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Image Reconstruction Based on the Combination of Orthogonal Matching Pursuit and Triplet Half-band Wavelets Based on Optimized Time-bandwidth Product

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Abstract: This paper presents compressive sensing scheme for image reconstruction based on the combination of orthogonal matching pursuit and the design of 13/19 triplet half-band filter bank (THFB) based on optimized time-bandwidth product. First, we obtain the half-band polynomial using Euler-Fraboneus polynomial (EFP). We achieve this by imposing vanishing moments and perfect reconstruction conditions. Then we obtain the required resultant polynomial by optimizing the resultant half-band polynomial using time-bandwidth product.. This obtained optimized half-band polynomial is used in three-step lifting scheme to get the required 13/19 triplet half-band wavelet filters bank. Next, the designed triplet wavelet filters are applied on the image to obtain the sparse image. The orthogonal matching pursuit (OMP) and Gaussian probability density function (measurement matrix) are used to reconstruct the image. The experimental results show that compressive sensing using combination of designed wavelet filters and OMP gives the better performance than the existing wavelet filters.

1. INTRODUCTION

Compressed sensing is an emerging field that has attracted considerable research interest in the signal processing community. It is known that Nyquist sampling theorem (where the sampling rate must be at least twice the maximum frequency of the signal) is frequently used to acquire the image. However, it increases the sampling frequency that leads to increase in the data storage and transmission cost [1]. The usual approach to acquire a compressible (digital) signal is to take measurements in the Dirac basis and then it uses a non-linear algorithm, such as JPEG coder to obtain a more efficient approximation. However, this approach is not practicable if the signal is presented at a higher rate. In order to solve these issues, [2, 28] proposed the compressed sensing theory in which a random linear projection is used to acquire efficient representation of compressible signals directly [28]. In this scheme, the band-limited model of the signal is replaced by the assumption that the signal is sparse or compressible with respect to some basis or dictionary of waveforms and enlarges the concept of sample to include the application of any linear function [3]. The compressive sensing (CS) merges the operations of data acquisition and compression by measuring sparse or compressible signal via a linear dimensionality reduction. Next, recover the signal by the use of sparse-approximation based algorithm. The signal is *K-sparse* if its coefficients are in some transform domain contains only K non-zero values. The signal is compressible if its coefficients decay rapidly when sorted by magnitude [4]. There are numerous other potential applications where sparse reconstruction for time sequences of signals/images may be needed. These applications can be found in [5]. The sparse reconstruction has been well studied for a while as given in [6]. The recent work on compressed sensing gives the conditions for exact reconstruction [7-10]. The compressive sensing can be applied effectively to the sparse signals but most of the signals encountered practically are not sparse [9]. Hence, DCT or

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wavelet transform (WT) can be first applied to the signals in order to obtain the sparse representation. The WT based compressive sensing becomes more successful because of its time-frequency localization and multi-resolution analysis property. It is observed from the literature that most of CS algorithms uses off-the-shelf wavelet basis i.e. orthogonal wavelet bases (Daubechies family), biorthogonal wavelet bases (9/7), and Haar basis. However, problem still exists in the field of image CS related to the choice of wavelets. The design of wavelet filters and investigations of their properties (near-orthogonality, regularity, time-frequency localization, linear phase, perfect reconstruction etc.) are not addressed fully in the literature for CS. It is also well known that the performance of wavelet based systems is highly dependent upon the choice of wavelets. Hence, this paper proposes compressive sensing scheme by the use of recent class of wavelet derived from the three-step lifting scheme using optimized time-bandwidth product based half-band polynomial and investigates their properties for CS [26].

Section-2 describes the review of the related work and the design of triplet half-band wavelet filter bank from the generalized half-band polynomial. In Section-3, a reconstruction algorithm based on orthogonal matching pursuit is presented. Section-4 presents the experimental results to evaluate the proposed scheme followed by conclusion in Section-5.

2. REVIEW OF 1-D BI-ORTHOGONAL WAVELET FILTERS

The biorthogonal wavelet is preferred over orthogonal wavelets for image processing due to its linear phase characteristics. Daubechies proposed the construction of orthogonal wavelets with compact support. However, such wavelets do not provide symmetric and linear-phase characteristics that are necessary for handling boundary distortions in images. Thus, linear phase symmetric filters obtained by relaxing orthogonality condition, called as biorthogonal wavelets[26]. Most of the popular biorthogonal FBs (e.g. Cohen-Daubechies-Feauveau (CDF) 9/7 and spline family of wavelet FBs) are designed by the factorization of Lagrange half-band polynomial (LHBP), which has the maximum number of zeros at $z = -1$ so as to achieve better regularity. However, LHBP filters do not have any degree of freedom and thus there is no direct control over frequency response of the filters. In order to have some independent parameters (which can be optimized to obtain some control over frequency response of the filter), Patil *et. al.*, [11] used general half-band filter factorization (not LHBP) to design two-channel biorthogonal wavelet FIR FBs (BWFB). However, factorization (decision of factorization of remainder polynomial and reassignment of zeros) improves the frequency response of one of the filters (analysis/synthesis) at the cost of the other filter (synthesis/analysis). The improvement in the frequency response of both the filters totally depends on the factorization of a half-band polynomial. This is somewhat tedious task for higher order polynomials. Lifting structure is also one of the attractive schemes to design FBs which provide structurally imposed PR property [27]. Based on the lifting scheme, Phoong *et. al.*, [12, 26] introduced a class of half-band pair filter bank which is defined by two kernels. However, it encounters certain restrictions for the control of its frequency response. In order to overcome this restriction, Ansari *et. al.*, [13,26] constructed a class of triplet half-band filter bank (THFB) using three kernels which has structural PR, feature-rich structure and simple design. In their work, two methods are proposed to design two-channel 1-D bi-orthogonal FBs based on the triplet of half-band filters. The first method is based on Lagrange half-band filters which results in maximal flatness filters (due to maximum regularity) with slow frequency roll-off. The second method uses Remez's algorithm, where equi-ripple filters with user-defined cut-off frequencies are obtained. It has sharp frequency roll-off but do not have the regularity condition. The shape parameter p is used to achieve a global shaping of the frequency response with greater flexibility. However, regularity order related to number of zeros at $z = -1$ is not specified. In order to bridge the gap between these two extremes of [13, 27], Tay and Palaniswami [14] recently introduced a novel approach to design a class of THFB. Parametric Bernstein polynomial is used to incorporate the arbitrary regularity condition (which is structurally imposed). However, Bernstein polynomial is suitable for nearly maximal flat frequency response rather than for ripple responses with sharp transition band. Chan and Yeung [15] presented a design of THFB with regularity using semi-definite programming (SDP). However, the constraints are approximated by large finite number of constraints which results in an inefficient design for higher order filters. Kha *et. al.*, [16] presented an efficient SDP method in order to design a class of THFB with optimal frequency selectivity for a given regularity condition. Amol *et. al.,* [17] investigated some problems in the design of the THFB and suggested a new class of THFB wavelets based on generalized half-band polynomial.

The importance of time-frequency localization property in wavelets and signal decomposition is given in [18, 22]. Tay [19,22] introduced an optimization of a balanced-uncertainty (BU) metric using PBP to design a class of HPFB. The designed filters have good balance of time-frequency localizations. Sharma *et. al.*, [20,22] proposed an Eigen filter based approach to design wavelet filter banks by optimizing time-frequency localization property. However, this proposed scheme is complicated and the optimization of second filter depends upon the optimization of first filter [21, 22]. Recently, Gawande *et. al.*, [22] introduced triplet-half band filter-bank based on balanced uncertainty optimization using Euler-Frobenius polynomial (EFP).

2.2.1. General Background

The two-channel BWFB is shown in Figure 1 where the analysis and synthesis high-pass filters (HPF) are obtained by quadrature mirroring the low-pass filters (LPF) so that aliasing cancellation is achieved as:

$$H_1(z) = z^{-1}G_0(-z);\ G_1(z) = zH_0(-z) \tag{1}$$

The analysis *scaling* and *wavelet* functions are given by the following equations:

$$\phi(t) = \frac{2}{H_0(\omega)|\omega = 0} \sum_n h_0(n)\,\phi(2t-n)$$

$$\Psi(t) = \frac{2}{G_0(\omega)|\omega = 0} \sum_n h_1(n)\,\phi(2t-n) \tag{2}$$

where, $h_0[n]$ and $h_1[n]$ are LP and HP filter coefficients.

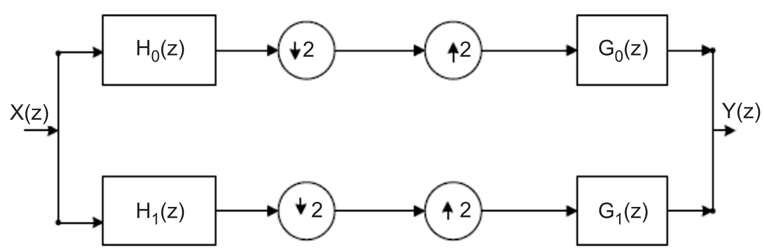

Figure 1: Two-channel filter bank

2.2.2. Triplet Half-band Filter Bank (THFB)

The analysis and synthesis LP filters of a class of THFB consists of three half-band filters given in [13, 27] are as follows:

$$H_0(z) = \frac{1+p}{2} + \frac{1}{2}T_1(z) - \frac{p}{2}T_0(z)\,T_1(z) \tag{3}$$

$$G_0(z) = \frac{1+pT_0(z)}{1+p} + \frac{1-p}{1+p}T_2(z)\left[\frac{1+p}{2} - \frac{1}{2}T_1(z)(1+pT_0(z))\right] \tag{4}$$

where, $T_0(z)$, $T_1(z)$, and $T_2(z)$ are the half-band filters which approximate as 1 in the pass-band and 0 in the stop-band. The analysis and synthesis HPFs are obtained using (1). The parameter p (degree of freedom) provides the flexibility in order to choose the magnitude at $\omega = 0.5\pi$.

2.2.3. Design of the THFB based on optimized time-bandwidth product [22]

The resultant half-band polynomial using EFP can be expressed as [22]:

$$P(z) = (1 + z^{-1})^{M} \, E(z)\sum_{i=0}^{L} \alpha_i z^{-i} \tag{5}$$

where, M is number of vanishing moments, E(z) is the EFP and α_i are the parameters to be determined such that perfect reconstruction is satisfied. The EFP is expressed as [22]:

$$E(z) = e(i+1)z^{-i}$$

where, N is the order of polynomial and its coefficients $e(i + 1)$ can be obtained from

$$e(i+1) = (-1)^k \binom{N+2}{k} (i+1-k)^{N+1}$$

Thus, the order of resultant P(z) is L + M + N, where L = (K/2) – 1. The sixth order half-band polynomial can be obtained by substituting M = 1, N = 3, and L = 2. With this, the resultant half-band polynomial can be written as:

$$P(z) = (1 + z^{-1})(1 + 11z^{-1} + 11z^{-2} + z^{-3})(\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2}) \tag{6}$$

The independent parameters are found by applying perfect reconstruction condition on the above equation is as follows:

$$\alpha_0 = -\frac{1}{240}, \alpha_1 = \frac{1}{20}, \alpha_2 = \frac{1}{240}$$

However, it is observed from the designed half-band filter that the resultant half-band polynomial is having the limited flexibility to control the frequency roll-off. In addition, it is also known that the different time-bandwidth product or time-frequency localization of half-band polynomial can be effective during the reconstruction process in compressive sensing application. Hence, this paper introduces the effect of the control of time-bandwidth product (TBP) in compressive sensing application. In order to achieve this, the resultant TBP polynomial can be written as:

$$Q(z) = P(z) + \beta_{L/2} R_{L/2}(z) \tag{7}$$

where, R(z) is the interpolating polynomial which is required to maintain the regularity and perfect reconstruction in Q(z). The detail design is given in [22] to obtain the optimized TBP half-band polynomial. Thus, the optimized time-bandwidth product half-band filter for sixth order half-band polynomial can be written as:

$$Q(z) = P(z) + \beta_1 R_1(z) \tag{8}$$

where the $\beta_1 = -0.3891$ is the value for optimized Time-bandwidth product. The detail on the computation of β_1 and R_1 is given in [22].

The resultant Q(z) is expressed for the specified value of β_1 is:

$$Q(z) = \frac{1}{32}\,[-0.0672 + 3\beta_1 + 8.0672 - 3\beta_1 z^{-2} - 3\beta_1 z^{-2} + 16z^{-3} + 8.0672 - 3z^{-4} - 0.0672 + 3\beta_1 z^{-6}]$$

Next, this designed optimized time-frequency localized half-band filter is used in three step lifting scheme to obtain the required 13/19 wavelet filter-banks. The transfer functions of the 13/19 wavelet filter banks are:

$$H_0(z) = -0.00098237 + 0.014690z^{-2} - 0.0375z^{-3} - 0.04027z^{-4} + 0.28051z^{-5} + 0.53915z^{-6} + 0.28051z^{-7} - 0.04027z^{-8} - 0.0375z^{-9} + 0.014696z^{-10} - 0.00098237z^{-12}$$

$$H_1(z) = 0.000028 - 0.00062z^{-2} + 0.00085z^{-3} + 0.0040z^{-4} - 0.0127z^{-5} - 0.0454z^{-6} + 0.0349z^{-7} + 0.291z^{-8} - 0.545z^{-9} + 0.291z^{-10} + 0.0349z^{-11} - 0.0454z^{-12} - 0.0127z^{-13} - 0.0040z^{-14} + 0.00085z^{-15} - 0.00062z^{-16} + 0.000028z^{-18}$$

where, $H_0(z)$ is the Low-pass filter and $H_1(z)$ is the high-pass filter. The frequency responses of these obtained filters are shown in Figure 2.

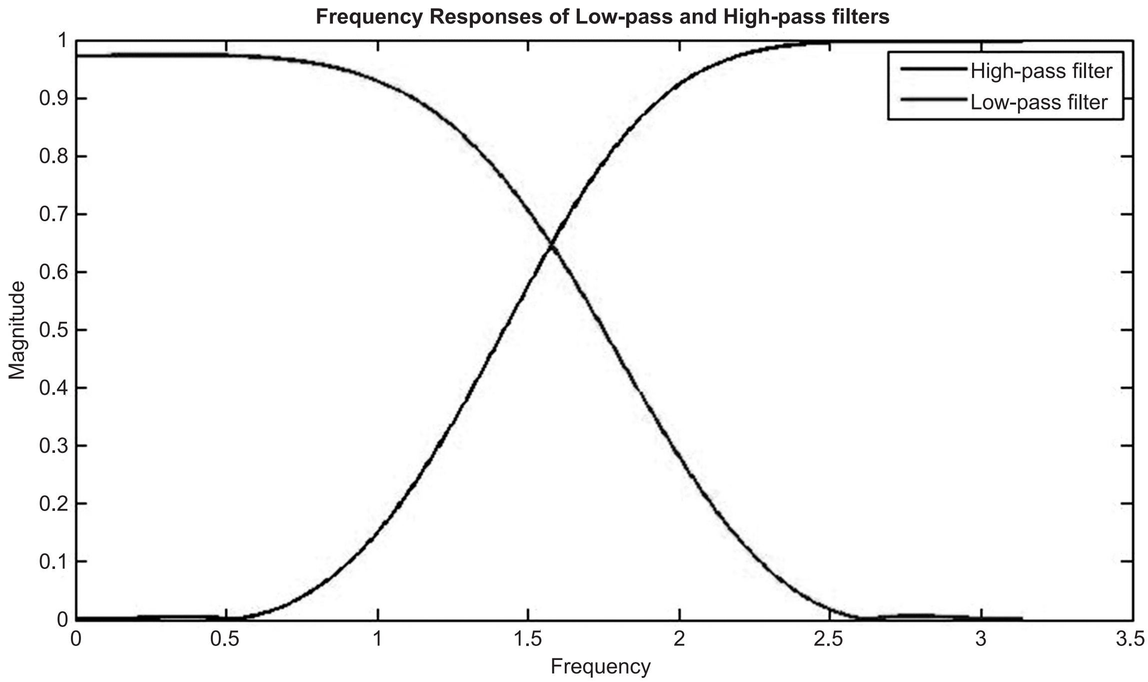

Figure 2: Frequency Response of Low-Pass and High-Pass filters

The corresponding scaling and wavelet functions are shown in Figure 3

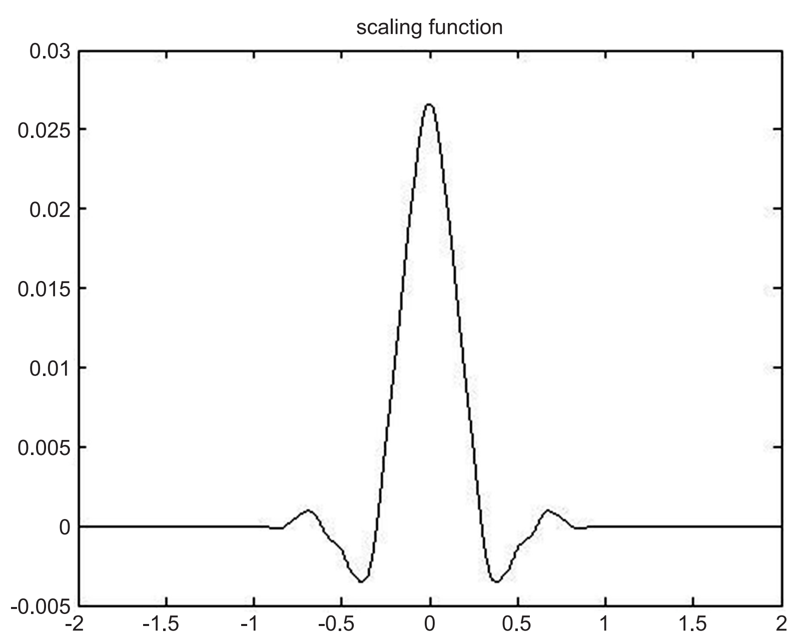

Figure 3: (a) Scaling Function

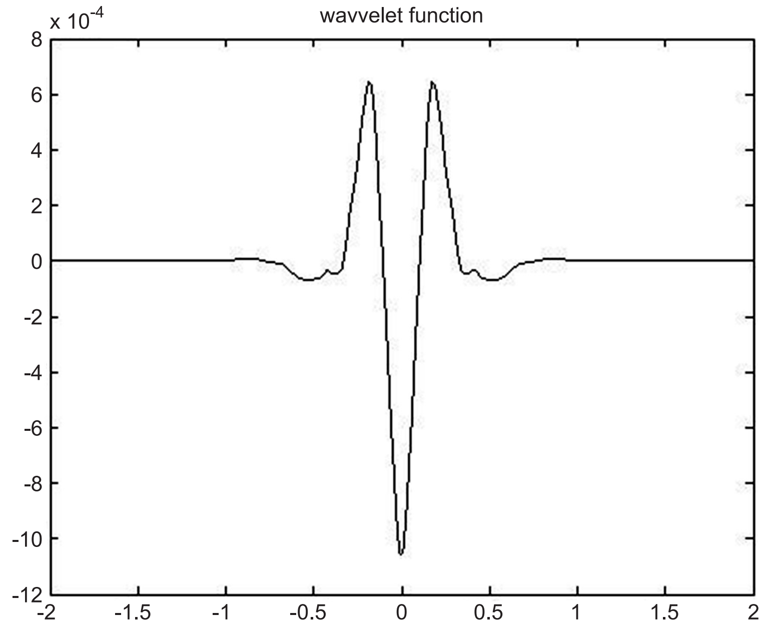

Figure 3: (b) Wavelet Function

In the next section, the properties of the above mentioned wavelet filters have been explored which are desirable for the compressive sensing scheme.

2.2.4. Properties of the 13/19 wavelet filters Desirable for CS [17, 26]

The desirable properties for CS are as follows [26]:

i) Linear phase (symmetry): These wavelets satisfy the linear-phase characteristics. This property can play an important role to avoid the boundary distortions.

ii) Near-orthogonality: These wavelets give near-orthogonality that can play a very important role on the quality of reconstructed output image [23-24].

iii) Frequency selectivity: These filters provide good frequency selectivity which is helpful to reconstruct the good quality image. This can be obtained by measuring the energy of error or energy of ripple [23-24].

vi) Regularity: It is also important to note that if regularity is imposed in the design, then the resultant wavelet basis has more approximation power in the decomposition section and is more regular in reconstruction [23]. This can be calculated using Sololev index [24]. This setting can be more desirable for image reconstruction.

v) Time-Frequency localization: The designed filters provide better time-frequency resolution, so it can be well adopted to characterize the variations in images.

3. PROPOSED IMAGE RECONSTRUCTION BASED ON COMPRESSIVE SENSING

The main objective of the CS scheme is to obtain the sparse image. This can be achieved using Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT), and Discrete Wavelet Transform (DWT). In this paper, DWT based on optimized TBP 13/19 wavelet filter is used to obtain the sparse image due to its multi-resolution analysis property. The researchers proposed some of the improved algorithms based on matching pursuit (MP) to reconstruct the images from the sparse image domain. It includes Basis Pursuit

(BP), Orthogonal Matching Pursuit (OMP), Sparsity Adaptive Matching Pursuit, etc. In this paper, OMP is used to reconstruct the image which has been suggested by Tropp and Gilbert [25]. As per the literature available, this is the first attempt to analyze the time-frequency localization property for the application of compressive sensing.

3.3.1. Orthogonal Matching Pursuit for Image Reconstruction

The cost for CS encoding and decoding significantly depend on the type of measurement matrix. Assume s is an arbitrary m-sparse image and $\{x_1, x_2, ..., x_N\}$ be a family of N-measurement vectors. The measurement matrix Φ can be obtained using $\Phi = N \times d$. It is observed that N-measurements of the signal can be collected in an N-dimensional data vector $v = \Phi S$, where $\{\Psi_1, \Psi_2, ..., \Psi_d\}$ are the column of the measurement matrix Φ [20]. As the signal S has only m-nonzero components, the data vector $v = \Phi S$ is the linear combination of m-columns from Φ. It means the data vector $v = \Phi S$ has *m-term* approximation over the dictionary Φ. Hence we can use the sparse approximation for image reconstruction. The objective is to determine which column of Φ participate in the data vector $v = \Phi S$ in order to identity the ideal signal S [20]. To achieve this, it is necessary to choose the column of Φ which is mostly correlated with the remaining part of data vector at each iteration. Next, subtract its contribution to data vector $v = \Phi S$ and iterate on the residue. Tropp and Gilbert [20] observed that after *m-iterations*, OMP can be identified as the correct set of column.

The proposed algorithm for Image reconstruction using OMP on 13/19 THFB wavelet filters based sparse image is as follows:

1. Acquire the image.
2. Design the 13/19 THFB based wavelet filters based on optimized time-frequency localization to solve the time-frequency uncertainty issue with existing wavelets.
3. Apply these wavelets on the acquired image to obtain the sparse image.
4. Generate the measurement matrix using Gaussian probability density function which is independent and identically distributed of size $N \times d$, where d is the number of rows of the original image.
5. Initialize the residual $r_0 = y$.
6. For each iteration (column of the sparse matrix) $t = 1 : N$ do
7. Find the column position i_t of $\Phi\Psi$ such that
$$i_t = \text{argmax}_i \, |\langle r_{t-1}, (\Phi\Psi)_i \rangle|$$
8. Augment the column position and the matrix of chosen $\Phi\Psi$.
9. Solve the least square problem in order to obtain the new residual as: $r_t = y - P_t y$
   where, P_t is the orthogonal projection onto the span of *t-columns* which are chosen from $\Phi\Psi$.
10. Increment and return to step 7 if $t < N$.

With this the reconstructed image from sparse approximation is obtained using designed wavelet filters and OMP.

4. EXPERIMENTAL RESULTS

The proposed CS method (combination of 13/19 THFB wavelet filters + OMP) is validated by comparing the peak signal to noise ratio (PSNR) values of the proposed scheme with the PSNR obtained from existing well-known cdf-9/7 wavelet filters. The performance of the proposed CS scheme has been tested over two8-bit standard images of size 256x256 for different values of N. The performance measure in terms of PSNR is measured in order to obtain an insightful analysis. The PSNR values are known to be mathematically convenient, so generally used for judging image quality.

In the first set of experimentation, we have directly used the CS theory for both of the images (Lena and cameraman) i.e. the random measurement directly applied to the test images instead of wavelet coefficients of the images. It is observed that the PSNR value is very small and the image cannot be reconstructed properly. The output reconstructed image is shown in Figure 4 (a). In order to improve the PSNR values and the quality of reconstructed image, we have designed 13/19 THFB wavelet filters and obtained the PSNR values for different values of N (measurement matrix). The results obtained from the proposed CS using the designed 13/19 THFB wavelet filters have been validated by comparing its result with the well-known cdf-9/7 wavelet filters. The experimental results for both the images using different values of measurement matrix generated with Gaussian probability density function is given Table 1 and Table 2.

Table 1
PSNR values (dB) of proposed CS method with existing Filters and Direct CS on Lena image

CS Methods	*Lena Image*			
	N = 150, d = 256	*N = 190, d = 256*	*N = 200, d = 256*	*N = 210, d = 256*
Direct method (image + OMP)	9.12	10.23	11.21	11.98
Standard 9/7 wavelet filter + OMP	24.092	25.53	25.98	26.25
Proposed method (11/9 wavelet filters + OMP)	24.82	25.75	26.094	27.98
Proposed method (THFB wavelet filters + OMP)	27.9574	30.6966	31.1941	31.7037

Table 2
PSNR values (dB) of proposed CS method with existing Filers and Direct CS on standard cameraman image

CS Methods	*Cameraman Image*			
	N = 150, d = 256	*N = 190, d = 256*	*N = 200, d = 256*	*N = 210, d = 256*
Direct method (image + OMP)	5.16	7.58	8.01	8.59
Standard 9/7 wavelet filter + OMP	23.123	25.47	26.32	27.12
Proposed method (11/9 wavelet filters + OMP)	24.08	26.08	26.66	27.96
Proposed method (THFB wavelet filters + OMP)	24.8943	28.5435	30.0256	30.6279

It is observed from Table 1 and Table 2 that the combination of DWT using the designed 13/19 filter with OMP yields slightly better performance than existing standard cdf-9/7 wavelet filters. This is due to its desired properties like flexible frequency response, near-orthogonality, regularity, better frequency selectivity, good time-frequency localization and linear-phase. Also, Figure 4 (a-c) shows the reconstructed image of Lena's picture using proposed scheme and existing technique.

Thus, the designed 13/19 THFB wavelet filters can be used for this compressive sensing application.

Figure 4 (a) Image Reconstruction using direct measurement for M = 200 (without applying DWT), (b) Image Reconstruction using cdf-9/7 wavelet filters using M = 200 and (c) Image reconstruction using designed 13/19 wavelet filters.

Figure 4: (a) Reconstruction using Direct (without DWT)(M = 200)

Figure 4: (b) Reconstruction using cdf-9/7 wavelet filters (M = 200)

Figure 4: (c) Reconstruction using designed 13/19 wavelet filters (M = 200)

5. CONCLUSION

A compressive sensing algorithm based on the use of a new class of wavelets and orthogonal matching pursuit is proposed in this paper. This paper investigates the importance of time-bandwidth product property of wavelets in the compressive sensing application. In this paper, a recent class of a wavelet basis has been designed, named as 13/19 THFB wavelet filters based on the optimized time-bandwidth product of weighted Euler-Frobenius half-band polynomial and investigated its properties which are desirable for compressive sensing. The performance of this scheme is compared with standard traditional wavelet cdf-9/7 and it is observed that the designed 13/19 wavelet filters gives better performance than the existing filter and direct image compressive sensing algorithm. Hence, the designed wavelet can be used in the field of compressive sensing with the combination of orthogonal matching pursuit due to its fast and easier implementation property.

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