

# A Numerical Model of Air Pollutants Emitted from a Line Source of Primary and Secondary Pollutants with Mesoscale Wind

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## ABSTRACT

A numerical model for air pollution due to primary and secondary pollutants emitted from a line source along with the mesoscale wind is presented. The Crank-Nicolson finite difference scheme is employed to solve the model numerically. The realistic forms of large-scale, mesoscale wind velocities and eddy diffusivity profiles are used in the model. The results have been analysed for the dispersion of air pollutants for stable and neutral conditions of the atmosphere in the presence of mesoscale wind. The effect of chemical reaction decreases the concentration of primary pollutants and increases the concentration of secondary pollutants everywhere.

**Keywords:** Numerical model, Line Source, Air Pollutants, Mesoscale wind, Chemical reaction, Finite difference method.

## 1. INTRODUCTION

The living system on earth is at risk, whether it is a human being, animal husbandry or vegetation, due to various contaminants occurring in the ecosystem. In particular, the grown cities or urban areas in developing and developed countries are majorly affected by this. One of the major contaminant affecting human life and its environment in recent past is air pollution because of considerable growth in industrialization and urbanization. The air pollutants are continuously emitted by combustion of hydrocarbon fuels in residential area, vehicular exhausts due to traffic flow and several other sources. These pollutants pollute a part or whole area of an urban environment. Janice J Kim et al. [1] reported that the traffic pollutants like particulate matter, black carbon, nitrogen oxides ( $\text{NO}_x$ ) etc., have association with respiratory symptoms, asthma exacerbations and decrements in lung function of children. Thus the impacts of vehicular emissions to the atmosphere are considerable and have contributions to air pollution.

Mathematical models are of prime importance to know the role of different meteorological parameters associated with the life cycle of the air pollutants and also in describing and understanding the dispersion of air pollutants on pollutant concentration. Arora et al [2] developed a one-dimensional numerical model with variable wind velocity and linear diffusivity profile to study the instantaneous and delayed removal process of air pollutant emitted from a line source. In an approximation to Gaussian plume model to study atmospheric dispersion John M Stockie [3] studied a special case of continuous line source emission and gave an analytical solution. All the above models did not take in to the account of mesoscale wind effect in addition to large scale wind and nothing about when the pollutants are chemically reactive.

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To predict concentration of pollutants in urban areas, the mesoscale wind is considered along with the large scale wind. The mesoscale wind is chosen to simulate a local wind produced by urban heat island effect [Dilley and Yen [4]]. They gave an analytical model for a pollutant distribution emitted from a line source considering the effect of mesoscale type wind. This model didn't consider the effect of chemically reactive pollutants. M Sheku and C Sulochana [5] represented a time dependent analytical model for chemically reactive primary pollutants emitted from an elevated line source with the parabolic eddy diffusion coefficient and the wind velocity as function of vertical height. But they didn't include the effect of mesoscale wind in the model.

Pandurangappa C et al [6] constructed an analytical model to study the dispersion of pollutants emitted from a line source and K Lakshminarayanachari et al [7] built a two dimensional numerical model to study the dispersion of pollutants emitted from an urban area source. These mathematical models considered the effect of mesoscale type wind and chemical reaction, but both models gave a solution to the concentration of primary pollutants only.

The conversion of air pollutants from gaseous to particulate form is an important atmospheric phenomenon that requires ones attention in mathematical modeling and reveals a lot on urban plume characteristics. This leads to formation of secondary pollutants from primary pollutants which have longer life period and are more hazardous to living environment. Sujith Kumar Khan et al [8] exhibited a three-dimensional unsteady mathematical model of primary as well as secondary pollutants with various types of time dependent sources. However this model did not consider the effect of mesoscale type wind.

Thus the present work aims at constructing a numerical model of air pollutants emitted from a line source of primary as well as secondary pollutants with mesoscale wind. As the mesoscale type wind and chemically reactive nature of pollutants have effect on concentration of pollutants. We took realistic form of variable wind velocity and eddy diffusivity profiles in the present problem and is solved by using Crank-Nicolson finite difference scheme. In this respect the proposed model is more realistic than the previous.

## 2. MODEL DEVELOPMENT

The problem consists of an infinite crosswind line source on the ground with finite downwind distance and infinite crosswind dimension. The pollutants are assumed to be transported horizontally in perpendicular

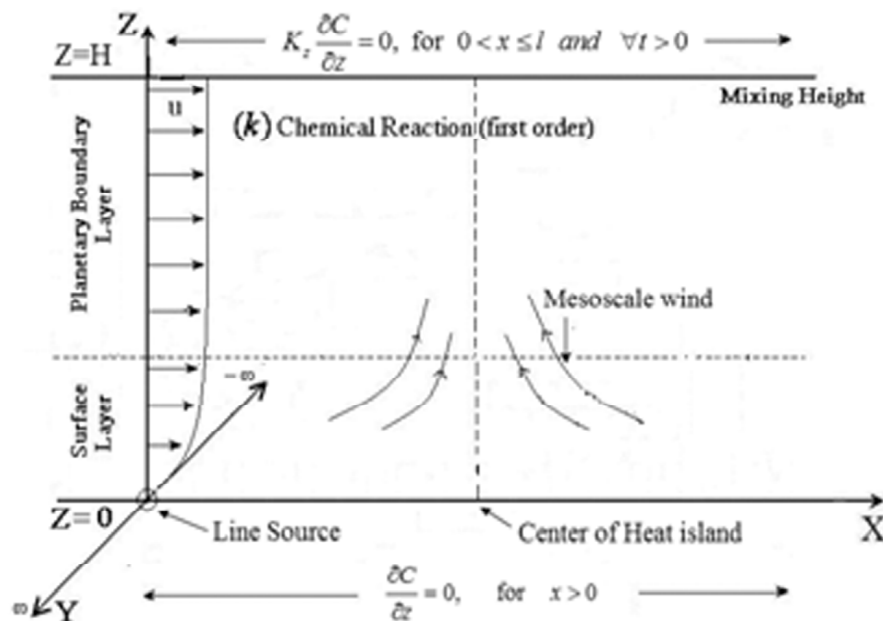


Figure 1: Physical layout of the model.

direction by a large-scale wind and horizontally as well as vertically by a local wind called mesoscale wind. The mesoscale wind is produced by urban heat island. Here the large-scale wind is considered as a function of vertical height ( $z$ ) and mesoscale wind is a function of both height ( $z$ ) and distance ( $x$ ). We have considered the center of heat island at a distance  $x = l/2$  that is at the center of the city, where  $l$  is the city length and  $l = 6\text{km}$  in this problem. The concentration of pollutants is computed in the region  $0 \leq x \leq l$ . The line source is kept at the beginning of the city, that is at  $x = 0$ . Here the pollutants are chemically reactive and are converted into secondary pollutants by means of first order chemical reaction rate. The physical description of this model is as shown in the figure 1.

The general species advection diffusion equation for air pollution is

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + W \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left( K_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial C}{\partial z} \right) - kC \quad (1)$$

where  $C$  is the pollutant concentration at any time  $t$  and location  $(x, y, z)$ ,  $U$ ,  $V$  and  $W$  are the velocity components along  $x$ ,  $y$  and  $z$  directions respectively,  $K_x$ ,  $K_y$  and  $K_z$  are the eddy – diffusivity coefficients along  $x$ ,  $y$  and  $z$  directions respectively and  $k$  is the chemical reaction rate coefficient.

## 2.1. Primary Pollutants

The basic governing partial differential equation (1) for primary pollutant can be written as

$$\frac{\partial C_p}{\partial t} + U(x, z) \frac{\partial C_p}{\partial x} + W(z) \frac{\partial C_p}{\partial z} = \frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_p}{\partial z} \right) - kC_p \quad (2)$$

where  $C_p = C_p(x, z, t)$  is the mean concentration of primary pollutant species,  $U(x, z)$  denotes the velocity along both horizontal and vertical direction and  $W(z)$  denotes the velocity in the vertical direction due to the effect of mesoscale wind,  $K_z(z)$  is the turbulent eddy diffusivity in  $z$ -direction,  $k$  is the first order chemical reaction rate coefficient of primary pollutant  $C_p$ . We derived the above equation (2) under the following assumptions

- Pollutants are chemically reactive.
- Wind velocity along the  $x$  direction is so large that the  $x$  direction diffusion is neglected.
- The lateral flux of pollutants along crosswind direction is assumed to be small i.e.,

$$V \frac{\partial C}{\partial y} \rightarrow 0 \text{ and } \frac{\partial}{\partial y} \left( K_y \frac{\partial C}{\partial y} \right) \rightarrow 0.$$

### 2.1.1. Initial and boundary conditions

We assume that at the beginning of the emission the region of interest is free from pollution. Therefore we get initial condition as:

$$C_p = 0, \text{ at } t = 0, \quad 0 \leq x \leq l \text{ and } 0 \leq z \leq H \quad (3)$$

We assume that no background pollution is entering the region of interest at  $x = 0$ , i.e.,

$$C_p = 0, \text{ at } x = 0, \quad 0 \leq z \leq H \text{ and } \forall t > 0 \quad (4)$$

For a continuous line source of strength  $Q$  located at origin (which has an infinitesimal small extension in the  $z$ -direction), we have the following boundary condition:

$$C_p = Q \frac{\delta(z)}{u(x, z)}, \text{ at } x = 0, z = 0 \text{ and } \forall t > 0 \quad (5)$$

We assume that there is no transfer or deposition of pollutants at the ground. Therefore there is no concentration gradient at the ground level i.e.,

$$\partial C_p / \partial z = 0, \text{ at } z = 0, x > 0 \quad (6)$$

The pollutants are confined within the mixing height and there is no leakage across the top boundary of the mixing layer. Thus

$$K_z \partial C_p / \partial z = 0, \text{ at } z = H, 0 < x \leq l \text{ and } \forall t > 0 \quad (7)$$

## 2.2. Secondary Pollutants

The basic governing partial differential equation (1) for secondary pollutant can be written as

$$\frac{\partial C_s}{\partial t} + U(x, z) \frac{\partial C_s}{\partial x} + W(z) \frac{\partial C_s}{\partial z} = \frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_s}{\partial z} \right) + k C_p \quad (8)$$

where  $C_s$  denote the mean concentration of secondary pollutants. In deriving eq. (8) we made similar assumptions as in the case of primary pollutant.

### 2.2.1. Initial and boundary conditions

At the beginning of the emission the region of interest is assumed to be free from pollution and no background pollution is entering the region of interest at  $x = 0$ . Thus the appropriate initial and boundary conditions are

$$C_s = 0, \text{ at } t = 0, 0 \leq x \leq l \text{ and } 0 \leq z \leq H \quad (9)$$

$$C_s = 0, \text{ at } x = 0, 0 \leq z \leq H \text{ and } \forall t > 0 \quad (10)$$

We assume that there is no transfer or deposition of pollutants at the ground. Therefore there is no concentration gradient at the ground level i.e.,

$$\partial C_s / \partial z = 0, \text{ at } z = 0, x > 0 \quad (11)$$

The pollutants are confined within the mixing height and there is no leakage across the top boundary of the mixing layer. Thus

$$K_z \partial C_s / \partial z = 0, \text{ at } z = H, 0 < x \leq l \text{ and } \forall t > 0 \quad (12)$$

## 3. METEOROLOGICAL PARAMETERS

In order to solve the equations (2) and (8) the profiles of large-scale, mesoscale wind speeds and eddy diffusivity for different atmospheric stability conditions and for various meteorological parameters such as surface roughness, friction velocity, stability length, net heat flux etc., are used in accordance with K Lakshminarayanachari et al [7].

It is assumed that for neutral atmospheric stability condition the surface layer terminates at  $z = 0.1k(u_* / f)$  and for stable atmospheric stability condition the surface layer extends up to  $z = 6L$ , where  $k = 0.4$  is known as the Karman's constant,  $f$  is the Coriolis parameter,  $u_*$  is the friction velocity and  $L$  is the Monin-Obukhov stability length parameter.

The wind velocity profiles used for neutral atmospheric stability condition are

$$U(x, z) = \left( \frac{u_*}{\kappa} - a(x - x_0) \right) \ln \left( \frac{z + z_0}{z_0} \right) \text{ and } W(z) = a \left[ z \ln \left( \frac{z + z_0}{z_0} \right) - z + z_0 \ln(z + z_0) \right] \quad (13)$$

where  $z < 0.1\kappa(u_* / f)$ .

The wind velocity profiles for stable atmospheric stability condition and for  $0 < z/L < 1$  are

$$U(x, z) = \left( \frac{u_*}{\kappa} - a(x - x_0) \right) \left[ \ln \left( \frac{z + z_0}{z_0} \right) + \frac{\alpha}{L} z \right] \text{ and } W(z) = a \left[ z \ln \left( \frac{z + z_0}{z_0} \right) - z + z_0 \ln(z + z_0) + \frac{\alpha}{2L} z^2 \right] \quad (14)$$

for  $1 < z/L < 6$ , we have

$$U(x, z) = \left( \frac{u_*}{\kappa} - a(x - x_0) \right) \left[ \ln \left( \frac{z + z_0}{z_0} \right) + 5.2 \right] \text{ and } W(z) = a \left[ z \ln \left( \frac{z + z_0}{z_0} \right) + z_0 \ln(z + z_0) + 4.2z \right]. \quad (15)$$

For both neutral ( $z \geq 0.1\kappa u_* / f$ ) and stable ( $z/L \geq 6$ ) atmospheric stability condition in the planetary boundary layer, above the surface layer, the wind profiles are given by

$$U(x, z) = \left[ (u_g - u_{sl}) - a(x - x_0) \right] \left( \frac{z - z_{sl}}{H - z_{sl}} \right)^p + (1 - a(x - x_0)) u_{sl} \text{ and } W(z) = a \left[ \frac{(z - z_{sl})}{p + 1} \left( \frac{z - z_{sl}}{H - z_{sl}} \right)^p + z u_{sl} \right] \quad (16)$$

where  $u_g$  is the geostrophic wind velocity,  $u_{sl}$  is wind at  $z_{sl}$ ,  $z_{sl}$  is the top of the surface layer,  $H$  is the mixing height and  $p$  is an exponent which depends on the atmospheric stability. Here  $p$  values are used as with Jones et al. [9]. All the above wind velocity profiles (13) to (16) are valid only for  $x \leq (u_* / a\kappa) + x_0$ .

For both surface layer and planetary boundary layer (entire boundary layer) the eddy diffusivity profiles used are  $K_z = 0.4u_* z \exp(-4z/H)$ , for neutral case (Shir [10])

$$(17)$$

$$\text{and } K_z = \left[ \kappa u_* z / (0.74 + 4.7 z/L) \right] \exp(-b\eta), \text{ for stable case (Ku et al. [11])} \quad (18)$$

where  $b = 0.91$ ,  $\eta = z/(L\sqrt{\mu})$ ,  $\mu = u_* / |fL|$  and  $f = 10^{-4}$ .

#### 4. METHOD OF SOLUTION

The objective of this model is to study and analyse the concentration of chemically reactive air pollutants and their secondary products emitted from a line source with mesoscale type wind. For this we need to solve eqs. (2) and (8) along with their corresponding initial and boundary conditions (3) to (7) and (9) to (12) respectively. One can note that due to the variable and complicated forms of wind speed and eddy diffusivity, it is tedious to obtain the analytical solution of eqs. (2) and (8). Thus we used the numerical method based on Crank-Nicolson finite difference scheme to obtain the solution. Now to apply Crank-Nicolson finite difference scheme the continuum region of interest is subdivided into a set of equal rectangles, by equally spaced grid lines, parallel to  $z$  axis, defined by  $x_i = (i-1)\Delta x$ ,  $i = 1, 2, 3, \dots$  and equally spaced grid lines parallel to  $x$  axis, defined by  $z_j = (j-1)\Delta z$ ,  $j = 1, 2, 3, \dots$  respectively, where  $\Delta x$  and  $\Delta z$  are sides of rectangles. Time is indexed as  $t_n = n\Delta t$ ,  $n = 0, 1, 2, 3, \dots$ , where  $\Delta t$  is the time step. Now the equation (2) is replaced by the equation valid at time step  $n+1/2$  and at the interior grid points  $(i, j)$ , therefore can be written as

$$\frac{\partial C_p}{\partial t} \Big|_{ij}^{n+1/2} + \frac{1}{2} \left[ U(x, z) \frac{\partial C_p}{\partial x} \Big|_{ij}^n + U(x, z) \frac{\partial C_p}{\partial x} \Big|_{ij}^{n+1} \right] + \frac{1}{2} \left[ W(z) \frac{\partial C_p}{\partial z} \Big|_{ij}^n + W(z) \frac{\partial C_p}{\partial z} \Big|_{ij}^{n+1} \right]$$

$$= \frac{1}{2} \left[ \frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_p}{\partial z} \right) \Big|_{ij}^n + \frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_p}{\partial z} \right) \Big|_{ij}^{n+1} \right] - \frac{1}{2} k (C_{pij}^n + C_{pij}^{n+1}),$$

$$i = 2, 3, 4, \dots, i \max, \quad j = 2, 3, 4, \dots, j \max - 1, \quad n = 0, 1, 2, \dots \quad (19)$$

Here the spatial derivatives are replaced by the arithmetic average of its finite difference approximations at the  $n^{\text{th}}$  and  $(n + 1)^{\text{th}}$  time steps and the time derivative is replaced by a central difference with time step  $n + 1/2$ .

Now consider

$$\frac{\partial C_p}{\partial t} \Big|_{ij}^{n+\frac{1}{2}} = \frac{C_{pij}^{n+1} - C_{pij}^n}{\Delta t}, \quad (20)$$

$$U(x, z) \frac{\partial C_p}{\partial x} \Big|_{ij}^n = U_{ij} \left[ \frac{C_{pij}^n - C_{pi-1j}^n}{\Delta x} \right], \quad (21)$$

$$U(x, z) \frac{\partial C_p}{\partial x} \Big|_{ij}^{n+1} = U_{ij} \left[ \frac{C_{pij}^{n+1} - C_{pi-1j}^{n+1}}{\Delta x} \right], \quad (22)$$

$$W(z) \frac{\partial C_p}{\partial z} \Big|_{ij}^n = W_j \left[ \frac{C_{pij}^n - C_{pij-1}^n}{\Delta z} \right], \quad (23)$$

$$W(z) \frac{\partial C_p}{\partial z} \Big|_{ij}^{n+1} = W_j \left[ \frac{C_{pij}^{n+1} - C_{pij-1}^{n+1}}{\Delta z} \right], \quad (24)$$

$$\frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_p}{\partial z} \right) \Big|_{ij}^n = \frac{1}{2(\Delta z)^2} \left[ (K_{j+1} + K_j)(C_{pij+1}^n - C_{pij}^n) - (K_j + K_{j-1})(C_{pij}^n - C_{pij-1}^n) \right], \quad (25)$$

$$\frac{\partial}{\partial z} \left( K_z(z) \frac{\partial C_p}{\partial z} \right) \Big|_{ij}^{n+1} = \frac{1}{2\Delta z^2} \left[ (K_{j+1} + K_j)(C_{pij+1}^{n+1} - C_{pij}^{n+1}) - (K_j + K_{j-1})(C_{pij}^{n+1} - C_{pij-1}^{n+1}) \right]. \quad (26)$$

Substituting equations (20) to (26) in eq. (19) and rearranging and simplifying we get finite difference equation for the primary pollutant concentration  $C_p$  in the form

$$B_j C_{pij+1}^{n+1} + D_{ij} C_{pij}^{n+1} + E_j C_{pij-1}^{n+1} = F_{ij} C_{pij}^n + G_j C_{pij-1}^n + M_{ij} C_{pij}^n + N_j C_{pij+1}^n - A_{ij} C_{pi-ij}^{n+1}. \quad (27)$$

for each  $i = 2, 3, 4, \dots$ , for each  $j=2, 3, 4, \dots, j\max-1$  and  $n = 0, 1, 2, 3, \dots$

Here

$$A_{ij} = -U_{ij} \frac{\Delta t}{2\Delta x} \quad F_{ij} = U_{ij} \frac{\Delta t}{2\Delta x}, \quad B_j = - \left[ \frac{\Delta t}{4\Delta z^2} (K_j + K_{j-1}) + \frac{\Delta t}{2\Delta z} W_j \right], \quad E_j = - \frac{\Delta t}{4\Delta z^2} (K_j + K_{j+1})$$

$$G_j = \left[ \frac{\Delta t}{4\Delta z^2} (K_j + K_{j-1}) + \frac{\Delta t}{2\Delta z} W_j \right], \quad D_{ij} = 1 + \frac{\Delta t}{2\Delta x} U_{ij} + \frac{\Delta t}{2\Delta z} W_j + \frac{\Delta t}{4\Delta z^2} (K_{j+1} + 2K_j + K_{j-1}) + \frac{\Delta t}{2} k,$$

$$M_{ij} = 1 - \frac{\Delta t}{2\Delta x} U_{ij} - \frac{\Delta t}{2\Delta z} W_j - \frac{\Delta t}{4\Delta z^2} (K_{j+1} + 2K_j + K_{j-1}) - \frac{\Delta t}{2} k, \quad N_j = \frac{\Delta t}{4\Delta z^2} (K_{j+1} + K_j).$$

and  $imax$  is the  $i$  value at  $x = l$  and  $jmax$  is the value of  $j$  at  $z = H$ .

The initial condition (3) can be written as

$$C_{pij}^0 = 0 \text{ for } j = 1, 2, \dots, jmax, i = 1, 2, \dots, imax. \tag{28}$$

The boundary conditions (4) and (5) together can be written as

$$C_{pij}^{n+1} = \begin{cases} Q/u_{ij} & \text{for } i = 1, j = 2 \\ 0 & \text{for } i = 1, j \neq 2 \end{cases} \text{ and } n = 0, 1, 2, 3, \dots, \tag{29}$$

The boundary condition (6) can be written as

$$C_{pi+1j}^{n+1} - C_{pij}^{n+1} = 0, \text{ for } j = 1, i = 2, 3, \dots, imax \text{ and } n = 0, 1, 2, 3, \dots \tag{30}$$

The boundary condition (7) can be written as

$$C_{pij-1}^{n+1} - C_{pij}^{n+1} = 0, \text{ for } j = jmax, i = 2, 3, \dots, imax \text{ and } n = 0, 1, 2, 3, \dots, \tag{31}$$

By adopting the similar procedure to obtain the finite difference equations for the secondary pollutant  $C_s$ , the partial differential equation (8) can be written as

$$\bar{B}_j C_{sij-1}^{n+1} + \bar{D}_j C_{sij}^{n+1} + \bar{E}_j C_{sij+1}^{n+1} = \bar{F}_{ij} C_{si-ij}^n + \bar{G}_j C_{sij-1}^n + \bar{M}_{ij} C_{sij}^n + \bar{N}_j C_{sij+1}^n + \Delta t k C_{pij}^n - \bar{A}_{ij} C_{si-ij}^{n+1} \tag{32}$$

for each  $i = 2, 3, 4, \dots$ , for each  $j = 2, 3, 4, \dots, jmax-1$  and  $n = 0, 1, 2, 3, \dots$

Here

$$\bar{A}_{ij} = A_{ij}, \quad \bar{B}_j = B_j, \quad \bar{D}_{ij} = 1 + \frac{\Delta t}{2\Delta x} U_{ij} + \frac{\Delta t}{2\Delta z} W_j + \frac{\Delta t}{4\Delta z^2} (K_{j+1} + 2K_j + K_{j-1}), \quad \bar{E}_j = E_j,$$

$$\bar{F}_{ij} = F_{ij}, \quad \bar{G}_j = G_j, \quad \bar{M}_{ij} = 1 - \frac{\Delta t}{2\Delta x} U_{ij} - \frac{\Delta t}{2\Delta z} W_j - \frac{\Delta t}{4\Delta z^2} (K_{j+1} + 2K_j + K_{j-1}), \quad \bar{N}_j = N_j.$$

The initial and boundary conditions on secondary pollutant  $C_s$  obtained from equations (9) to (12) can be written as

$$C_{sij}^0 = 0 \text{ for } j = 1, 2, \dots, jmax, i = 1, 2, \dots, imax \tag{33}$$

$$C_{sij}^{n+1} = 0 \text{ for } i = 1, j = 2, \dots, jmax, n = 0, 1, 2, 3, \dots, \tag{34}$$

$$C_{si+1j}^{n+1} - C_{sij}^{n+1} = 0 \text{ for } j = 1, i = 2, 3, \dots, imax, n = 0, 1, 2, 3, \dots, \tag{35}$$

$$C_{sij}^{n+1} - C_{sij-1}^{n+1} = 0 \text{ for } j = jmax, i = 2, 3, \dots, imax, n = 0, 1, 2, 3, \dots, \tag{36}$$

Equations (27) and (32) along with their corresponding initial and boundary conditions form a coupled system of equations. We first solve the system of equations (27) to (31) for  $C_{pij}^n$ , which is independent of the system of equations (32) to (36) at every time step  $n$ . This result is used at every time step in eq. (32) and solved for  $C_{sij}^n$ . Both the systems of equations are solved using the Thomas algorithm for tri-diagonal equations and therefore the solutions are obtained for primary and secondary pollutant concentrations.

## 5. RESULTS AND DISCUSSION

A two dimensional numerical model has been developed to compute the air pollutants concentration along downwind and vertical directions emitted from a line source with mesoscale type wind and chemical reaction. The model presented here allows the estimation of concentration distribution for more realistic meteorological conditions. The model has been solved by using Crank-Nicolson finite difference method which is unconditionally stable. We considered  $\Delta x = 75$  meter and  $\Delta z = 1$  meter as grid size. We used first order back difference scheme to approximate the advective terms and central differences to approximate the diffusion terms present in the basic equation. The discretized algebraic equations are in tri-diagonal matrix form and the Thomas algorithm is used to solve it efficiently. The results of this model have been exhibited graphically in figures 2 – 9 to analyse the dispersion of primary and secondary air pollutants for stable and neutral conditions of atmosphere.

In figures 2 and 3 the effect of mesoscale wind on primary and secondary pollutants with respect to height is studied for stable and neutral cases. The concentration of primary and secondary pollutants is less in the presence of mesoscale wind at the distance  $x = 975$  meter compared to that in the absence of mesoscale wind, whereas the concentration is more in the presence of mesoscale wind at  $x = 5025$  meter compared to that in the absence of mesoscale wind for both stable and neutral cases. This is due to increase in the velocity of large-scale wind in the upwind side of center of heat island by mesoscale wind and decrease in velocity of large-scale wind in the downwind side of center of heat island. Thus the mesoscale wind reduces the concentration of primary and secondary pollutants in the upwind side and increases the concentration in the downwind side of the center of heat island. The neutral case carries the concentration of pollutants to greater heights compared to that of stable case for both primary and secondary.

In figures 4 and 5 the concentration of primary and secondary pollutants with respect to distance is presented for stable and neutral cases. The concentration of pollutants is less on the upwind side and more on the downwind side in the presence of mesoscale wind compared to that in the absence. This is due to the horizontal component of mesoscale wind which is along the large scale wind on the left and against on the right. The secondary pollutant concentration is reduced in the right side of the center of heat island as the magnitude of secondary pollutant concentration is less when compared to that of primary pollutant.

The effect of chemical reaction on concentration of primary and secondary pollutant with respect to height for stable and neutral atmospheric condition at  $x = 975$  meter are depicted in figures 6 and 7. As the chemical reaction rate coefficient increases the concentration of primary pollutant decreases, whereas the

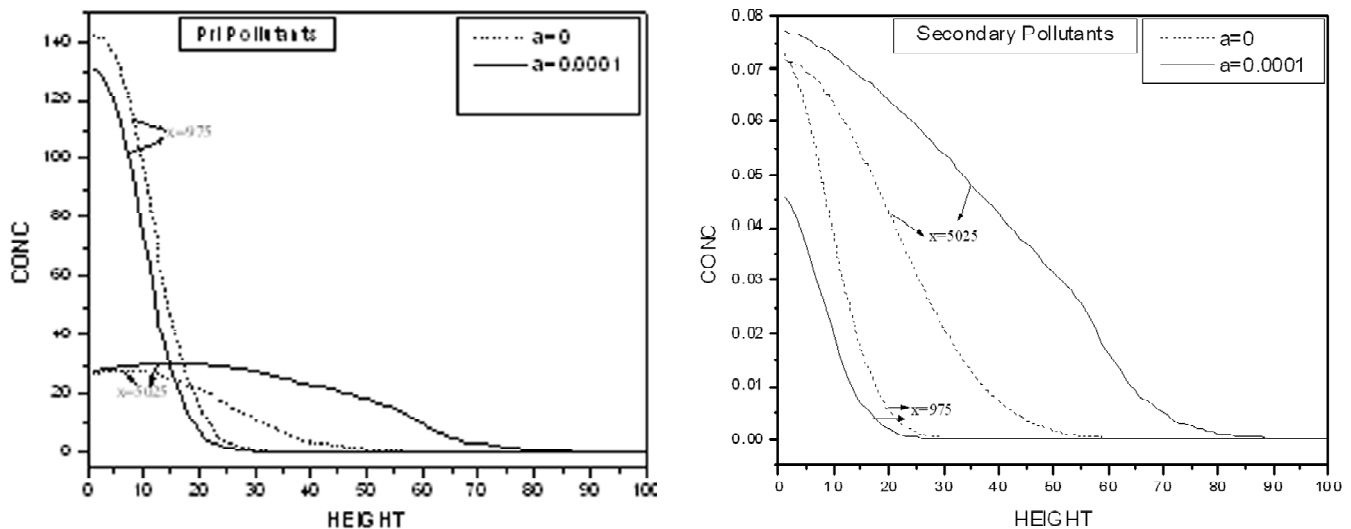


Figure 2: Concentration versus Height for stable case.



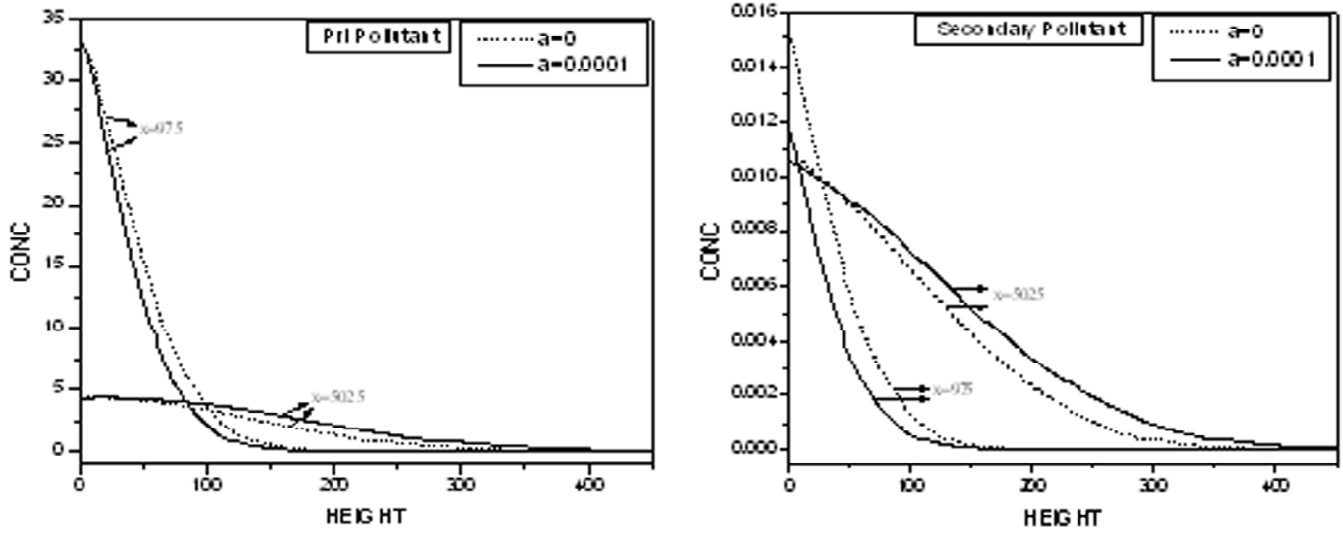


Figure 3: Concentration versus Height for neutral case.

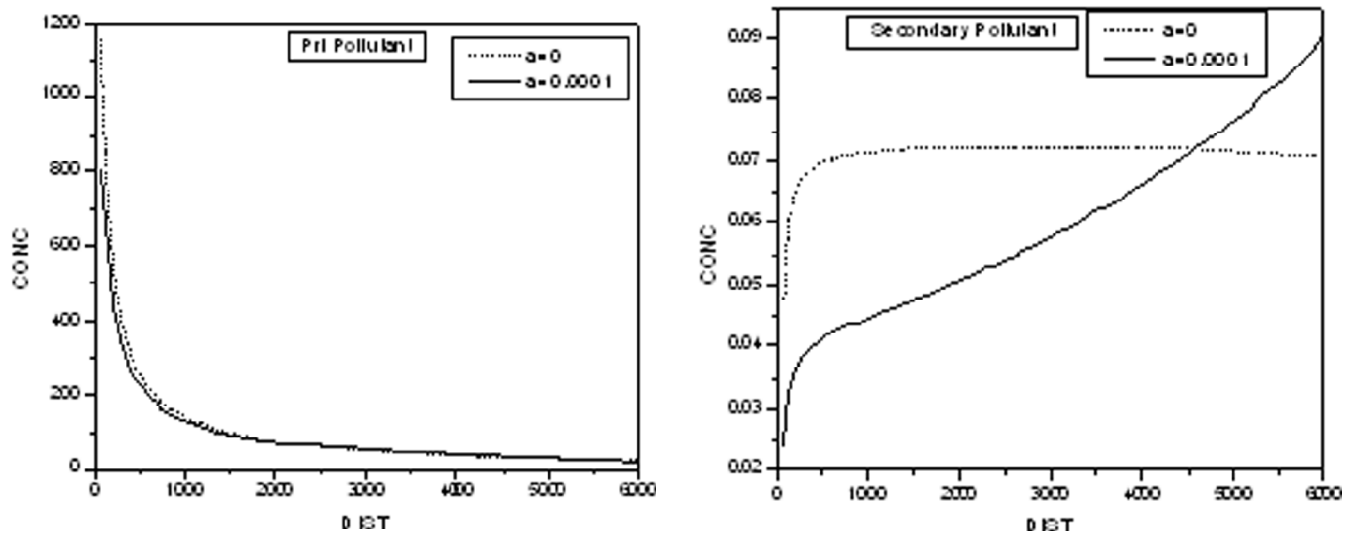


Figure 4: Concentration versus Distance of Primary and Secondary pollutants for stable case.

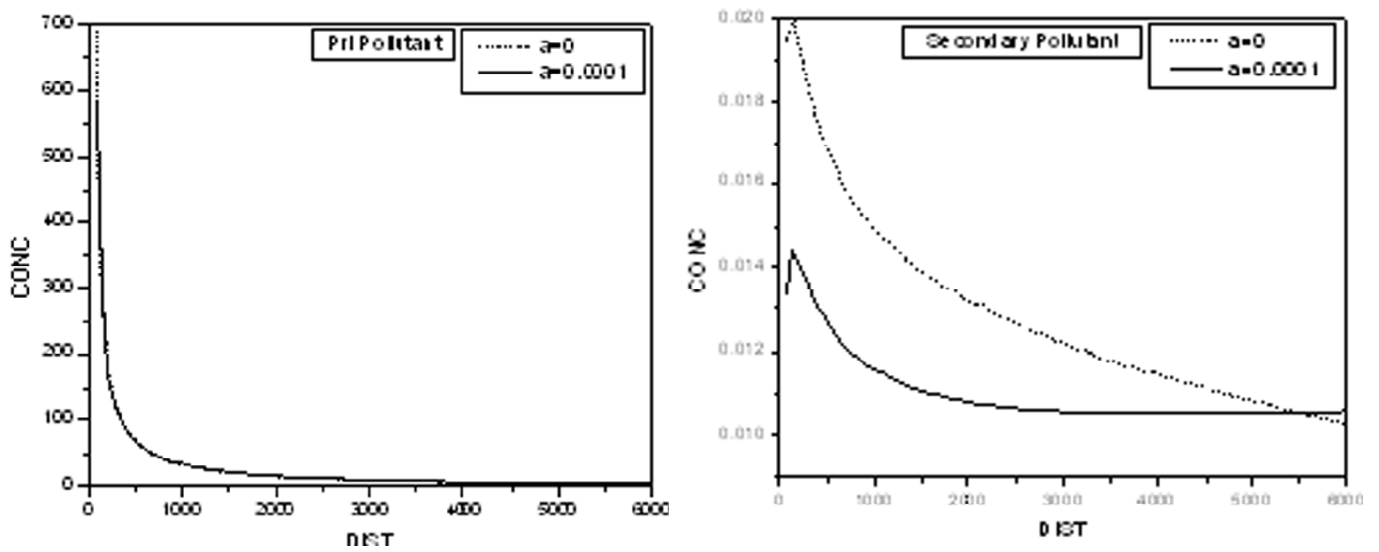


Figure 5: Concentration versus Distance for neutral case.

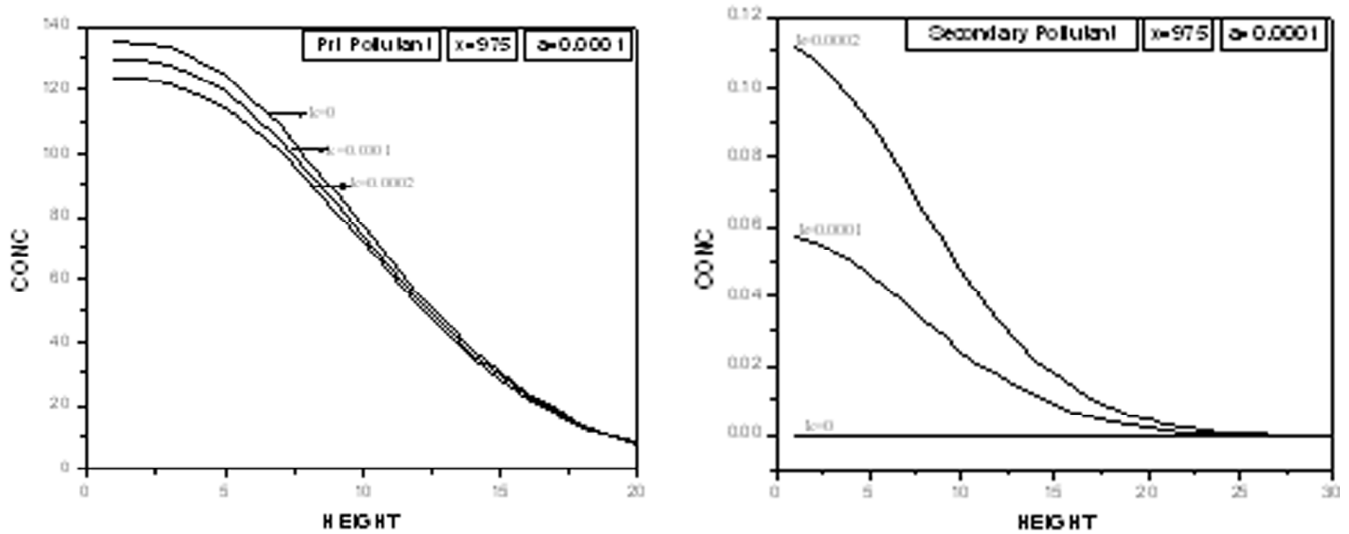


Figure 6: Concentration versus Height for different chemical reaction rate coefficient for stable case.

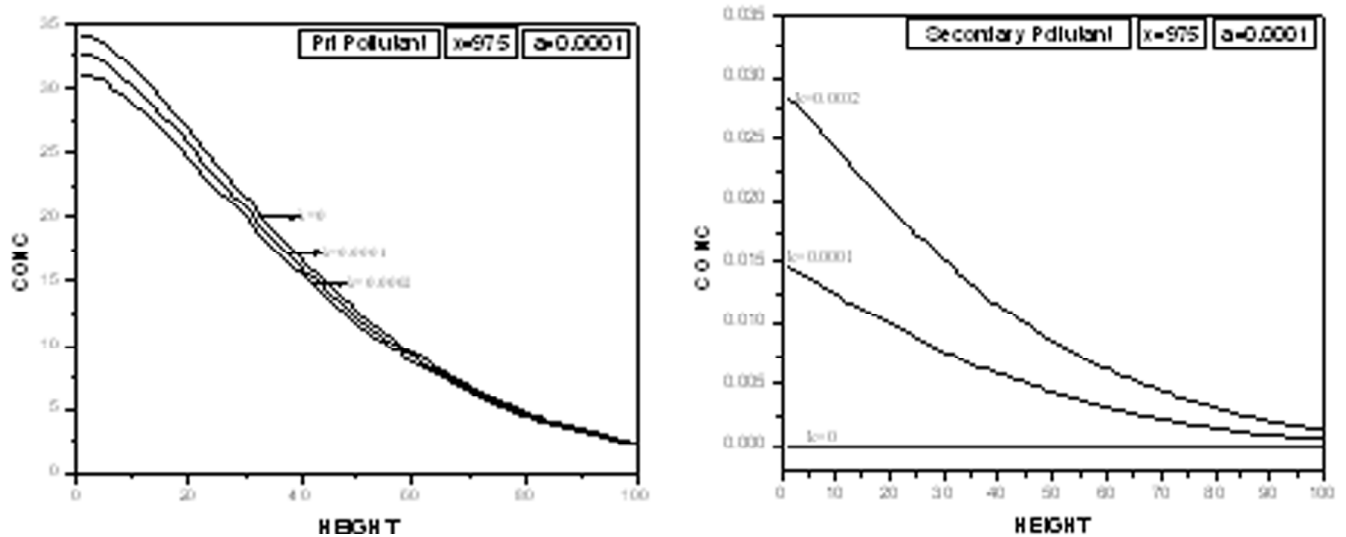


Figure 7: Concentration versus Height for different chemical reaction rate coefficient for Neutral case.

concentration of secondary pollutant increases with increase in chemical reaction. This behavior is due to conversion of primary air pollutants to secondary through chemical reaction. Clearly when there is no chemical reaction ( $k = 0$ ) there is no concentration of secondary pollutants.

## 6. CONCLUSION

A numerical model has been developed to compute the ambient concentration of primary and secondary air pollutants emitted from a line source along with the effect of mesoscale type wind and chemical reaction. The results have been analyzed for stable and neutral atmospheric conditions of atmosphere for the dispersion of air pollutants in the downwind and the vertical direction. From the figures we reason out that the mesoscale wind aggravates the air pollutants concentration for stable and neutral conditions of atmosphere. Also, the results of this model incur that, as the chemical reaction rate coefficient increases the concentration of primary pollutants decreases whereas concentration of secondary pollutants increases everywhere. We also found that the magnitude of concentration is less in neutral case and more in stable case and the concentration level reaches maximum height in neutral case compared to stable case, pointing that the neutral stability condition enhances the vertical diffusion of pollutants in the atmosphere.

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