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Sierpinski Curve for Agarwal Orbit

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Abstract: In this paper we mathematically discuss the structure of the Mandelpinski Necklaces for the Agarwal iteration. We will present the geometrical analysis for the relational maps $z \to (z^n + c/z^n)$. This paper presents the generalization of this relational map for special case when $n = 2$, using Agarwal iteration. Further we mathematically analyzed the characteristics of the fractal images in the complex c planes using Agarwal iterative process.

Keywords: Agarwal Iteration, Complex dynamics,Sierpinski curve, Mandelpinski Necklace.

1. INTRODUCTION

In previous year there have been many research papers which discuss the structure of the family of the complex relational maps $z \to (z^n + c)$ $n \geq 2$ and $z \to (z^n + c/z^n)$ where c is a parameter with complex values. There are various iteration procedures which are used to approximate the fixed points. We are going to explore Agarwal iteration, which is an example of two-step feedback procedure for these relational maps.The objective of this paper is to discuss the structure of Julia Set in parameter plane.

2. PRELIMINARIES

There are many other iterative processes which are used to find the fixed points of different operators.

2.1. Agarwal iteration [1]

Agarwal iteration process can be defined as

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)Tx_n + \alpha_n Ty_n \\ y_n &= (1-\beta_n)x_n + \beta_n Tx_n \end{aligned} \qquad (2.1)$$

Let suppose C be a closed compact convex subset of a real Hilbert space H, and let $T : K \to K$ be a mapping. Let sequences α_n and β_n lie in [0, 1]. Let $\{x_n\}$ be the sequence defined by process (2.1). If $\lim_{n\to\infty} \alpha_n = 1$, $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n \beta_n = \infty$, then for any arbitrary point $x_0 \in C$, the sequence generated using Agarwal iteration process converges to a fixed point x_f [6].

All that follows, for the sake of simplicity we have taken $\alpha_n = \alpha$ and $\beta_n = \beta$.

2.2. Mandelbrot Set [6]

The Mandelbrot set arises as the complex plane for quadratic family $z^2 + c$. The map is the set of all complex parameter $c \in C$ for which the orbit of the point 0 does not escape to ∞. The Mandelbrot Set was given by the Benoit Mandelbrot in 1980. Mandelbrot set is one of the most interesting objects in mathematics. For $n > 2$ in the family $z^n + c$ is termed as Multibrot set.

2.3. Julia Sets [2 4]

For the family $z^n + c$, the set of all points for which orbit tends to infinity is called the basin of attraction. The Julia set is the boundary of this basin. In other word Julia set is boundary of all set of points that escapes to infinity. The filled Julia set is the set of all points whose orbit does not tend to infinity.

3. EXPERIMENTAL ANALYSIS OF RELATIONAL MAPS

Parameter planes can be defined with three different components. The first is the set of parameters for which all critical values of relational map fallsin the immediate basin of infinity. Second component is the set of parameters c for which the critical values escapes to ∞. The final component is that when critical orbits does not escape to infinity. The result of first component is Cantor set, and the second result is Sierpinski Curve. The first is when the critical orbits lies on a boundary curve of the escaping set; the second is where the critical orbits lie on buried points in the Julia set; third is where we have quadratic like components. Based on the Devaney's work in [3], a curve which is similar to the Sierpinski curve fractal is Sierpinski curve. It is a universal plane such that it contains a copy of every compact, connected, single dimensional planer set. Two Sierpinski curves are always homeomorphic[8].

We now consider other type of relational map of the form

$$F_c(z) = z^n + c / z^n$$

where $n = 2$. If $c \to 0$, and $n = 2$ the Julia set seem to converge to a unit disk. When $c = 0$, we have the simple map $F_c(z) = z^2$, for which the Julia set is a unit circle. But when we analyze this map to any form of $F_c(z) = z^2 + c/z^2$

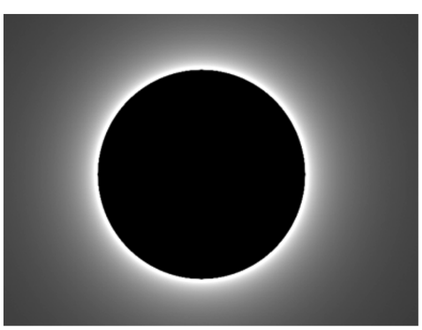

Figure 1: When c = 0

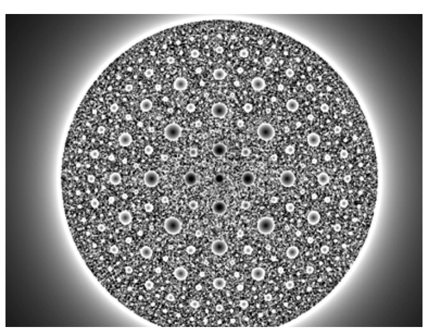

Figure 2: Julia Set for c = -0.001

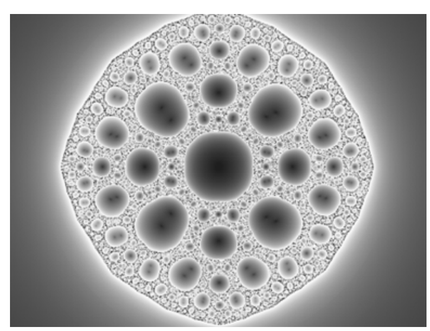

Figure 3: Julia Set for α = 0.1, β = 0.1, c = -0.0625

The Julia set structure is quite very different. See Figure 2

In Fig 3 external region contains parameter for which the Julia set is similar to Cantor set, the holes are Sierpinski holes. The external region where, the Julia sets are equivalent to Cantor sets.The central region is called McMullen domain [7].

The relational map is differentiable [9] and hence continuous, since differentiable functions are continuous. In theorem 1 we have proved the escape criteria for this relational family using Agarwal orbit, which shows that the orbit of z tends to infinity as n tends to infinity.

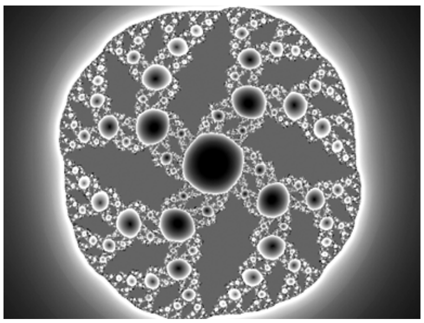

Figure 4: for $\alpha = 0.1$, $\beta = 0.25$, $c = 0.025 + 0.0375i$

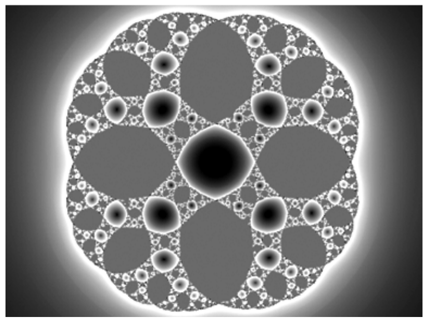

Figure 5: for $\alpha = 0.1$, $\beta = 0.25$, $c = 0.0625$

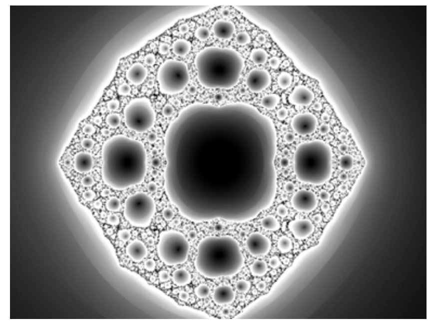

Figure 6: for $\alpha = 0.5$, $\beta = 0.1$, $c = -0.254$

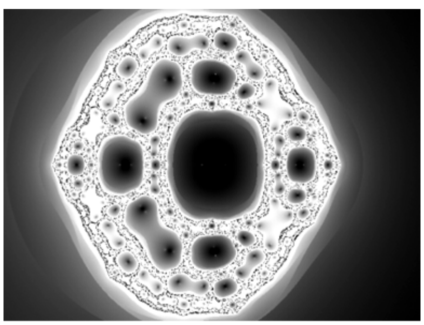

Figure 7: for $\alpha = 0.5$, $\beta = 0.5$, $c = 0.0625$

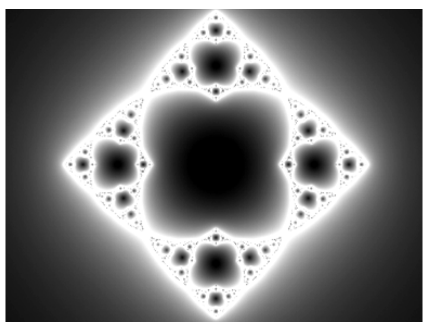

Figure 8: for $\alpha = 0.0$, $\beta = 0.0$, $c = -0.36875$

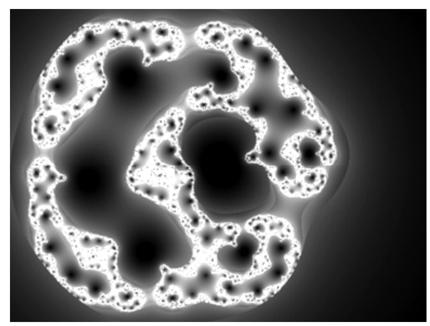

Figure 9: for $\alpha = 0.9$, $\beta = 0.5$, $c = -0.36875-0.25i$

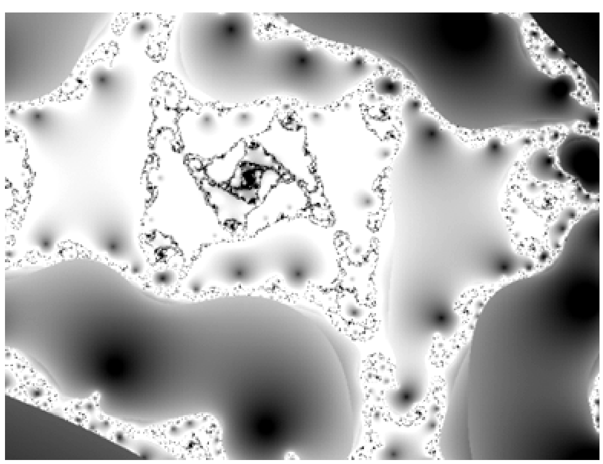

Figure 10: Magnification of Fig 9

When c = 0, we have $F_c(z) = z^2$, if the initial point is 0 or 1, then the orbits are constant. If $x_0 > 1$, the orbit diverges to infinity. If $0 < x_0 < 1$, then orbit of Julia set converges to fixed point 0. For $\alpha = 0.5$ and $\beta = 0.1$, for $\alpha = 0.5$ and $\beta = 0.5$ fig 6 and fig 7 shows that the necklaces are symmetrical about x-axis. If we set α and β both to 0 zero then the Julia sets are symmetrical about both the axis (Figure 8). For $\alpha = 0.9$ and $\beta = 0.5$ the Julia set contains four part, the boundary of each is Cantor set. In figure 4 and 5 the inner red colored regions are filled in Julia set.

Theorem 1 (the Escape Criterion). Suppose $|z| \geq |c| > 2/\alpha$ and $|z| \geq |c| > 2/\beta$ where $0 < \alpha < 1$ and

$0 < \beta < 1 and\ |c| < 1, c \in C$

Define the sequence

$|z_1| = (1-\alpha)F(z) + \alpha F'(z)$

•

•

•

$|z_n| = (1-\alpha)F(z_{n-1}) + \alpha F'(z_{n-1})$

Where $F(z)$ and $F'(z)$ are relational maps of the form $f = z^2 + c/z^2$.

Then we have $|z_n| \to \infty$ as $n \to \infty$.

Proof: If $|z| \geq 2$, then we have

$$|F_c(z)| \geq |z^2| - \frac{|c|}{|z^2|}$$

$$\geq |z||z| - \frac{1}{|z||z|}$$

$$\geq 2|z| - \frac{1}{4}$$

$$> \frac{3}{2}|z| \qquad (1)$$

$|F_c'(z)| = |(1-\beta)z + \beta F_c(z)|$

$$= (1-\beta)|z| + \frac{3}{2}\beta|z| \qquad \text{using (2)}$$

$$= |z| - \beta|Z| + \frac{3}{2}\beta|z|$$

$$= |z| + \frac{1}{2}\beta |z|$$

$$\geq \frac{3}{2}\beta |z|$$

i.e. $|F(z)| \geq \frac{3}{2}\beta |z|$ (2)

Since $|z_n| = (1-\alpha)F(z_{n-1}) + \alpha F^{'}(z_{n-1})$

We have

$|z_1| = (1-\alpha)F(z) + \alpha F^{'}(z)$

$= (1-\alpha)\frac{3}{2}|z| + \alpha\beta\frac{3}{2}|z|$

$= \frac{3}{2}|z| - \frac{3}{2}\alpha|z| + \alpha\beta\frac{3}{2}|z|$

$= \frac{3}{2}|z|(1-\alpha+\alpha\beta)$

$\geq \alpha\beta\frac{3}{2}|z|$

Since $|z| > 2$ and $0 < \alpha < 1$ an $0 < \beta < 1$, inductively we have

$|z_n| \geq \left(\frac{3}{2}\right)^n |z|$.

Hence the orbit of z escapes to infinity. This completes the proof.

4. CONCLUSION

We have extended the Devaney's work and constructed a series of Necklaces and the Cantor set of the complex analytical map using Agarwal iteration. Again we have mathematically proved the escape criteria for given maps.

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