# COST BENEFIT ANALYSIS OF STOCHASTIC MODEL OF A SYSTEM WITH PROVISO OF SWITCH RECTIFICATION AND OPERATING UNIT TIME THRESHOLD 

R. K. BHARDWAJ, MANDEEP KAUR*


#### Abstract

A probabilistic model of a cold-standby system is developed in the current research paper. The system model takes into account switch rectification as well as the operating unit's maximum operation time limit. The model is built on the theory of semi-Markov processes. Using the regenerative point technique, some performance measures are derived. For numerical representations of the results, a data set is considered.


## 1. Introduction

The long sustaining of a working system demands for the cost-benefit analysis. The availability of the system and hence the profit depends upon various remedial strategies adopted. The frequency of remedial actions as well as the amount of time spent by the server are key components contributing to system running cost. So there are some of the factors that affect the running cost of the system. One way of improving the system performance is to use standby unit. In the literature researchers have tried to develop probabilistic system models to reveal the scope of improvement. Some studies such as (Singh \& Bhardwaj, 2017), (Yongjin et al., 2018) etc., emphasizes the cold standby systems. The provision of preventive maintenance is studied by (Garg \& Kadyan, 2016), (Yang et al., 2018). In the present research paper we evaluated the cost-benefit of a two identical unit cold-standby system. The operating unit gets preventive maintenance after surpassing a threshold limit, called maximum operation time. Upon failure the operating unit needs replacement by the standby unit that may or may not found operable. Similarly, for doing replacement task the switch also may or may not found operable. A service facility, called server, is responsible to remedial or rectification activities in the system. The semi-Markov processes (Limnios, 2012) and regenerative point technique (Smith, 1955) are used to develop the system model. The system performance measures such as availability, busy period, frequency of remedial tasks etc. are evaluated to study the system profit.

## 2. Acronyms

The notations and acronyms in this paper are used that of (Bhardwaj \& Singh, 2017). Some additional notations as given below.

[^0]\[

$$
\begin{array}{ll}
a / b & \begin{array}{l}
\text { Probability that repair/ replacement is feasible after } \\
\text { inspection }
\end{array} \\
u_{p m t} / U_{P M T} & \begin{array}{l}
\text { Unit under Preventive Maintenance (PM)/ under PM } \\
\text { continuously from previous state }
\end{array} \\
w_{p m t} / W_{P M T} & \begin{array}{l}
\text { Unit waiting for PM/ waiting for PM continuously from } \\
\text { previous state }
\end{array} \\
p_{m}(t) / P_{m}(t) \quad \begin{array}{l}
\text { pdf/ cdf of preventive maintenance (PM) time }
\end{array}
\end{array}
$$
\]

## 3. System State Transition Diagram



Figure: State transition diagram

## 4. Transition Probabilities

Simple probabilistic considerations yield the following expressions for the nonzero elements

$$
\begin{aligned}
& p_{i j}=Q_{i j}(\infty)=\int_{0}^{\infty} q_{i j}(t) d t=\widetilde{Q}_{i j}(0) \\
& p_{01}=\int_{0}^{\infty} p z(t) \bar{S}(t) \bar{O}(t) d t, \quad p_{02}=\int_{0}^{\infty} p o(t) \bar{Z}(t) \bar{S}(t) d t, \quad p_{03}=\int_{0}^{\infty} s(t) \bar{Z}(t) \bar{O}(t) d t, \\
& p_{04}=\int_{0}^{\infty} q z(t) \bar{S}(t) \bar{O}(t) d t, \quad p_{05}=\int_{0}^{\infty} q o(t) \bar{Z}(t) \bar{S}(t) d t, \quad p_{10}=\int_{0}^{\infty} f(t) \bar{Z}(t) \bar{O}(t) d t, \\
& p_{1,14}=\int_{0}^{\infty} o(t) \bar{F}(t) \bar{Z}(t) d t, \quad p_{1,15}=\int_{0}^{\infty} z(t) \bar{F}(t) \bar{O}(t) d t, \quad p_{20}=\int_{0}^{\infty} p_{m}(t) \bar{O}(t) \bar{Z}(t) d t,
\end{aligned}
$$

$$
\begin{aligned}
& p_{2,11}=\int_{0}^{\infty} z(t) \overline{P_{m}}(t) \bar{O}(t) d t, \quad p_{2,12}=\int_{0}^{\infty} o(t) \overline{P_{m}}(t) \bar{Z}(t) d t, \quad p_{30}=\int_{0}^{\infty} b g(t) \bar{Z}(t) \bar{O}(t) d t, \\
& p_{31}=\int_{0}^{\infty} a g(t) \bar{Z}(t) \bar{O}(t) d t, \quad p_{39}=\int_{0}^{\infty} z(t) \bar{G}(t) \bar{O}(t) d t, p_{3,10}=\int_{0}^{\infty} o(t) \bar{Z}(t) \bar{G}(t) d t, \quad p_{41}=\int_{0}^{\infty} h(t) \bar{S}(t) d t, \\
& p_{47}=\int_{0}^{\infty} s(t) \bar{H}(t) d t, \quad p_{52}=\int_{0}^{\infty} h(t) \bar{S}(t) d t, \quad p_{5,13}=\int_{0}^{\infty} s(t) \bar{H}(t) d t, \quad p_{61}=\int_{0}^{\infty} f(t) d t, \quad p_{78}=\int_{0}^{\infty} h(t) d t, \\
& p_{83}=\int_{0}^{\infty} f(t) d t, \quad p_{91}=\int_{0}^{\infty} b g(t) d t, \quad p_{96}=\int_{0}^{\infty} a g(t) d t, \quad p_{10,2}=\int_{0}^{\infty} b g(t) d t, \quad p_{10,16}=\int_{0}^{\infty} a g(t) d t, \\
& p_{11,1}=\int_{0}^{\infty} p_{m}(t) d t, \quad p_{12,2}=\int_{0}^{\infty} p_{m}(t) d t, \quad p_{13,17}=\int_{0}^{\infty} h(t) d t, \quad p_{14,2}=\int_{0}^{\infty} f(t) d t, \quad p_{15,1}=\int_{0}^{\infty} f(t) d t, \\
& p_{16,2}=\int_{0}^{\infty} f(t) d t, \quad p_{17,3}=\int_{0}^{\infty} p_{m}(t) d t, \quad p_{1,1,15}=p_{1,15}[c] p_{15,1}, \quad p_{1,2,14}=p_{1,14}[c] p_{14,2}, \\
& p_{2,1,111}=p_{2,11}[c] p_{11,1}, \quad p_{2,2.12}=p_{2,12}[c] p_{12,2}, p_{3,1.9}=p_{39}[c] p_{91}, \quad p_{3,19,96}=p_{39}[c] p_{96}[c] p_{61}, \\
& p_{3,2.10}=p_{3,10}[c] p_{10,2}, \quad p_{3,2.10,16}=p_{3,10}[c] p_{10,16}[c] p_{16,2}, \quad p_{4,3,7,8}=p_{47}[c] p_{78}[c] p_{83}, \\
& p_{5,3,13,17}=p_{5,13}[c] p_{13,17}[c] p_{17,3}
\end{aligned}
$$

## 5. Mean Sojourn Times

The mean sojourn time in the state $S_{i}$ is given by-
$\mu_{i}=E(t)=\int_{0}^{\infty} P(T>t) d t$, where T denotes the time to system failure.
$\mu_{0}=\int_{0}^{\infty} \bar{Z}(t) \bar{O}(t) \bar{S}(t) d t, \mu_{1}=\int_{0}^{\infty} \bar{F}(t) \bar{Z}(t) \bar{O}(t) d t, \mu_{2}=\int_{0}^{\infty} \overline{P_{m}}(t) \bar{Z}(t) \bar{O}(t) d t, \mu_{3}=\int_{0}^{\infty} \bar{G}(t) \bar{Z}(t) \bar{O}(t) d t$, $\mu_{4}=\mu_{5}=\int_{0}^{\infty} \bar{H}(t) \bar{S}(t) d t, \mu_{6}=\mu_{8}=\mu_{14}=\mu_{15}=\mu_{16}=\int_{0}^{\infty} \bar{F}(t) d t, \mu_{7}=\mu_{13}=\int_{0}^{\infty} \bar{H}(t) d t, \mu_{9}=\mu_{10}=\int_{0}^{\infty} \bar{G}(t) d t$, $\mu_{11}=\mu_{12}=\mu_{17}=\int_{0}^{\infty} \overline{P_{m}}(t) d t$,

## 6. Cost-Benefit Analysis

## Steady State Availability

Let $A_{i}(t)$ be the probability that the system is in up-state at instant ' t ' given that the system entered regenerative state $S_{i}$ at $\mathrm{t}=0$. The steady state availability is given below.

$$
\begin{aligned}
& A_{0}(\infty)=\lim _{s \rightarrow 0} s A_{0}^{*}(s) \\
& \quad\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+\mu_{0}\right\}+\left\{1-p_{30}\right. \\
& \left.\quad\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}\left(p_{20}+p_{2,1.11}\right)+\mu_{2} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}+\right. \\
& =\frac{\left.\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}-p_{20} \mu_{1}\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
& +\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
& + \\
& \left.\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{aligned}
$$

## Busy Period of Server due to inspection

Let $B_{i}^{I}(t)$ be the probability that the server is busy in inspection of the unit due to cold-standby failure at an instant ' $t$ ' given that the system entered the state $S_{i}$ at time t=0. The steady state busy due to inspection given below.

$$
\begin{aligned}
B_{0}^{I}(\infty)= & \lim _{s \rightarrow 0} s B_{0}^{* I}(s) \\
= & \frac{W_{3}^{* I}(0)\left[\left\{1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right\}\left\{p_{20}\left(1-p_{1,1.15}\right)+p_{2,1.11} p_{10}\right\}\right]}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\right.} \\
& \left.\mu_{0}\right\}+\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}\right. \\
& \left.+p_{05} p_{52}+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}^{\prime}\right\}
\end{aligned}
$$

## Busy Period of Server due to Repair of Unit

Let $B_{i}^{R}(t)$ be the probability that the server is busy in repairing the unit due to failure at an instant ' $t$ ' given that the system entered the regenerative state $S_{i}$ at time $\mathrm{t}=0$. We get the time for which server is busy due to repair in steady state.

$$
\begin{aligned}
& \quad B_{0}^{R}(\infty)=\lim _{s \rightarrow 0} s B_{0}^{* R}(s) \\
& \\
& =\frac{W_{1}^{* R}(0)\left[p_{20}\left(p_{01}+p_{04} p_{41}\right)+p_{2,1.11}-\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\left\{p_{2,1.11} p_{30}-p_{20}\right.\right.}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3,}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}+} \\
& \left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}+\right. \\
& \left.\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{aligned}
$$

## Busy Period of Server due to Repair of Switch

Let $B_{i}^{R S}(t)$ be the probability that the server is busy in repairing the switch due to failure at an instant ' $t$ ' given that the system entered the regenerative state $S_{i}$ at time $\mathrm{t}=0$.

$$
\begin{aligned}
& B_{0}^{R S}(\infty)=\lim _{s \rightarrow 0} s B_{0}^{* R S}(s) \\
& =\frac{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{W_{4}^{* R S}(0) p_{04}+W_{5}^{* R S}(0) p_{05}\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}+} \\
& \left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}+\right. \\
& \left.\left(p_{3,2.10}+p_{3,2,10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{aligned}
$$

## Busy Period of Server due to Preventive Maintenance

Let $B_{i}^{P M}(t)$ be the probability that the server is busy in preventive maintenance at an instant ' $t$ ' given that the system entered the regenerative state $S_{i}$ at time t=0.

$$
\begin{aligned}
& B_{0}^{P M}(\infty)=\lim _{s \rightarrow 0} s B_{0}^{* P M}(s) \\
& \\
& \left.=\frac{W_{2}^{* P M}(0)\left[\left(1-p_{1,1.15}\right)-\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\left\{p_{30}\left(1-p_{1,1.15}\right)+p_{10}\left(p_{31}+p_{3,1.9}\right.\right.\right.}{\left.\left\{\left(1-p_{3,196,6}\right)\right\}-p_{10}\left(p_{01}+p_{04} p_{41}\right)\right]} p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}+ \\
& \left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}+\right. \\
& \left.\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{aligned}
$$

## Expected Number of Replacements

Let $R_{i}^{P}(t)$ be the expected number of replacements of the unit by the server in $(0, \mathrm{t}]$ given that the system entered the regenerative state $S_{i}$ at time t=0.

$$
\begin{aligned}
& \quad R_{0}^{P}(\infty)=\lim _{s \rightarrow 0} s \widetilde{R}_{0}^{P}(s) \\
& =\frac{\left\{p_{30}+p_{3,1.9}+p_{3,2.10}\right\}\left\{1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right\}\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
& +\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
& \left.+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{aligned}
$$

## Expected Number of Repairs (Unit)

Let $R_{i}^{U}(t)$ be the expected number of repairs of the failed unit by the server in $(0, \mathrm{t}]$ given that the system entered the regenerative state $S_{i}$ at $\mathrm{t}=0$.

$$
\begin{gathered}
R_{0}^{U}(\infty)=\lim _{s \rightarrow 0} s \widetilde{R}_{0}^{U}(s) \\
p_{2,1.11}-\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\left[p_{30}\left(p_{20}+p_{2,1.11}\right)+p_{20}\left(p_{3,2.10}-p_{3,1.9,6}\right)\right] \\
+p_{20}\left(1-p_{02}+p_{04}-p_{04} p_{41}-p_{05} p_{52}\right)+\left\{p_{10} p_{2,1.11}-p_{20} p_{1,1.15}\right\}\left\{\left(p_{3,1.9,6}+p_{3,2.10,16}\right)\right. \\
=\frac{\left.\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04}\left(1-p_{41}\right)\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
+\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
\left.+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}\right\}
\end{gathered}
$$

## Expected Number of Repairs (Switch)

Let $R_{i}^{S}(t)$ be the expected number of repairs of the failed switch by the server in $(0, \mathrm{t}]$ given that the system entered the regenerative state $S_{i}$ at $\mathrm{t}=0$.

$$
\begin{aligned}
& R_{0}^{S}(\infty)=\lim _{s \rightarrow 0} s \widetilde{R}_{0}^{S}(s) \\
&= \frac{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{p_{04} p_{41}+p_{05} p_{52}\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
&+\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
&\left.+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}^{\prime}\right\}
\end{aligned}
$$

## Expected Number of Inspections

Let $I_{i}^{I}(t)$ be the expected number of inspections of the failed unit in $(0, \mathrm{t}]$ given that the system entered the regenerative state $S_{i}$ at $\mathrm{t}=0$.

$$
\begin{aligned}
& I_{0}^{I}(\infty)=\lim _{s \rightarrow 0} s \widetilde{I}_{0}^{I}(s) \\
= & \frac{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right\}}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
& +\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
& \left.+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}^{\prime}\right\}
\end{aligned}
$$

## Expected Number of preventive Maintenances (PM)

Let $P_{i}^{M}(t)$ be the expected number of PM of the failed unit in $(0, \mathrm{t}]$ given that the system entered the regenerative state $S_{i}$ at $\mathrm{t}=0$.

$$
\begin{gathered}
P_{0}^{M}(\infty)=\lim _{s \rightarrow 0} s \widetilde{P}_{0}^{M}(s) \\
=\frac{\left.+p_{04} p_{41}-p_{2,1.11} p_{05}\left(1-p_{52}\right)+\left(p_{31}+p_{3,1.9}+p_{3,1.9,6}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right]}{\left\{\left(1-p_{1,1.15}\right) p_{20}+p_{10} p_{2,1.11}\right\}\left\{\mu_{3}^{\prime}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)+p_{04} \mu_{4}^{\prime}+p_{05} \mu_{5}^{\prime}+\mu_{0}\right\}} \\
+\left\{1-p_{30}\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{\mu_{1}^{\prime}\left(p_{20}+p_{2,1.11}\right)+\mu_{2}^{\prime} p_{1,2.14}\right\}+\left\{p_{02}+p_{05} p_{52}\right. \\
\left.+\left(p_{3,2.10}+p_{3,2.10,16}\right)\left(1-p_{01}-p_{02}-p_{04} p_{41}-p_{05} p_{52}\right)\right\}\left\{p_{10} \mu_{2}^{\prime}-p_{20} \mu_{1}^{\prime}\right\}
\end{gathered}
$$

## 7. Profit

The Profit incurred to the system model in $(0, t]$ is given as
$P_{0}=K_{0} A_{0}-\left(C_{1} B_{0}^{I}+C_{2} B_{0}^{R}+C_{3} B_{0}^{R S}+C_{4} B_{0}^{P M}+C_{5} R_{0}^{P}+C_{6} R_{0}^{U}+C_{7} R_{0}^{S}+C_{8} I_{0}^{I}+C_{9} P_{0}^{M}\right.$
$K_{0}=$ Revenue per unit up-time of the system
$C_{1}=$ Cost per unit time for which server is busy in the inspection
$C_{2}=$ Cost per unit time for which server is busy in the repair of failed unit
$C_{3}=$ Cost per unit time for which server is busy in the repair of switch
$C_{4}=$ Cost per unit time for which server is busy in the PM
$C_{5}=$ Cost per unit replacement of the unit,
$C_{6}=$ Cost per unit repair of the unit
$C_{7}=$ Cost per unit repair of the switch,
$C_{8}=$ Cost per unit inspection of the standby unit, $C_{9}=$ Cost per unit PM of unit

## 8. Illustration using Weibull Distribution

As a special case Weibull density function with common shape parameter and different scale parameters is used as follows:
$z(t)=\alpha \eta t^{\eta-1} \exp \left(-\alpha t^{\eta}\right), g(t)=\lambda \eta t^{\eta-1} \exp \left(-\lambda t^{\eta}\right)$,
$f(t)=\beta \eta t^{\eta-1} \exp \left(-\beta t^{\eta}\right), h(t)=\gamma \eta t^{\eta-1} \exp \left(-\gamma t^{\eta}\right)$,
$s(t)=\mu \eta t^{\eta-1} \exp \left(-\mu t^{\eta}\right), o(t)=v \eta t^{\eta-1} \exp \left(-\nu t^{\eta}\right)$,
$p_{m}(t)=\omega \eta t^{\eta-1} \exp \left(-\omega t^{\eta}\right)$, Where $t \geq 0$ and $\alpha, \lambda, \beta, \gamma, \mu, \nu, \omega, \eta>0$.
The following numerical results are obtained
Table: Effect of various parameters on the Profit

| Failur e rate ( $\alpha$ | Profit ( $\eta=0.5$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{p}=0.4, \mathrm{q}=0.6, \mathrm{a}=0.3 \\ & , \mathrm{~b}=0.7, \beta=0.6, \\ & \gamma=0.7, \lambda=0.3, \mu=0.1 \\ & , v=0.02, \omega=0.8 \end{aligned}$ | $\begin{gathered} \hline \mathrm{p}=0.6, \\ \mathrm{q}=0.4 \end{gathered}$ | $\beta=0.7$ | $\lambda=0.5$ | $v=0.03$ | $\omega=1.0$ |
| 0.01 | 29064.65 | 29107.90 | 29128.30 | 29460.93 | 28850.96 | 29194.70 |
| 0.02 | 28841.39 | 28903.75 | 28914.16 | 29331.65 | 28612.75 | 28971.88 |
| 0.03 | 28601.98 | 28685.25 | 28685.23 | 29188.53 | 28359.43 | 28732.37 |
| 0.04 | 28347.44 | 28453.26 | 28442.55 | 29031.64 | 28092.00 | 28477.27 |
| 0.05 | 28078.80 | 28208.60 | 28187.13 | 28861.09 | 27811.47 | 28207.65 |
| Failur | $\boldsymbol{\eta}=1.0$ |  |  |  |  |  |
| e rate <br> ( $\alpha$ ) | $\begin{gathered} \mathrm{p}=0.4, \mathrm{q}=0.6, \mathrm{a}=0.3 \\ \mathrm{~b}=0.7, \beta=0.6, \\ \gamma=0.7, \lambda=0.3, \mu=0.1 \\ , \nu=0.02, \omega=0.8 \end{gathered}$ | $\begin{aligned} & \mathrm{p}=0.6 \\ & \mathrm{q}=0.4 \end{aligned}$ | $\beta=0.7$ | $\lambda=0.5$ | $v=0.03$ | $\omega=1.0$ |


| 0.01 | 26526.04 | 26706.36 | 26788.99 | 27415.98 | 26211.96 | 26881.77 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.02 | 26192.33 | 26427.20 | 26449.34 | 27116.39 | 25887.26 | 26523.32 |
| 0.03 | 25867.03 | 26153.22 | 26120.18 | 26819.43 | 25570.41 | 26175.51 |
| 0.04 | 25549.63 | 25884.07 | 25800.77 | 26525.28 | 25260.96 | 25837.59 |
| 0.05 | 25239.67 | 25619.41 | 25490.46 | 26234.08 | 24958.50 | 25508.88 |
| Failur | $\boldsymbol{\eta}=\mathbf{2 . 0}$ |  |  |  |  |  |
| e rate | $\mathrm{p}=0.4, \mathrm{q}=0.6, \mathrm{a}=0.3$ | $\mathrm{p}=0.6$, | $\beta=0.7$ | $\lambda=0.5$ | $\hat{y}=0.03$ | $\omega=1.0$ |
| $(\alpha)$ | $, \mathrm{b}=0.7, \boldsymbol{\beta}=0.6$, | $\mathrm{q}=0.4$ |  |  |  |  |
|  | $\gamma=0.7, \lambda=0.3, \mu=0.1$ |  |  |  |  |  |
|  | $, \nu=0.02, \omega=0.8$ |  |  |  |  |  |
| 0.01 | 23631.81 | 23945.07 | 24130.91 | 25102.58 | 23419.33 | 24004.44 |
| 0.02 | 23393.29 | 23784.49 | 23850.92 | 24790.42 | 23192.36 | 23728.05 |
| 0.03 | 23166.47 | 23625.76 | 23590.24 | 24497.05 | 22975.50 | 23468.86 |
| 0.04 | 22949.76 | 23468.86 | 23345.82 | 24219.79 | 22767.51 | 23224.27 |
| 0.05 | 22741.91 | 23313.81 | 23115.32 | 23956.53 | 22567.38 | 22992.20 |

## 9. Discussion on Results

The above table depicts the behavior of profit in relation to the failure rate and various values of the shape parameter. The table shows a decreasing trend in system profit as the unit failure rate increases. We can also see that the index begins to rise as the repair rate rises from 0.6 to 0.7 , the inspection rate rises from 0.3 to 0.5 , and the PM rate rises from 0.8 to 1.0 , while a downward trend can be seen. The numerical results show that system performance is highly dependent on standby and switch failures. As a result, adequate design and corrective strategies are required to make such systems more reliable and profitable.

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R. K. Bhardwaj, Mandeep Kaur, Department of Statistics, Punjabi

University Patiala-147002, India
E-mail address: mndp13@gmail.com


[^0]:    Received: 2019-6-20, Accepted: 2019-10-17; Communicated by guest editor George Yin. 2010 Mathematics Subject Classification. Primary 60K20; Sceondary 90B25
    Keywords. Probabilistic model, semi-Markov process, switch, operation time.

    * Corresponding Author

