I J C T A, 10(9), 2017, pp. 303-314 ISSN: 0974-5572

Dual Tree Complex Wavelet Transform based Robust Video Watermarking

Neeta Deshpande*

ABSTRACT

Watermarking of digital data for copyright protection is the current need of the society. Scheming robust algorithms for embedding watermarks in the media resolves copyright infringements. A robust video watermarking technique that integrates two concepts, the Dual Tree Complex Wavelet Transforms (DT-CWT) and the Just Noticeable Difference (JND) for embedding the robust watermarks in videos is proposed in the paper. When applied to images, the DT-CWT decomposes the image into six sub bands based on frequency scales and orientations. The proposed algorithm initially computes the just noticeable difference and then embeds the watermark in the luminance component of first level sixth band of 2D DT-CWT of the video frame. Block DCT of the sixth sub band is calculated and watermark is embedded in the midband DCT coefficients of this block. The proposed algorithm also gives successful results when the watermark is embedded in the fourth band of the luminance adapted JND frame. The designed algorithm is capable to resist the video attacks, signal processing attacks and geometrical attacks to verify the robustness.

Keywords: Image Denoising, robust, algorithm, videos, attacks

1. INTRODUCTION

The foremost technique intended for protection of Intellectual Property Rights [IPR] and copyright protection is Digital watermarking [1]. The Dual Tree Complex Wavelet Transforms (DT-CWT) and the Just Noticeable Difference (JND) are two well known concepts in watermarking. A robust video watermarking algorithm deploying the assimilation of these two techniques is proposed in the paper.

The dual tree complex wavelet transform was developed by Kingsbury [2]. When applied to images, the DT-CWT decomposes the image into six sub bands based on frequency scales and orientations. The proposed algorithm preferred the sixth and fourth sub bands from the first scale and sixth sub band from the second scale for embedding the watermarks in the video frame.

Human visual perception is not perceptive to the Just Noticeable difference (JND) [3]. JND can be combined with watermarking for better image quality evaluation as in [4, 5]. It can also be deployed for video compression [6, 7, and 8]. The luminance adaptation represents the fact that the HVS is sensitive to the luminance difference. Consequently, in the JND model of [9] the luminance adaption function is deduced according to Weber‘s law. The proposed algorithm utilizes this concept to compute the JND estimation.

In this paper, the subsequent section elaborates the mathematical model for the watermarking process. Further section steps towards focusing on DT-CWT followed by the elucidation of the JND. The fourth section explicates the proposed technique. Further section elaborates the results and discussions of embedding the watermarks in the sixth and fourth bands of the luminance adapted DT-CWT. A comparative analysis with the various combinations of analysis and synthesis filters using the proposed algorithm for robustness is also elaborated. The comparison of the proposed video watermarking technique with the DT_CWT techniques available in literature is justified with results in the second last section followed by conclusion.

* Associate Professor Department of Computer Engineering D.Y. Patil College of Engineering Akurdi
Manuscript received XX, XX, 20XX; accepted XX, XX, 20XX.

2. MATHEMATICAL MODEL FOR WATERMARKING PROCESS

The basic components involved in robust watermarking are watermark embedding and watermark detection. The watermarking system can be defined as a set *WS* = {*E*, *V*} where *E* represents the set of parameters {*e0, e1, e2, e3, e4, e5*} and V is the set of functions {*v0, v1, v2, v3*}. Let *Fe* be a rule of *E* into *V*, such that for given parameter, the function *Fe (e)* → *v* or *Fe (e)* → *e'*, where *e* and *e'* € *E* and *v* € *V*. For example *Fe (e0)* → *{e1, e2}* € *E* or *Fe (e0, e1, e2)* → *v2* such that *v2* € *V*. The mathematical model of watermarking process is shown in Figure 1.

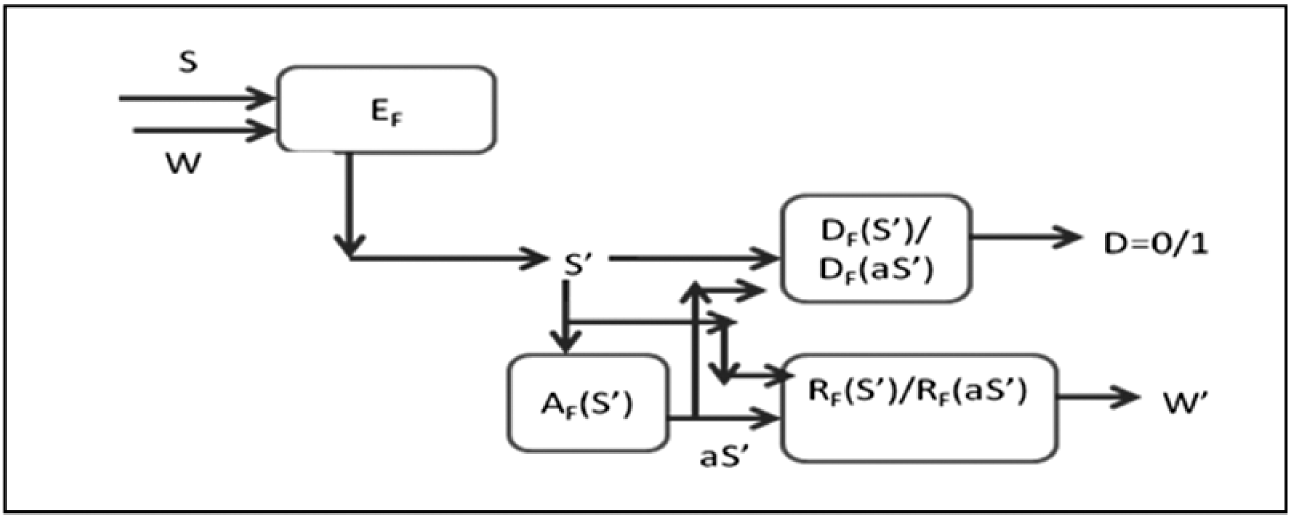


Figure 1: Mathematical model for watermarking process

Here *E = {e0, e1, e2, e3, e4, e5}* is a set of parameters

Where

e0 ≡ *S* → Signal in which watermark is to be embedded

e1 ≡ *W* → The Watermark to be embedded in the base signal

e2 ≡ *S'* → The Watermarked signal

e3 ≡ *aS'* → Watermarked attacked Signal

e4 ≡ *D* → Detected Watermark

e5 ≡ *W'* → Extracted Watermark

and V is the set of functions {v0, v1, v2, v3}

v0 ≡ *EF(S)* → Embedding Function

v1 ≡ *AF(S')* → Attacking Function

v2 ≡ *DF(S')* || *DF(aS')* → Detector Function

v3 ≡ *RF(S')* || *RF(aS')* → Retrieval or Extraction Function

Figure 2 shows the activity diagrams for the various functions involved in the mathematical model when the watermark *W* is an image i.e. it is a set of pixels $\{W11, W12\ldots W_{m*n}\}$. The signal for embedding a watermark is a frame *S*, $\{S11, S12\ldots S_{M*N}\}$ of the video where *m*n* and *M* N* are the widths and heights of the watermark image and the video frame respectively.

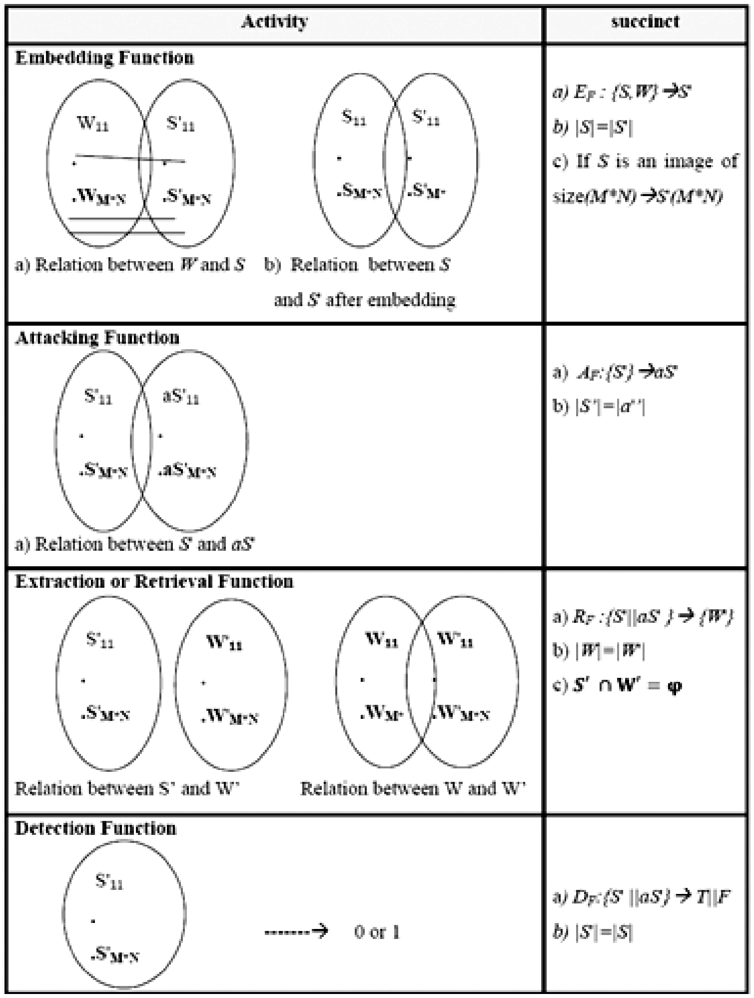


Figure 2: Activity Diagram for functions involved in watermarking process

3. DUAL TREE COMPLEX WAVELET TRANSFORM (DT-CWT)

An analytic wavelet transform, DT-CWT was first introduced by Ivan W. Selesnick et al. in 1998 [2]. In this, two real DWT's are combined together to form a dual tree. The real part of the transform is given by first DWT and the imaginary part is given by the other.

When a 2D DWT is applied to an image, it generates output in three orientations giving the features at angles of 90^0, $\pm 45^0$ and 0^0. An analytic wavelet $\psi(x)$ is composed of two real wavelets $\psi(x) = \psi h(x) + j\psi g(x)$, where $\psi h(x)$ and $\psi g(x)$ form a Hilbert transform pair. i.e. they are orthogonal. Eventually, a 2D-DTCWT will generate six directional sub bands per level. These sub bands give the details of the image at $\pm 15^{0,}$ $\pm 45^0$, $\pm 75^0$. Thus it achieves a 4:1 redundancy. In [10], it is mentioned that by doubling the sampling rate, approximate shift invariance is possible with standard DWT. This is possible by eliminating the down sampler after the 1- level filter and the samples are evenly spaced. Thus it can be summarized by saying that the discrete real wavelet transform (DWT) uses one tree per dimension whereas the CWT uses two trees so a quad-tree is achieved for a 2D image. The main properties of DT-CWT as in [2] are

- Approximate shift invariance
- Good directional selectivity
- Fast implementation algorithm with complexity O(N) and
- 4:1 redundancy in 2-D.

4. JUST NOTICEABLE DIFFERENCE (JND)

The perceptivity threshold of Human Visual System (HVS) below which any change cannot be detected is known as the JND. Weber‘s law states that the just noticeable difference between two stimuli is proportional to the magnitude of the stimuli. The luminance adaption based on Weber‘s law and the spatial masking functions derived from luminance difference are the two factors essential for computing the just noticeable difference. The Human visual system is sensitive to the luminance difference and the luminance adaptation models are based on this concept [11]. In most of the models, the Weber‘s law is helpful in deriving the luminance adaption function in spatial domain. The spatial masking is estimated based on luminance contrast. Various models are proposed in the literature where spatial masking is computed in a variety of ways. Maximum signal along four directions is explored for computing the effect of spatial masking in Chou and Li‘s model [11]. The concept of bigger luminance contrast in horizontal and vertical directions is explored in Chiu and Berger‘s model [12] for determining the spatial masking. In the proposed algorithm, the architecture of just noticeable difference in [13] is adapted for computing the JND matrix.

5. PROPOSED DT-CWT BASED VIDEO WATERMARKING

The proposed algorithm embeds the watermark in the luminance component of first level sixth band of 2D DT-CWT of the image. Block DCT of the sixth sub band is calculated and watermark is embedded in the midband DCT coefficients of this block as in [14]. The proposed algorithm also gives successful results when the watermark is embedded in the fourth band of the luminance adapted JND frame.

Embedding Algorithm

1. Comprehend the frame of the video
2. Set the filter to be used as analysis and synthesis filters in dual tree complex wavelet transform.
3. Set the decomposition level. The proposed scheme is being tested with 3 levels of decomposition.
4. Determine the threshold alpha for embedding the watermark
5. Read the image *Wi* to add as a watermark and convert it to vector. Calculate its size for embedding it as a watermark *(mr, mc)* and store this in a vector.
6. Select the frame of the video and extract the luminance component.
7. Estimate the background luminance distortion and compute JND matrix. The procedure for computing the JND is as follows. For all frames *F(xi, yi)*

   {

   a) Spatial masking estimation:

$$B_{lum(x,v)} = \{[mask[5,5] * F(xi, yi)]\} / 32 \tag{1}$$

   b) Luminance adaption function is deduced as in [13]

   B(*x*) = *background luminance pixel*, *Bo* = 127

   γ = 3/128, T0 = 17

$$lum(xi, yi) = \begin{cases} T0 * \left(1 - \sqrt{\dfrac{B(x)}{B0}}\right) + 3 & if\ B(x) < 0 \\ \gamma * (B(x) - B0) + 3, & else \end{cases} \tag{2}$$

   c) To compute the overall JND estimation of the pixel combine the spatial masking estimation and the luminance adaption function.

$$JND(xi, yi) = \left\{Tlum(xi, yi) * \left(B_{lum(x,y)} + 1\right)\right\} \tag{3}$$

}

8. Consider the JND matrix obtained in step 7 as the cover object for embedding the watermark. The steps for watermark embedding are
   a) Compute size of the cover object.
   b) Using the above selected filter, and perform a 2 level DT-CWT decomposition on the JND matrix.
   c) For watermark embedding

   {

   i) Select the analysis and the synthesis filters required for the DT-CWT

   ii) Compute the six bands of 2D- DT-CWT adapting the method by Prof Kingsbury

$$\psi_{1,1}(x, y) = \psi h(x)\, \psi h(y),$$
$$\psi_{2,1}(x, y) = \psi g(x)\, \psi g(y),$$
$$\psi_{1,2}(x, y) = \psi h(x)\, \psi h(y),$$
$$\psi_{2,2}(x, y) = \psi g(x)\, \psi g(y),$$
$$\psi_{1,3}(x, y) = \psi h(x)\, \psi h(y),$$
$$\psi_{2,3}(x, y) = \psi g(x)\, \psi g(y),$$

   iiii) Extract the sixth band, Band_host i.e. the low energy band for embedding the watermark.

   iv) Generate two random sequences, PN sequence 0 and PN sequence 1 considering the size of the Band_host.

   v) Compute 8x8 block DCT of the Band host

   vi) Insert PN sequence 0 in the mid band components of the DCT block if the message bit contains 0; else insert PN sequence 1 in the mid band components of the DCT block. The equation used for watermark embedding is

$$Band\_host(x, y) = Band\_host(x, y) + k * wm(x, y) \tag{4}$$

   vii) Transform every block to spatial domain to get the watermarked image by performing inverse DCT.

   viii) Replace the modified Band_ host coefficients with the original

   ix) Perform inverse DT-CWT.

   }

9. Reconstruct the video from frames.

Extraction Algorithm

1. Comprehend the image to be embedded as a watermark and calculate its size.
2. Follow the steps of the embedding algorithm till step 8iii to extract the sixth sub band.
3. The sixth sub band image is broken up into same 8×8 blocks, and a DCT is performed.
4. Compare the same PN sequence to the middle frequency values of the transformed block in the above embedding process.
5. If the correlation between the sequences exceeds some threshold T, a 1 is detected for that block; otherwise a 0 is detected.

6. RESULTS AND DISCUSSIONS

The DT-CWT is explored for embedding the watermark in the sixth band of the first level of DT-CWT of the luminance adapted JND frame. Also successful results are obtained for embedding the watermark in fourth band of first level of DT-CWT. Satisfactory results are obtained for both the bands and the algorithm is able to resist the video attacks, signal processing attacks and geometrical attacks to verify the robustness. Attempts are made to embed the watermark in second level's sixth band. The watermark embedded in the sixth band of second level DT-CWT gave adequate results with a PSNR in the range of 34 to 39 for all the frames. The watermark is extracted from sixth band of second level of DT-CWT with NC value of 0.68. However in the second level, the watermark is degraded to a recognizable extent on application of attacks.

6.1. Stepwise Implementation Results of the Algorithm

6.1.1. Preprocessing for Luminance Adapted JND Frame

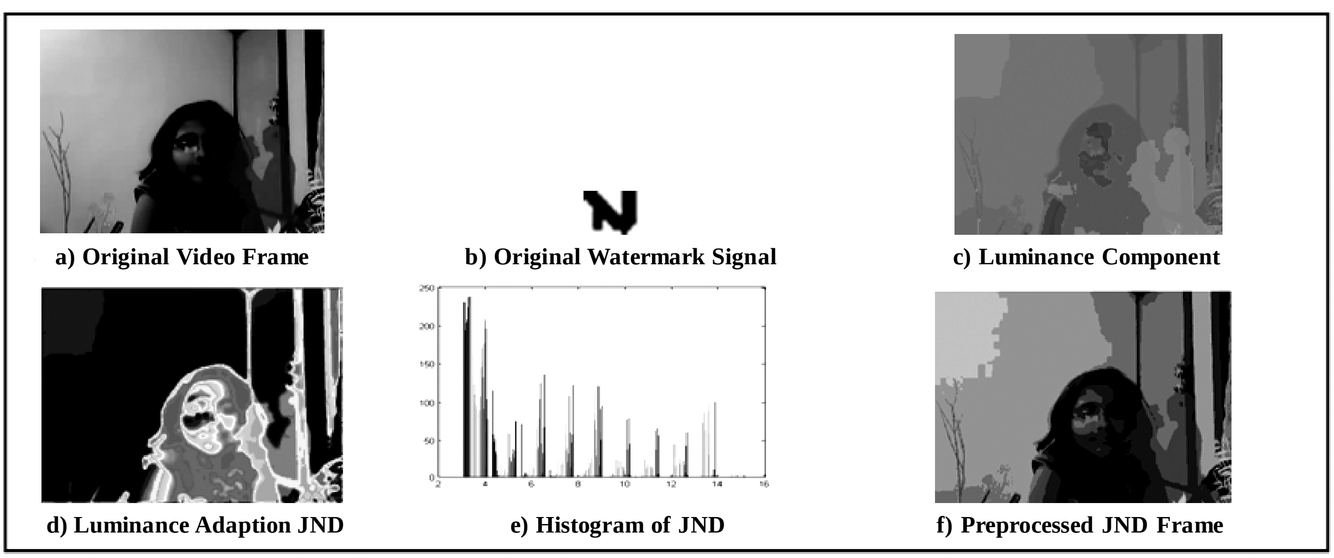
Fig 3 shows the results of the preprocessing step required to achieve the preprocessed luminance adapted JND frame for embedding the watermark. The original frame of the video and the original watermark are shown in Figure 3a) and Figure 3(b) respectively. Figure 3(c) shows the next part of implementation i.e. the result of the luminance component of the frame. The luminance adapted JND matrix and its histogram are revealed in Figure 3(d) and Figure 3(e) respectively. Finally the preprocessed frame required for watermark embedding is shown in Figure 3(f).

6.1.2. Computing the 2D DT-CWT

A DT-CWT is applied to this preprocessed watermarked frame and the six oriented images in six sub bands are obtained that are shown in Figure 4(a) to Figure 4(f). The sixth oriented frame i.e. the sixth sub band is selected for embedding the watermark. Figure 4(g) displays the frequency contents of the sixth sub band.

6.1.3. Watermark Embedding

A block DCT is computed for the sixth sub band frame and watermark is embedded in mid level DCT coefficients of the block. Figure 5(a) shows the value of a particular block on a circular grid. This value of the block is before computing the DCT. The results of DCT computed block are shown in Figure 5(b). The watermark is embedded in the midband components of this block and IDCT is computed. The IDCT computed



a) Original Video Frame
b) Original Watermark Signal
c) Luminance Component
d) Luminance Adaption JND
e) Histogram of JND
f) Preprocessed JND Frame

Figure 3: Preprocessing Step to achieve the Luminance Adapted JND Frame

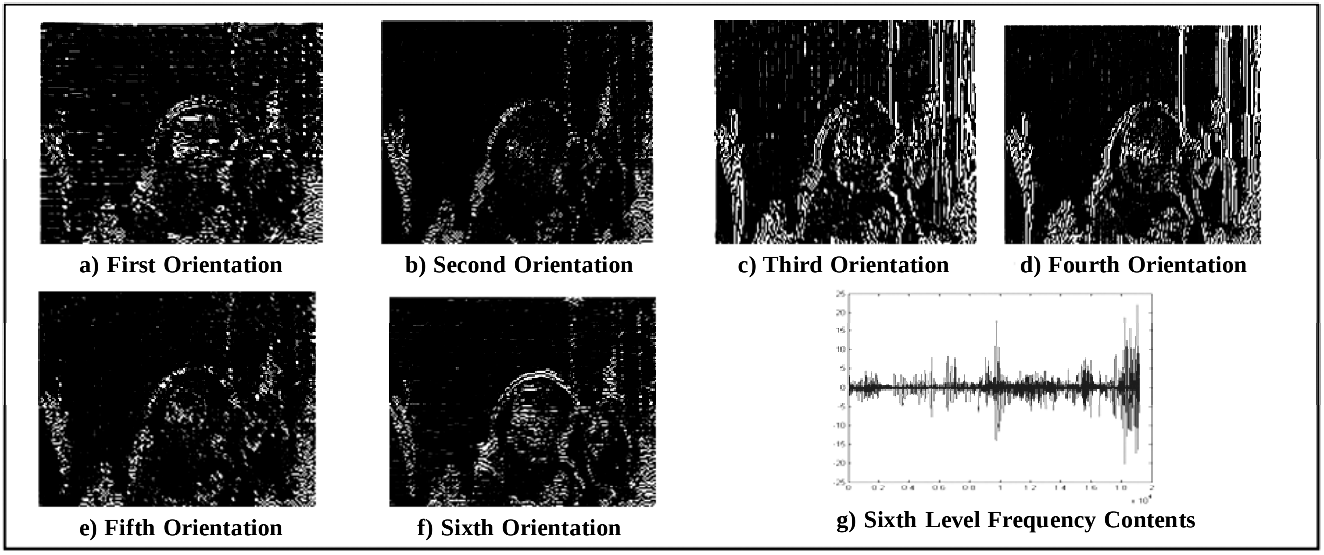


Figure 4: Six Oriented Sub bands of Preprocessed Luminance Adapted JND Frame

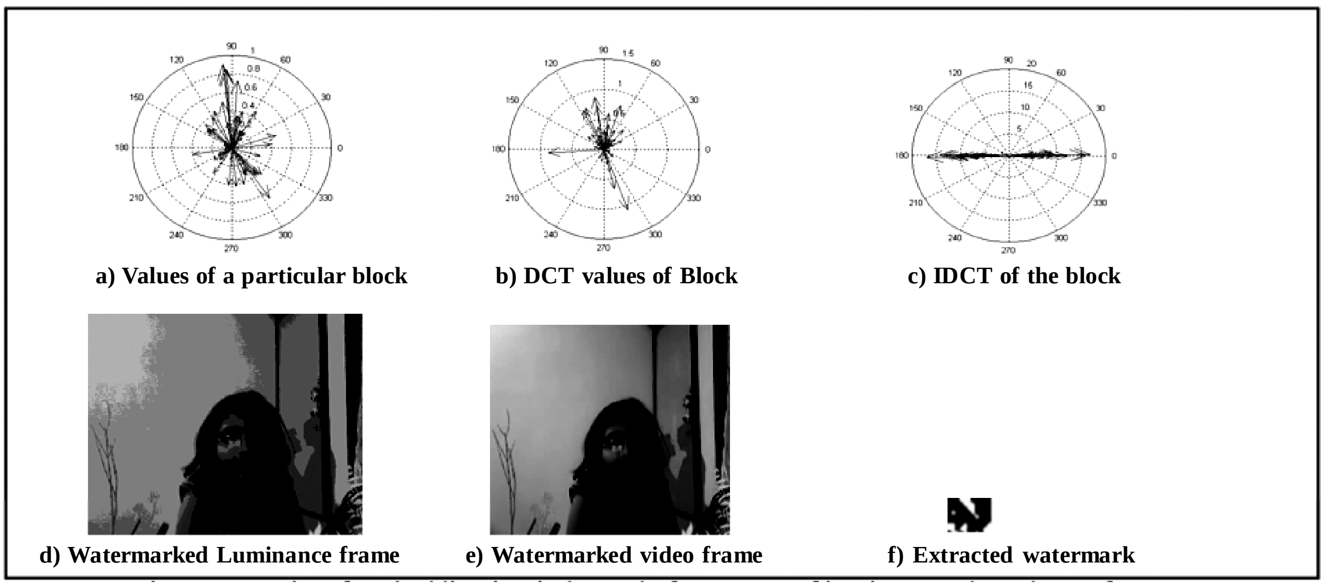


Figure 5: Results of Embedding in Sixth Band of 2D-CWT of luminance adapted JND frame

block is shown in Figure 5(c). An inverse DT-CWT is applied to get the watermarked luminance frame shown in Figure 5(d). The converted RGB watermarked video frame and the extracted watermark are shown in Figure 5(e) and Figure 5(f) respectively.

6.2. Robustness Test

6.2.1. Robustness Results of First Level Sixth Band Embedded Watermark

The proposed algorithm is tested for robustness after the application of various attacks. Table 1 shows the extracted watermark after application of various attacks and the corresponding NC value obtained after extraction from the sixth band of first level of DT-CWT.

The algorithm is robust to the flipping and frame dropping attack. It is also able to withstand a cropping 10 % and 20 % of the frame given in the table as Cropping_90 andCropping_80 respectively. A 3 × 3 Gaussian filter is applied to test the filtering attack andwatermark is successfully extracted with 80 % accuracy. The geometrical attack of rotation by 2^0 and 90^0 are sustained with approximately 60 % accuracy.

Table 1
NC value and Extracted Watermarks for level one sixth sub band of DT-CWT

Attack	*NC value*	*Extracted Watermark*
No attack	0.7969	
Flipping	0.5727	
Frame Dropping	0.833	
Frame _Averaging	0.6790	
Cropping_90	0.3714	
Cropping_80	0.5687	
3×3 Gaussian Filter	0.8597	
Rotation_2	0.5556	
Rotation_90	0.5802	
Contrast Stretching	0.7071	
Gaussian Noise_0.02	0.5261	
Salt and Pepper noise 0.02	0.5154	
Scaling_1.5	0.7968	
Scaling_1	0.7875	
Sharpening	0.4738	
Translation_40	0.6222	
swap frames	0.7869	

70 % accuracy is achieved with contrast stretching attack. The algorithm also resists against the Gaussian noise and salt and pepper noise of 0.02 density yielding success rate above 50%. The scaling attacks with parameters 1 and 1.5 gave efficiency near to 80 %. The sharpening attack could only give 47 % of accuracy while extracting the watermarks. The algorithm also survived against the translation attack with 62 % accuracy and the frame swapping attack with 78% accuracy.

6.2.2. Robustness Results of First level Fourth Band Embedded Watermark

The proposed algorithm also successfully embedded the watermarks in the fourth sub band of the DT-CWT of the preprocessed frame with luminance adapted JND values. The extracted watermark after application of various attacks and the corresponding NC value obtained after extraction from the fourth band of DT-CWT are elaborated in Table 2.

In the fourth band the algorithm could withstand the same attacks as that of sixth band embedding except the cropping attack and the rotation_90 attack. Figure 8 elaborates the comparative analysis of robustness by embedding watermarks in these bands.

6.2.3. Evaluation with Various Combinations of Analysis and Synthesis Filters

The proposed DT_CWT algorithm is tested with several combinations of the filters at different levels. Successful watermark extraction is possible with the combinations illustrated in Table 3. With the achieved results we can say that that Legall 5, 3 tap filter at first level and Q_shift_c 16,16 tap filter at level two and three gave us the most perfect extracted watermark for the supplied video with almost 84 % efficiency. The results shown for the first level band four embedding and first level band six embedding in Figure 6 are achieved using the Legall and Q_shift_c filters. _32 Filters.

Table 2
NC value and Extracted Watermarks for level one Fourth sub band of DT-CWT

Attack	*NC value*	*Extracted Watermark*
No attack	0.7549	
Flipping	0.4738	
Frame Dropping	0.7416	
Frame_Averaging	0.6748	
Cropping_90	**	**
Cropping_80	**	**
3×3 Gaussian Filter	0.7802	
Rotation_2	0.589	
Rotation_90	**	**
Contrast Stretching	0.6776	
Gaussian Noise_0.02	0.5806	
Salt and Pepper noise_0.02	0.5815	
Scaling_1.5	0.8111	
Scaling_1	0.8181	
Sharpening	0.8081	
Translation_40	0.5220	
swap frames	0.796	

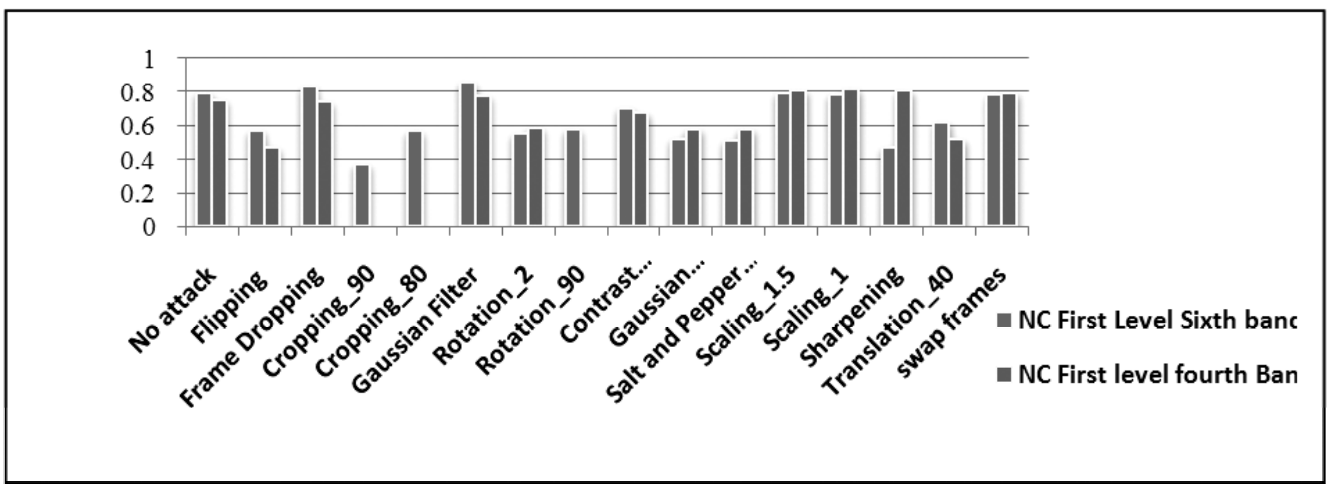


Figure 6: NC Value Comparisons for Fourth and Sixth Band of DT-CWT with Legall and Qshift

6.3. Evaluation of Quality

The proposed algorithm is tested for the evaluation of quality by estimating the PSNR values of the watermarked and the attacked video frames. Figure 7 elaborates the PSNR values obtained by the algorithm when watermark is embedded in the fourth and the sixth sub bands of DT-CWT. The PSNR of the original and attacked frame was in the range of 26 – 45 db for all the successful attacks yielding to an acceptable quality of the video.

Table 3
NC Value and Extracted Watermarks for Various Filters at Different Levels of DT-CWT

Filter Combination	*NC value*	*Extracted Watermark*
Antonini at level 1 and Qshift_06 for other levels	0.7969	
Antonini at level 1 and Qshift_a for other levels	0.8111	
Antonini at level 1 and Qshift_b for other levels	0.7550	
Antonini at level 1 and Qshift_c for other levels	0.812	
Antonini at level 1 and Qshift_d for other levels	0.7727	
Nearsymb at level 1 and Qshift_06 for other levels	0.762	
Nearsymb at level 1 and Qshift_a for other levels	0.7857	
Nearsymb at level 1 and Qshift_b for other levels	0.7723	
Nearsymb at level 1 and Qshift_c for other levels	0.7840	
Nearsymb at level 1 and Qshift_d for other levels	0.7985	
Nearsyma at level 1 and Qshift_06 for other levels	0.7857	
Nearsyma at level 1 and Qshift_a for other levels	0.786	
Nearsyma at level 1 and Qshift_b for other levels	0.7856	
Nearsyma at level 1 and Qshift_c for other levels	0.7985	
Nearsyma at level 1 and Qshift_d for other levels	0.7725	
Legall at level 1 and Qshift_06 for other levels	0.8333	
Legall at level 1 and Qshift_a for other levels	0.8095	
Legall a at level and Qshift_b for other levels	0.7726	
Legall at level 1 and Qshift_c for other levels	0.8343	
Legall at level 1 and Qshift_d for other levels	0.785	

6.4. Comparison with Existing Techniques in Literature

The algorithms deploying the DT-CWT concept for watermarking are available in [15, 16 and 17]. In [15], the authors embedded the watermark in the DT-CWT coefficients of level 3 and 4. The watermark was a 2D array that was 64 times smaller than the size of the frame. The algorithm proposed by authors of [16]

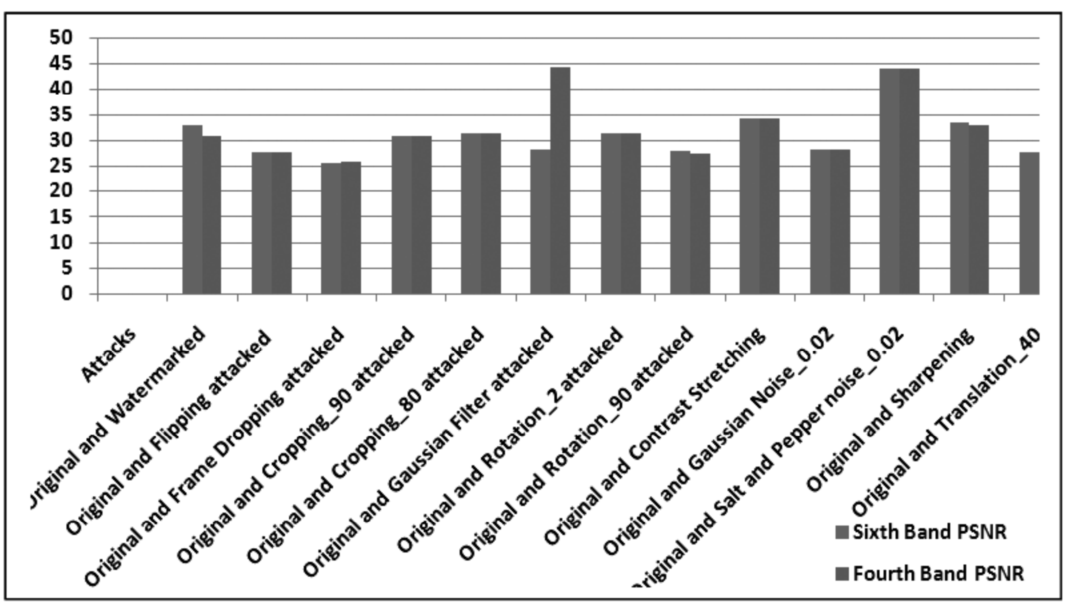


Figure 7: PSNR of Original and Attacked Frame, Watermark in Fourth and Sixth Bands

Table 4
Comparison of Proposed DT-CWT Algorithm with Algorithms from Literature

	Ref(15)	*Ref(16)*	*Ref(17)*	*Proposed*
Watermark	2D array 64 times smaller than size of the frame	Watermark sequence	Image	Video
Watermark embedded in	Level 3 and 4 coefficients	Six subbands with same JND value	Pixel values capable of sustaining the most water-mark energy i.e. low energy pixels	Sixth band and fourth band of the luminance component
Base Media	Video	Image	Image	Video
Limitation	Temporal synchronization	–	Severe attacks cause degradation of host image	Could overcome Frame averaging attack with approx 60 % accuracy
Attacks	Geometric distortions and lossy compression	Gaussian low pass filter-ing, Gaussian noise, Jpeg compression , rotation and scaling	Blurring, cropping, jpeg compression, Median filtering, Rotation and resizing	Video attacks: Frame averaging, Frame dropping, Frame cropping, Geometrical attacks rotation, scaling, translationSignal processing attacks like Gaussian filtering, Gaussian noise and salt and pepper noise
DT-CWT	4 level DT-CWT on real image with near symmetric 13 , 19 tap filters	4 level DT-CWT to host image and one scale decomposition to obtain six bands	4 level DT-CWT with antonini filter to host and watermark image	3 level DT-CWT with 20 combination of filters to luminance adapted JND frame
Watermark	Detected	Detected	Extracted	Extracted
JND Estimation for	–	Six subbands	–	For luminance component of frame

computed the JND for the six subbands, and the watermark sequence was embedded in the six subbands with the same JND value. The authors of [17] embedded the image watermark in pixel values capable of sustaining the most watermark energy i.e. low energy pixels of the DT-CWT. Table 4 portrays the comparison of the techniques in literature with the proposed algorithm.

7. CONCLUSION

The proposed watermarking technique gave better results for robustness. A binary image was successfully embedded in the fourth and the sixth sub bands of the first level of DT-CWT. From the sixth sub band adequate extraction of the embedded watermark was possible with 17 attacks with approximately 60 % efficiency for all. Similar results were achieved for the fourth sub band embedding except for some geometrical attacks. The technique embedding in fourth level sustained 15 attacks. The proposed algorithm was also experimented by applying various combinations of filters at different levels of the DT-CWT. An accuracy of 75 % was accomplished for 20 various combinations of the filters. The quality of the proposed algorithm also gave significant results giving acceptable PSNR for all the attacks.

ACKNOWLEDGEMENT

The authors express due acknowledgement to Prof Nick Kingsbury for his help regarding Dual Tree Complex Wavelet Transforms.

REFERENCE

[1] F. Hartung and M. Kutter, “Multimedia Watermarking Techniques”, Proceedings of IEEE, vol. 87, pp. 1079–1107, July 1999.

[2] Ivan W. Selesnick, Richard G. Baraniuk, and Nick G. Kingsbury, "Dual Tree Complex Wavelet Transforms", in IEEE Signal Processing Magazine, pp 123-151, Nov 2005.

[3] N.Jayant, J. Johnston, R. Safranek, "Signal Compression Based on Models of Human Perception", Proc. IEEE pp 1385–1422, 1993.

[4] Y. Niu , S. Krishnan, Q. Zhang, "Combined Just Noticeable Difference Model Guided Image Watermarking", in IEEE Int. Conf. Multimedia and Expo (ICME), pp1679–1684, 2010.

[5] I. Hontsch, L. Karam, "Adaptive Image Coding with Perceptual Distortion Control", IEEE Transaction on Image Processing. 11 (3), pp 213–222, 2002

[6] Z.Chen, C. Guillemot, "Perceptually-Friendly H.264/AVC Video Coding Based on Foveated Just-Noticeable-Distortion Model", IEEE Transactions, Circuits Systems, Video Technology, 20 (6) pp 806–819, 2010.

[7] T. Colleu, S. Pateux, L. Morin, C. Labit, "A Polygon Soup Representation for Multiview Coding" , J. Vis. Commun. Image Represent. 21 (5–6), pp 561–576, 2010

[8] W. Lin, C.J. Kuo, "Perceptual Visual Quality Metrics: A Survey", J. Vis. Commun. Image Represent. 22 (4), pp 297–312, 2011

[9] C .H. Chou, Y.-C. Li, "A Perceptually Tuned Subband Image Coder Based on the Measure of Just-Noticeable Distortion Profile", IEEE Transactions Circuits Syst. Video Technol. 5 (6) pp 467–476, 1995.

[10] N.G Kingsbury, "Complex Wavelets for Shift Invariant Analysis and Filtering of Signals", Journal of Applied and Computational Harmonic Analysis, 10(3) pp 234-253, 2001.

[11] C.-H. ChouY, C. Li, "A Perceptually Tuned Sub band Image Coder Based on the Measure of Just-Noticeable Distortion Profile", IEEE Transactions Circuits Syst. Video Technol. 5 (6) pp 467–476, 1995.

[12] Y.J. Chiu, T. Berger, A Software-only Video Codec Using Pixelwise Conditional Differential Replenishment and Perceptual Enhancement', IEEE Transactions on Circuits Syst. Video Technol. 9 (3), pp 438–450, 1999.

[13] Jinjian Wu, Fei Qi, Guangming Shi, Self-similarity Based Structural Regularity for Just Noticeable Difference Estimation', Journal of Vis. Commun. Image R. 23, pp 845–852, 2012.

[14] Christ Shoemaker, "Hidden Bits: A Survey of Techniques for Digital Watermarking", Independent study EER-290.

[15] A. Bouridane, F Kurugollu, M Byrne and S. Boussakta , Host Adaptive Color Image Watermarking Using Complex Wavelets', Irish Machine Vision and Image Processing Conference, Belfast , UK, 30 -31, August 2005.

[16] Lino E. Coria, Mark Pickering, Panos Nasiopoulos, Rabab K. Ward, "A Video Watermarking Scheme Based on the Dual-Tree Complex Wavelet Transform, IEEE Transactions on Information Forensics and Security", Volume 3(3), pages 466-474, September 2008.

[17] Jinhua Liu, Kun She, Robust Image Watermarking Using Dual Tree Complex Wavelet Transform Based on Human Visual System', Image Analysis and Signal Processing (IASP), 2010 International Conference, pg 675-679, April 2010.