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FORECASTING PERFORMANCE WITH THE HARMONIC MEAN: LONG-TERM INVESTMENT HORIZONS IN SHANGHAI STOCK EXCHANGE

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Abstract: *Portfolio managers favor long-term investment horizons. Their performance is usually forecasted using either the arithmetic mean or the geometric mean. The harmonic mean is generally ignored as an instrument of financial and/or portfolio management. We examine the performance of the harmonic mean employing real life data on SSE180 Index and we compare it with the corresponding performances of arithmetic and geometric means. In all cases, the harmonic mean gave us the best performance.*

Keywords *Portfolio evaluation; Performance; Long-term investment horizon; Arithmetic mean; Geometric mean; Harmonic mean; SSE180 Index.*

1. INTRODUCTION

Measures of expected value provide essential information when preparing projections of the behaviour of financial investments into the future. Therefore, measurement and evaluation of investment performance has received considerable attention in the finance literature. That is why many academic studies focus on the performance of portfolios and try to establish whether such portfolios succeed in earning abnormal returns.

Very often, portfolio managers and investors as well, must assess the long-run expected rates of return of their investment portfolio. They would typically base their assessment of future expected rates of return upon past experience. Perhaps the simplest way to forecast future returns is to use some average of past returns. Very naturally, this method has been favoured by many investors and analysts. In portfolio management, the majority of future forecasts are based either on the arithmetic mean (by summing the percentage return for each year then dividing by the number of years), or on the geometric mean (by compounding the annual returns

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and putting this to the power of the inverse of the number of years), or on some sort of weighted mixture of the two. Consequently, many studies have been conducted to identify which mean will provide the most accurate forecasts for a given series of data. Such generalizations are important because analysts often rely upon a single mean.

Another very well known mean, but never used in finance or in economics in general, is the harmonic mean. We intent to investigate its forecasting performance with respect to long-term investment horizons.

Portfolio managers usually forecast the value of their investment at the end of the selected investment horizon. Long-term investment horizons give portfolio managers the chance to achieve their strategic goals without undue pressure for short-term results. The conventional wisdom in the professional investment community seems to be that investors with a long time horizon should invest more heavily in stocks than investors with a short time horizon. We may say that in practice, the longer the investment horizon the better for the portfolio manager. It is, therefore, the objective of this paper to examine the comparative performance of harmonic mean and its rivals, the geometric mean and the arithmetic mean, with respect to long-term investment horizons. In recent years, China's stock market capitalization brought significant gains and, therefore, many worldwide investors are investing in China. Such popularity directed us to employ the Chinese stock market as the environment for our empirical analysis.

The rest of the paper is organized as follows: We first present the harmonic mean. We then discuss portfolio performance, long-term investment horizons, and the importance of the Chinese stock market. Next, we present our empirical results and, finally, we conclude.

2. THE HARMONIC MEAN

Let us consider an elementary physics problem. Suppose we are traveling by car a 500 kilometer distance from one city to another. Somehow, we manage to keep the speed of our vehicle absolutely constant for 100 km. Then, the speed suddenly changes but remains absolutely constant for another 100 km. The same happens for the next 100 km, and the next 100 km, and the next last 100 km. Every time the speed changes, we record it, so we have five speed observations, say, 90 km/h, 80 km/h, 100 km/h, 120 km/h and 110 km/h. The problem is to calculate the trip's average traveling speed.

An impatient or ignorant person will immediately answer 100 km/h, which is the arithmetic mean of the five speed observations. Unfortunately, or fortunately, this is a wrong answer. Neither the geometric mean, 98.99 km/h, is correct. The correct answer is 97.97 km/h, which is the harmonic mean of the five speed observations.

The harmonic mean is found by dividing the number of data elements by the sum of the reciprocals of each data element. In formula, let us suppose we have a positive arithmetic sequence, such as the T annual returns of a stock, say X:

$$\{X_1, X_2, \ldots, X_T\} = X_t,\ t = 1, \ldots, T \text{ with } X_t \geq 0 \text{ for all } t,$$

then, the harmonic mean of X_t, say h, is

$$h = \frac{T}{\sum_{t=1}^{T} \left(\frac{1}{X_t} \right)} .$$

As a historical note, the name harmonic mean was introduced by Archytas (428-350 BC) of Tarentum, a well-known Greek mathematician, statesman and philosopher of the Pythagorean School. In earlier times, the harmonic mean was called the sub-contrary mean but Archytas renamed it *harmonic* since the ratio proved to be useful for generating harmonious frequencies on string instruments. Archytas was working on the "doubling of the cube" problem (the Delian Problem); that is to find the side of a cube with a volume twice that of a given cube. This problem had been worked on by Hippocrates, but Archytas derived an elegant geometric solution using the harmonic mean.

3. PORTFOLIO PERFORMANCE

Although the harmonic mean has the same time span, of more than 2,500 years, with the arithmetic mean and the geometric mean – all three means were suggested and used by Pythagoras (circa 530 BC) and his followers, the Pythagoreans (Heath 1981) – it is seldom used. In finance, the case is even worse. The harmonic mean is never used, and none modern financial analyst has investigated the consequences of using it. There are two straightforward reasons for this situation. First, there is no direct financial interpretation of the harmonic mean, such as the one exists for the geometric mean. The geometric mean is interpreted as the compound growth rate of return of an investment. We do not argue whether this interpretation is right or wrong. It is more than enough for us, that it is, indeed, a financially meaningful interpretation. The second reason is that financial literature is full of the definite and absolute statement "*the arithmetic mean overestimates the true compound rate of return, and the geometric underestimates it*". See, for example, Bodie, Kane and Marcus (2002, p. 810) and Reilly and Brown (2003, p. 9) to quote a couple. It is, therefore, logical and sensible to look for a measure that is between the arithmetic and the geometric mean. As a consequence, we have suggestions such as the ones made by Blume (1974) and Cooper (1996). Unfortunately, the harmonic mean is always less than the geometric mean since the inequality: arithmetic mean ^{3} geometric mean ^{3} harmonic mean, always holds. Consequently, the harmonic mean is ruled out as it is expected to underestimate even further the true compound rate of return.

Active stock portfolio management requires an effective stock selection strategy. No matter the selection strategy, no matter the selection tools used, i.e. fundamental and/or technical analysis, the portfolio manager will be judged by the actual portfolio performance at the end of the investment period. He will be judged, e.g. now, for decisions he made years ago. So, some questions arise. On what basis the portfolio manager took these past decisions. What were the expectations he had at that past time, and how well or bad those expectations were fulfilled.

When a portfolio manager is building a portfolio, he selects stocks that he expects to perform quite well after a given investment period. He mainly bases his expectations on past performance. In fact, he assumes that tomorrow will be like yesterday, next year will be like last year, and generally, future events can be predicted from the past.

The majority of future forecasts in portfolio management are based either on the arithmetic mean, or on the geometric mean, or on some sort of weighted mixture of the two. With respect to the differences between arithmetic and geometric means, a plethora of papers have been written on the pros and cons of each one of them and the relative merits of using the one against the other. See Fabozzi (1999) and Francis and Ibbotson (2002) to quote a few. For more detailed information see our earlier work Missiakoulis, Vasiliou and Eriotis (2007 and 2010). The main drawback of both means is that they produce biased results for long investment horizons (Indro and Lee, 1997). Some mixtures of both have been suggested as better alternatives, with Blume (1974) and Cooper (1996) being the classical ones.

As an important parenthesis, let us inform the reader that in the science of Computer Architecture, "*to find a single number to summarize overall performance over a benchmark suite is critical in performance evaluation*". Computer scientists, in a similar to finance scholars, measure performance as a rate and they are interested in a non-misleading single number performance measure. They investigated the performance of arithmetic, geometric and harmonic means and they concluded that (a) the arithmetic mean can be used as an accurate measure of performance expressed as time, (b) the harmonic mean should be used for summarizing performance expressed as a rate, and (c) Geometric mean should not be used for summarizing performance expressed as a rate or as a time. For details see Bose and Conte (1998), Fleming and Wallace (1986), Giladi (1996), John (2004), Mashey (2004) and Smith (1988).

In the science of mathematics, all three means are treated as of equal importance. For us, the harmonic mean has one crucial property. It is the only one that associates all three of them. It is the mathematical tool for transforming the arithmetic mean into the geometric mean. Let us consider the arithmetic-harmonic mean ***ahm**(a,b)* of two positive numbers a and b which is defined by starting with $a_0 \equiv a$ and $b_0 \equiv b$, and then iterating

$$a_{n+1} = \frac{a_n + b_n}{2} \text{ and } b_{n+1} = \frac{2}{\frac{1}{a_n} + \frac{1}{b_n}} = \frac{2a_n b_n}{a_n + b_n},$$

then

$$ahm(a,b) = \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \sqrt{ab}$$

which is the geometric mean of a and b. That is, using the harmonic formulation on the arithmetic mean we end up to the geometric mean.

Let us now consider a financial problem: What is the average daily rate of return of an investment, say a given stock, in a week's time. This financial problem is analogous to the above-mentioned physics one as long as we make the following syllogisms:

(a) Instead of having length distances we consider time distances.

(b) The distance between the two cities is one working week (time distance measure), i.e. the five working days of the week.

(c) Which in turn implies that each of the 100 km intervals is equated to one working day.

(d) The speed of the vehicle is now the rate of daily return.

We may then interpret the financial rate of return as the speed with which the initial value of an investment will be transformed to its terminal value.

4. LONG INVESTMENT HORIZONS

The horizon of an investment, the planned liquidation date, is very crucial for each asset's selection. Investors prefer long-term investment horizons so they can weather any short-term fluctuations in exchange for potentially higher long-term returns. The prevailing rule says that the higher the return that the investor requires, the greater the risk that he is willing to accept. He can offset necessary risk by setting a longer investment period during which he is able to leave his investment untouched - the investment horizon.

Given a sufficiently long investment horizon overall risk significantly decreases and there is greater probability, though never a certainty, that the investor will achieve the expected return. Peters (1991), for example, claims the stability of the market exists because of different expectations and different investment time horizons. An investor with a long-term investment horizon is going to react differently to short term information than a short-term investor. If there is a negative short-term change in expectations, the short investment horizon investors will sell the security. However, because there is not a change in the expectations of the long investment horizon investors, they will take advantage of the lowered prices to buy additional securities.

Long-term investment horizons give portfolio managers the chance to achieve their strategic goals without undue pressure for short-term results. In fact, the choice of investment horizon is the very first crucial decision a portfolio manager has to take during his portfolio managerial duties. At one extreme, for example, day-traders have an investment horizon of a few hours, whereas at the other extreme, institutions such as pension funds have investment horizons of the order of many years. The length of the investment horizon, in turn, will determine the market or markets of securities, as well as the asset allocation that best matches the investor's needs.

Portfolio managers usually forecast the value of their investment at the end of the selected investment horizon. They project the performance of an investment on the basis of the planned investment horizon and expectations concerning reinvestment rates and future market yields. By doing so, they evaluate which of several potential investments considered for acquisition will perform the best over the planned investment horizons.

With our reader's permission, we will be, for a moment, unscientific and we will address our problem from its practical point of view. Portfolio managers, like all managers, are interested in results. Given a method, they are interested more in its effectiveness with respect to outcome, rather than in its theoretical interpretation. If the method produces "good results", who cares for its "virtual theory"? Furthermore, in long and uncertain investment horizons, is it safer for a professional portfolio manager to use the harmonic mean for forecasting future portfolio performance? What is the risk of such a use? We will answer this question with the empirical investigation that follows and, in particular, we will focus on the Chinese stock market.

5. WHY CHINA?

China has rapidly increased in economic importance. During the first decade of the current millennium, China grew in economic size and geo-political stature. China's stock market returns outpaced most of developed western markets, with the U.S. one being a prime example. As a result, its stock market capitalization brought significant growth. Today many worldwide investors redirect their investments to China because they realized that China has become the world's growth engine. Although the Chinese growth is mainly based on exports, presently is focusing on its domestic expansion as well.

The Shanghai Stock Exchange (S.S.E.) "*has become the most preeminent stock market in Mainland China in terms of number of listed companies, number of shares listed, total market value, tradable market value, securities turnover in value, stock turnover in value and the T-bond turnover in value. As at the end of 2010, S.S.E. boasted 1,500 listed securities and 894 listed companies, with a combined market capitalization of RMB 17,900.724 billion and a total of 98.51 million trading accounts. A large number of companies from key industries, infrastructure and high-tech sectors have not only raised capital, but also improved their operation mechanism through listing on Shanghai stock market*".

For S.S.E. the index and indexation investment is very important. Therefore, the number of indices headed with «SSE» has reached nearly 80. The variety of SSE indices goes from the traditional SSE Composite Index (a whole market index) to the benchmark SSE180 Index, and, to various specialized sector and/or fund indices such as, among others, Dividend, Sector, Fund, Government Bond Index, and Corporate Bond indices. Thus, S.S.E. indices, provide investors with benchmark systems for different investment portfolios. The SSE Composite Index tracks the performance of all listed stocks on the S.S.E. Companies are weighed according to their size in terms of market capitalization which represents the total market value of all companies listed on S.S.E.

The S.S.E., "*in order to promote the long-term infrastructure construction and the standardization process of the security market, constructed the SSE Constituent Index*", known nowdays as the **SSE180 Index**. "*The number of companies in SSE180 Index is 20% of that of all S.S.E. listed companies, while their market capitalization and turnover account for 73% and 55% of that of the total, respectively*". "*Its objective is to select constituents that best represent Shanghai market through scientific and objective method, to establish a benchmark index that will reflect Shanghai market and serve as a performance benchmark and a basis for financial innovation*".

The SSE180 listed companies (180 in total) are classified into 10 industries. The classification is based on Global Industry Classification System developed by Morgan Stanley and Standard & Poors, and corresponding adjustments have been made to adapt to the characteristics of listed companies in China. The relevant industry groups are: Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunication, and Utilities. The SSE180's constituents are adjusted every 6 months following the principle of Stability and Dynamic Tracking. Constituents adjusted each time will not exceed 10%. Temporary adjustment can be made under certain circumstances if necessary.

6. EMPIRICAL RESULTS

For the fairness of our results, we employed real life and easily accessible data on SSE180 index in our analysis. We selected SSE180 due to its importance as an indicator of China's overall market performance and, therefore, it consists the most important benchmark of active portfolio managers when they evaluate their performance compared to the overall stock market. Furthermore, it has little systematic risk.

Since we wanted a real data testing and analysis procedure, we rejected the idea of constructing portfolios and then having examined the performance of each measure. By doing so, we eliminate the existence of personal prejudice in our analysis. Instead we select to base our investigation on the above index, which is, in fact, a portfolio. It may be considered as a conservative portfolio but it is the most representative, and since we are aiming to impartial conclusions the choice of SSE180 was inevitable. Furthermore, the use of SSE180 has three supplementary benefits. First, it is easily identified. Second, the stocks it is consisted of represent, almost certainly, the portfolio manager's top selection, and third, by order of weight, it has the largest impact on portfolio's performance.

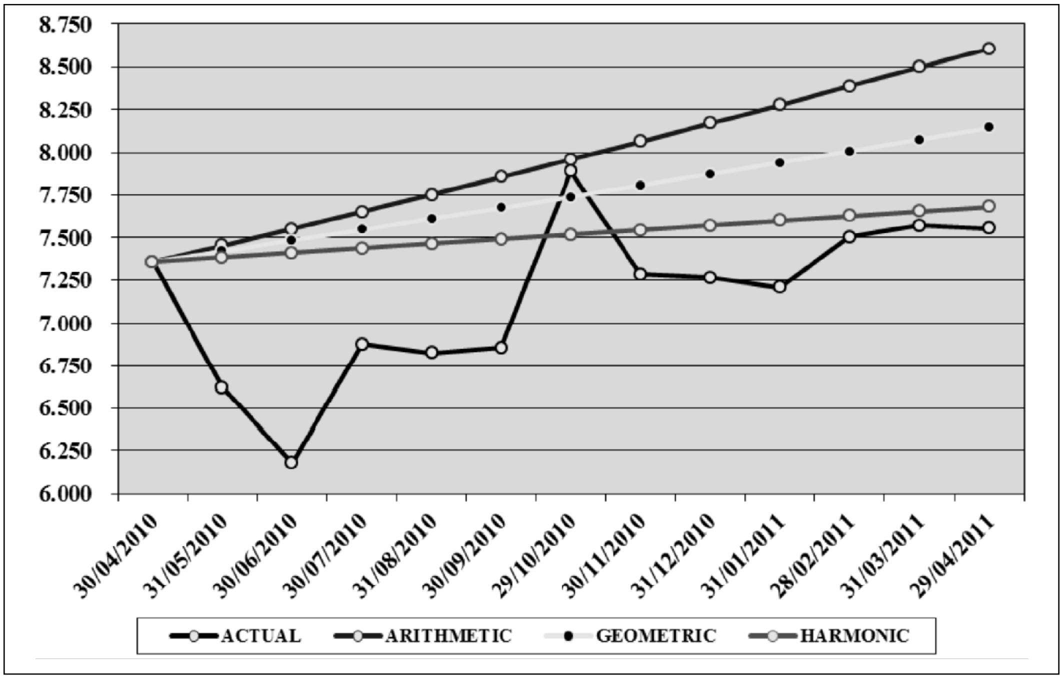
We decided to use monthly data, although we had the daily data. By doing so, we minimize, if not eliminate, the possible effects of serial correlation over time and similar dependencies. The monthly rates of return used in our empirical analysis have been adjusted for stock splits and dividends. The data set (since 28th of June 2001) used is readily available in electronic form, and is easily accessible by anyone. All data used are publicly available to the individual investor in the relevant page of S.S.E. internet site. So, any interested person could duplicate our results and findings.

We design our experiment as follows. First, we travel back in time and we assumed we are living in May 1, 2010. Therefore, the most resent observation we have on SSE180 is that of April 30, 2010. We further assumed that we are interested in forecasting the values of this index for an investment horizon of twelve months, i.e. April 29, 2011. The available data are the monthly adjusted closing prices from June 28, 2002 to April 30, 2010. Now, each mean of the monthly returns serves as the expected return. Finally, we compute the various mean values using the existed data. Now, each mean of the monthly returns serves as the expected return. Finally, we forecast the performance of SSE180 for each month of the interval May 2010-April 2011 using each one of the three means.

The computed results are shown diagrammatically in Figure 1. As we can see, all estimates (with only one exemption) overestimate the true value, with harmonic mean always being the closest to the actual. The geometric mean is the second best, and the arithmetic mean always being the worst.

The performance of a predicting method or model is measured by using loss functions or error statistics. The most commonly used (see Balaban *et al.*, 2006 and Brailsford and Faff, 1996) ones are the mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE), which are defined as

Figure 1: SSE 180 Index Total Return Forecasts



$$ME = \frac{1}{N}\sum_{j=T+1}^{T+N}\left(\hat{r}_j - r_j\right) ,$$

$$MAE = \frac{1}{N}\sum_{j=T+1}^{T+N}\left|\hat{r}_j - r_j\right| ,$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=T+1}^{T+N}\left(\hat{r}_j - r_j\right)^2} ,$$

$$MAPE = \frac{1}{N}\sum_{j=T+1}^{T+N}\left|\frac{\hat{r}_j - r_j}{r_j}\right| ,$$

where r_j is the actual value of index of return at time t, $\hat{r}_j$ is the corresponding predicted value, T is the number of time-periods on which we based our prediction, and N is the investment horizon measured with the same time unit as T.

Given that we are interested in long-term investment horizons, we investigated the forecasting performance of each mean in 7, 8, 9, 10, 11 and 12 months ahead, considering that investment horizons of 7 to 12 months are long enough. All results are tabulated in Tables 1 to 4.

Table 1
Mean Error

Horizon	*Arithmetic*	*Geometric*	*Harmonic*
7	822,09	678,14	531,27
8	832,27	669,16	503,26
9	858,72	676,17	491,09
10	861,12	658,85	454,44
11	866,81	644,54	420,64
12	882,44	639,91	396,36

Table 2
Mean Absolute Error

Horizon	*Arithmetic*	*Geometric*	*Harmonic*
7	822,09	720,35	636,80
8	832,27	706,09	595,60
9	858,72	709,00	573,17
10	861,12	688,40	528,31
11	866,81	671,40	487,80
12	882,44	664,53	457,92

Table 3
Root Mean Square Error

Horizon	*Arithmetic*	*Geometric*	*Harmonic*
7	898,02	789,32	699,46
8	898,72	768,83	663,24
9	919,37	764,85	638,93
10	915,77	742,83	607,42
11	916,49	724,22	579,69
12	928,77	713,92	556,26

Table 4
Mean Absolute Percentage Error

Horizon	*Arithmetic*	*Geometric*	*Harmonic*
7	0,123	0,108	0,095
8	0,123	0,105	0,088
9	0,126	0,104	0,085
10	0,125	0,101	0,078
11	0,125	0,097	0,072
12	0,126	0,096	0,067

In all tables, we are looking for the best performer, that is, for each error statistic we are looking for the mean that produces the closest to zero value of the statistic. In all 24 (= six investment horizons × four error statistics) experiments, the harmonic mean has the best performance and produces the lowest statistics.

From all tables we see that the harmonic mean always produces forecasts that are the closest to the corresponding actual value. The harmonic mean results error statistics which are on average 30% better than those based on the geometric mean and 60% better than those based on the arithmetic mean.

For us there is one obvious explanation which has to do with the mathematical construction of each mean. Both arithmetic and geometric mean have a monotone (additive or multiplicative) mathematical formula. Real life, however, taught us that nothing is monotone. Real life is full of ups and downs. So are the financial markets. The harmonic mean, which is the reciprocal of the arithmetic mean of the reciprocals of the observations, is definitely not a monotone one.

Another thing we observe is that the performance of the harmonic mean is always closer to the geometric's rather than the arithmetic's. This was expected as it is consistent with the mathematics of the three means.

7. CONCLUSION

We have investigated three different mean measures in the context of forecasting future portfolio performance for long-term investment horizons. Our empirical analysis showed us that the answer to the question which averaging method leads to the best forecasts is the harmonic mean irrespective of the chosen error statistic. In all cases, the harmonic mean always resulted as the best choice with the geometric mean being second and the arithmetic third.

Therefore, in long and uncertain investment horizons, we may say that it is safer for a professional portfolio manager to use the harmonic mean for forecasting future portfolio performance.

On the basis of our empirical results, we argue strongly in favor of using the harmonic mean as the appropriate instrument when evaluating the long-term investment horizon forecasting performance of different portfolio management strategies. Although our analysis has been conducted entirely on SSE1800 index, we may state, without loss of generality, that our results apply to other similar cases.

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