findings, which should be used to determine the blueprint for sterilization policy.

References

1. Biswas, S. (2004): Applied stochastic processes: A Biostatistical and population-oriented approach. New Central Book Agency (P) Ltd., Kolkata, India
2. Biswas, S. and Pachal, T. K. (1987): A multistage Markov chain model for evaluating a sterilization policy. Biometrical Journal, Vol. 24, 57-67.
3. Biswas, S. and Pachal, T. K. (1990): On Martingale approach to a problem on sterilization policy. Stochastic modelling in decision making, edited by S. Biswas, Khama Publishers, New Delhi.
4. Karlin, S. and Taylor, H. M. (1975): A first course in stochastic processes. Academic press (2nd edition), New York.
5. Ledbetter, R. (1984): Thirty years of family planning in India. Asian Surv., 24(7), 736-58.
6. International Institute for Population Sciences (IIPS) and Macro International (2007).
7. National Family Health Survey (NFHS-3), 2005–06, volume 1. Mumbai, India.
8. International Institute for Population Sciences (IIPS) and ICF (2017): National family health survey (NFHS-4), 2015–16, Mumbai, India.
9. Mishra, V., Roy, T. K. and Retherford, R.D. (2004): Sex differentials in childhood feeding, healthcare, and nutritional status in India. Popul Dev Rev., 30(2), 269–95.
10. Ram, F. and Roy, T. K. (2004): Comparability issues in large sample surveys-some observations: Population, health and development in India-changing perspectives. International Institute for Population Sciences, Mumbai, New Delhi, pp. 40–56.
11. Matthew, Z., Padmadas, S. S., Hutter, I., McEachran, J. and Brown, J. J. (2009): Does early childbearing and a sterilization-focused family planning programme in India fuel population growth? Demogr. Res., 20, 693–720.
12. Ramanathan, M., Dilip, T. R. and Padmadas, S. S. (1995): Quality of care in laparoscopic sterilisation camps: observations from Kerala, India. Reprod. Health Matters. 3(6), 84–93.

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