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A Modeling on Frequency of Rectangular Plate

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ABSTRACT

A Modeling emphasises on natural frequency on non-uniform and non-homogeneous rectangular plate. For non-uniformity of material, thickness of plate is considered linear and for non-homogeneity in the plate, Poisson ratio vary circular which is not discussed earlier. To make the problem realistic, temperature variation is taken to be bi-parabolic in nature. Rayleigh Ritz technique is used for solving governing differential equation and to obtain both modes of natural frequencies. MAPLE software is used for all the calculations in the present study. A comparison of results with existing result is also done with the help of tables, which shows the importance, existence and usability in the further research.

Keywords: Circular Variation, Rectangular Plate, Vibration

1. INTRODUCTION

All the engineering structures (mechanical and electrical) have vibrational phenomenon while performing. It is desirable as well as undesirable (in machine tools). These days, studies based on vibrations have become a great interest among researchers and scientists. Tapered plates with temperature plays an essential role in moderen engineering structures like automobiles industries, nuclear reactor, misiles and power plants.

The main object of scientists and researchers are to optimize vibration and get desired frequency. For this, physical properties of material can not be disdain. Various studies based on effect of temperature (one dimensional) as well as non-homogeneity (linear or parabolic) have been done. However very less work is done on two dimensional temperatures and no work has been reported on circular or other variation as non-homogeneity effect.

Transverse vibration analysis of rectangular plate having edges elastically against rotation and having two direction variations in thickness is discussed by Laura et. al [1]. Vibrational analysis of rectangle plate having thickness variation (linear and parabolic) along both the axes is studied by Gupta and Khanna [2-3]. An effect of bi-directional exponential variation in thickness on vibrational modes using rectangle plate have described by Gupta et. al [4]. Lal and Dhanpati [5] have depicted the effect of non-homogeneity on vibration of orthotropic rectangular plates having varying thickness variation resting on pasternak foundation. Effect of temperature, variation in Poisson ratio as non-homogeneity and simultaneous variation in density as well as in Poisson ratio to vibrational behevior of rectangular plate have described by Khanna and Kaur [6-8]. The transverse vibrations on simply supported plate with an oblique cut and generalized anisotropy have studied by Avalos and Laura [9]. Gupta and Singhal [10] studied parabolic thickness and temperature effect on vibrational frequencies of non-homogeneous rectangle plate. Effect of variation in Poisson ratio with linear temperature variation in both the axes

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have discussed by Khanna et. al [11]. Different combination of variation in parameter and shapes is provided by Leissa [12] in his monograph. An evaluation of time function using tapered visco-elastic plate has studied by Khanna and Singhal [13]. Khanna and Kaur [14] have discussed the vibration of tapered rectangular plate under non-uniform temperature field. They gave the comparison of linear tapering and exponential tapering. An orthotropic rectangular plate by taking two dimensional thickness and temperature variation has studied by Sharma et. al [15]

In this study, the authors have attempted to study the effect of circular variation in Poisson ratio as non-homogeneous constants to frequency modes of vibrations and compare the results with [6] having exponential variation in Poisson ratio on non-uniform rectangular plate. The authors have considered linear variation in thickness and bi-parabolic temperature variation in the present study. First two modes of vibrations are obtained using Rayleigh Ritz technique as numerical data which is represented in the form of tables for different combination of value of thermal gradient, non-homogeneous constant, tapering constant and aspect ratio.

2. ANALYSIS OF MODEL AND SOLUTION

2.1. Material

Due to non-uniformity in material, thickness vary linearly as

$$j = j_0\left(1+\frac{\beta x}{a}\right) \tag{1}$$

and β is known as tapered constant. Thickness becomes constant at β = 0.

Bi-parabolic variations in temperature in both the dimensions are

$$T = T_0\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right) \tag{2}$$

Where a, b and T_0 represents the length, breadth and temperature at origin of the plate.

The temperature dependent modulus of elasticity is taken as

$$K = K_0(1-\gamma T) \tag{3}$$

Where K_0 is the Young's modulus at reference temperature and γ is known as slope variation.

From eqn. (2), eqn. (3) becomes

$$K = K_0\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right] \tag{4}$$

and $\alpha = \gamma T_0, (0 \le \alpha < 1)$ is known as thermal gradient.

As material of the plate is considered to be non-homogeneous, circular variation is taken in Poisson ratio as

$$\nu = \nu_0\left[1-m_1\left(\sqrt{1-\frac{x^2}{a^2}}\right)\right] \tag{5}$$

Where $m_1\,(0 \le m_1 \le 1)$ are known as constants of non-homogeneity.

For obtaining natural frequency, two term deflection function along with boundary condition is given by

$$\Phi = \left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2 \left[C_1 + C_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \quad (6)$$

$$\Phi = \Phi_{,x} = 0,\ x = 0,\ a \text{ and } \Phi = \Phi_{,y} = 0,\ y = 0,\ b \quad (7)$$

2.2. Solution to Find Vibrational Frequency

To solve the problem, Rayleigh Ritz technique is applied. The method based upon the phenomena that the maximum potential energy (P) must equal to maximum kinetic energy (K). Therefore,

$$\delta(P - K) = 0 \quad (8)$$

Where expressions of P and K is given by

$$P = 0.5\int_0^a\int_0^b D\left\{(\Phi_{,xx})^2 + (\Phi_{,yy})^2 + 2\nu\Phi_{,xx}\Phi_{,yy} + 2(1-\nu)(\Phi_{,xy})^2\right\} dydx \quad (9)$$

$$K = (0.5)p^2\rho\int_0^a\int_0^b j\Phi^2\, dydx \quad (10)$$

And $D = Kj^3/12(1-\nu^2)$ is known as plate's flexural rigidity and ρ is the density of the plate.

Using eqns. (1), (4) and (5), the value of flexural rigidity becomes

$$D = \frac{K_0 j_0^{\,3}\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right]\left(1+\frac{\beta x}{a}\right)^3}{12\left(1-\nu_0^{\,2}\left[1-m_1\left(\sqrt{1-\frac{x^2}{a^2}}\right)\right]^2\right)} \quad (11)$$

On using eqns (1), (5), (9), (10) and eqn (11), eqn (8) converted into

$$\delta\left(P_{max} - \lambda^2 K_{max}\right) = 0 \quad (12)$$

Where

$$P_{max} = \int_0^a\int_0^b\left[\frac{K_0 j_0^{\,3}\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right]\left(1+\frac{\beta x}{a}\right)^3}{24\left(1-\nu_0^{\,2}\left[1-m_1\left(\sqrt{1-\frac{x^2}{a^2}}\right)\right]^2\right)}\left\{\begin{array}{l}(\Phi_{,xx})^2 + (\Phi_{,yy})^2 + 2\nu_0\left[1-m_1\left(\sqrt{1-\frac{x^2}{a^2}}\right)\right]\Phi_{,xx}\Phi_{,yy} \\ +2\left(1-\nu_0\left[1-m_1\left(\sqrt{1-\frac{x^2}{a^2}}\right)\right]\right)(\Phi_{,xy})^2\end{array}\right\}\right]dydx \quad (13)$$

$$K_{max} = (0.5)p^2\rho j_0\int_0^a\int_0^b\left(1+\frac{\beta x}{a}\right)\Phi^2\, dydx \quad (14)$$

and

$$\lambda^2 = \frac{12\rho_0 p^2 a^4}{K_0 j_o^{\,2}} \quad (15)$$

Eqn (15) represents parameters of frequencies. Eqn (12) consists of two unknowns i.e., C_1, C_2 because of eqn (6) which can evaluated by

$$\frac{\partial\left(P_{max} - \lambda^2 K_{max}\right)}{\partial C_i} = 0,\ i = 1, 2 \quad (16)$$

On simplifying eqn (16), we get

$$q_{i1}C_1 + q_{i2}C_2 = 0,\ i = 1, 2 \quad (17)$$

Where, $q_{i1}, q_{i2}\ (i = 1, 2)$ comprises with frequency λ^2.

To obtained vibrational frequencies, it is necessary that determinant of coefficient matrix from eqn. (17) must be zero

$$\begin{vmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{vmatrix} = 0 \quad (18)$$

Eqn (18) gives vibrational frequency modes as λ_1 and λ_2.

3. RESULTS DISCUSSION AND COMPARISON

Duralium material is considered for all the numerical calculations. Vibrational frequencies are obtained corresponding to different value of non-homogeneity constants, taper constant, aspect ratio and thermal gradient. The parameter used for the calculations are as follows.

$$K_0 = 7.08 * 10^{10}\ N/M^2,\ \nu_0 = 0.34, \rho_0 = 2.80 * 10^3\ Kg/M^3\ \&\ j_0 = 0.01m$$

The findings are presented in tabular form.

Table 1
Frequency (λ) vs Thermal Gradient (α) for a/b = 1.5

α	$m_1 = \beta = 0.0$		$m_1 = \beta = 0.6$	
	λ_1	λ_2	λ_1	λ_2
0.0	64.77	255.98	81.42	321.79
0.2	61.73	244.03	77.99	308.12
0.4	58.52	231.47	74.36	293.82
0.6	55.11	218.19	70.53	278.80
0.8	51.46	204.05	66.44	262.93

Table 1 contains both natural vibrational frequency modes and display the facts that frequency vibration decreases with the continuous increment in thermal gradient for two cases (m_1 = β = 0.0) and (m_1 = β = 0.6). For case two (m_1 = β = 0.6) natural frequencies of vibration is higher than as compared to case one (m_1 = β = 0.0).

Table 2 provides natural frequencies for both the modes and showed the fact that both natural vibrational frequency modes decreases with the continuous increment in non-homogeneity for two cases (α = β = 0.0) and (α = β = 0.6). Table 2 also provides that vibrational frequencies increases as combined value of taper constant and temperature gradient increases form 0.0 to 0.6.

Table 3 accommodates both modes of natural frequencies. Authors conclude that vibrational frequencies are increasing as taper constant increases for two cases (α = 0.2, m_1 = 0.0) and (α = 0.2, m_1 = 0.6). But both natural frequencies modes are decreases as the value of non-homogeneity increasing form 0 to 0.6. For m_1 = 0.0, authors obtained same frequencies for both modes as in [6] but for m_1 = 0.6, frequencies are less than as compared to [6].

Table 2
Frequency (λ) vs Non-homogeneity (m_1) for a/b = 1.5

m_1	$\alpha = \beta = 0.0$		$\alpha = \beta = 0.6$	
	λ_1	λ_2	λ_1	λ_2
0.0	64.77	255.98	73.74	291.50
0.2	63.49	250.98	72.40	286.19
0.4	62.49	247.07	71.34	281.99
0.6	61.73	244.16	70.53	278.80
0.8	61.21	242.16	69.96	276.55
1.0	60.90	241.02	69.60	275.17

Table 3
Frequency (λ) vs Taper constant (β) for α =0.2, a/b = 1.5

β	$m_1 = 0.0$		$m_1 = 0.6$	
	λ_1	λ_2	λ_1	λ_2
0.0	61.73{**61.73**}	244.03{**244.03**}	58.86 {**58.86**}	232.82{**232.82**}
0.2	68.16{**68.17**}	269.38{**269.39**}	65.01{**72.42**}	257.14{**286.76**}
0.4	74.83{**74.83**}	295.60{**295.60**}	71.41{**79.63** }	282.31{**315.44**}
0.6	81.67{**81.68**}	322.47{**322.48**}	77.95{**87.02**}	308.12{**344.82**}
0.8	88.65{**88.66**}	349.85{**349.86**}	84.69{**94.54**}	334.43{**374.75**}
1.0	95.74{**95.74**}	377.63{**377.64**}	91.50{**102.17**}	361.13{**405.09**}

The value in bold bracket {} are from [6]

Table 4 displays both modes of frequencies for aspect ratio. One can easily conclude that with the increment in aspect ratio, frequencies are increasing for both the cases ($\alpha = 0.2$, $\beta = 0.2$, $m_1 = 0.0$) and ($\alpha = 0.2$, $\beta = 0.2$, $m_1 = 0.6$). On other hand, frequencies for both modes are decreasing as the value on non-homogeneous constant increasing from 0 to 0.6. For $m_1 = 0.0$, authors get the same frequencies for both modes as in [6] and for $m_1 = 0.6$, frequncies are less than as compared to [6]

Table 4
Frequency (λ) vs Aspect ratio (a/b) for α =0.2, β = 0.2

a/b	$m_1 = 0.0$		$m_1 = 0.6$	
	λ_1	λ_2	λ_1	λ_2
0.25	25.76{**25.76**}	105.15{**105.16**}	24.73{**27.92**}	101.08{**114.46**}
0.50	27.76{**27.77**}	111.22{**111.23**}	26.62{**29.99**}	106.82{**120.72**}
0.75	32.33{**32.34**}	127.33{**127.33**}	30.95{**34.75**}	122.06{**137.43**}
1.0	40.44{**40.44**}	158.28{**158.29**}	38.64{**43.22**}	151.44{**169.76**}
1.25	52.41{**52.42**}	205.80{**205.81**}	50.02{**55.80**}	196.62{**219.70**}
1.5	68.16{**68.17**}	269.38{**269.39**}	65.01{**72.42**}	257.14{**286.76**}

The value in bold bracket {} are from [6]

4. CONCLUSION

From the above comparison and results, authors concludes that in the present study, frequencies are less than due to effect of circular variation in Poisson ratio as compared to [6] (exponential variation in Poisson ratio) as shown in table 3 and table 4. When non-homogeneity constant become zero, frequencies from [6] are obtained from the present study as shown in table 3 and table 4. Therefore, authors conclude that desired frequencies can be obtained by taking appropriate variation in the parameters. The objective of the study is to give some numerical data to researchers or scientists to fulfill the requirement of modern world.

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