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AN IMPACT OF GROWTH HORMONE RELEASING HORMONE (GHRH) ON GROWTH HORMONE (GH) RESPONSE IN WOMEN OF REPRODUCTIVE AGE ASSOCIATED WITH OBESITY– A STOCHASTIC MODEL OF EXPONENTIATED GUMBEL TYPE

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Abstract: The Gumbel distribution is a well accepted statistical model due to its broad applicability. We sought to introduce a generalization version of the standard Exponentiated Gumble (EG) type-2 distribution in life data analysis. A class of univariate distributions (EG) is one of the lifetime or stochastic model with some statistical and reliability properties. In this study, we have adopted the lifetime data ie., the association between GHRH- stimulated GH release in premenopausal women with obesity. The results clearly states that an increased levels of probability density function (pdf) and reliability function of GH secretion in obese premenopausal women after stimulation with GHRH, in other hand the level of hazard function was decreased in corresponds with acquired lifetime data. Further, it has been noticed that the application part is well fitted with the mathematical model and this could be an accepted stochastic model to express the life time data in a reputable manner.

Key words: Gumbel, GH, Exponentiated, Parameters, Hazard function

Mathematical Subject Classification: 92B25, 60H35

1. INTRODUCTION

The Gumbel distribution, it has been known that the type-1 extreme value distribution has significant research attention in the past years particularly, in extreme value analysis of extreme events. Pinheiro and Ferrari, 2015 have developed some important real lifetime applications of the Gumbel distribution [1]. Exponentiating distributions are more attractive method in statistical modelling that offers an efficient way of introducing an additional shape parameter for the standard distribution to achieve flexibility and robustness.

Gupta et al. [2] has introduced an Exponentiated distribution that developed form the standard exponential distribution which formed the cumulative density function to a positive constant power. Nadarajah and Kotz [3], Nadarajah [4] introduced the Exponentiated Fr´echet distribution as a generalization of the standard Fr´echet distribution. The Exponentiated Gumbel (EG) type-2 distribution has been developed from the Gumbel type-2 distribution [5], Exponentiated Fr´echet distribution [6], and the Fr´echet distribution. The EG type-2 distribution has α and ϕ are the shape parameters and θ is the scale parameter [3]. The exponentiated gumbel type- 2 distribution is developed for the uses of modelling data sets that arise from complex phenomena. Hence, in the present study, we were interested to develop a stochastic model using life time data ie., the GH secretion in obese premenopausal women after stimulation with GHRH increased levels.

There are many known cardiometabolic risk factor in the general population. In particular, visceral adipose tissue is strongly associated with cardiometabolic risk factors. Since, it is indistinct whether body fat distributions are also vital in obese women and men. However, GH deficiency in adults is associated with abnormalities in body composition, metabolic derangements, and suboptimal physical performances; these impairments improve with GH replacement therapy [7-9]. Increased cardiovascular mortality is associated with hypopituitarism, women are disproportionally affected, and GH deficiency has been considered a contributory factor. The increased mortality is reflected in elevated cardiovascular risk markers [10-12] and GH replacement results in improvement in these markers [13-15]. The mostly discussed area is GH secretion test in obesity is GHRH-induced GH secretion. It should be very interesting to study its relationship with different cardiovascular risk factors in a homogenous population. Hence, we have chosen the lifetime data to investigate the associations between measures of GH response in women with obesity using EG type -2 distributions. In this study, we have utilized the data from Cordido. F, et al., [16] study subjects like women of reproductive age, since data in this group are particularly lacking as well as this will be an importantly identified risk factors for future cardiovascular disease that leads to death in the general population. Here, we have discussed the published lifetime data from Cordido F, et al., [16] using exponentiated gumbel type-2 distribution for selected parameters with modifications to acquire the clear interpretations on hazard function and reliability functions. This is to express the data in well accepted as well as in understandable appearance.

2. METHODOLOGY

2.1. MATHEMATICAL MODEL

2.1.1. EXPONENTIATED GUMBEL TYPE-2 DISTRIBUTION

The Exponentiated Gumbel (EG) type-2 distribution is one of the new lifetime distributions. Recently, it has been developed many applications of the Gumbel distribution [1]. The Gumbel type-2 distribution is not commonly utilized in statistical modelling and the reason may not be far from its lack of fits in data modelling. In probability theory, the moments of a random variable are one of the popular properties of a distribution. It has been used to derive other essential properties such as mean, variance, skewness, and kurtosis statistics that describes a probability distribution.

The cumulative density function cdf (F(x) of the exponentiated family of distributions according to Nadarajah and Kotz [3] is defined by

$$F\ (x;\omega;\ \alpha)= 1 - (1 - G(x;\ \omega))^{\alpha} \qquad x \in R;\ \alpha > 0;\ \omega \in \Omega;$$

differentiating (1) with respect to x gives the corresponding probability density function pdf (f(x)) as

$$f(x;\omega;\ \alpha)= \alpha g(x;\ \omega)\ (1 - G(x;\omega))^{\alpha-1} \qquad ,x \in R;\ \alpha > 0;\ \omega \in \Omega,$$

where ω and Ω are the vector of parameters and parameter space of the baseline distribution (G(x; ω)), respectively.

2.1.2 Exponentiated Gumbel Type-2 Distribution

Definition

A random variable X is said to follow the Gumbel type-2 distribution if its cumulative density function [9–11], (cdf) G(x) is given by

$$G(x)= e\text{-}\theta\ x^{\phi} \quad ,x > 0;\ \phi,\theta > 0, \qquad ..(1.1)$$

while the corresponding probability density function (pdf) g(x) is given by

$$g\ (x) = \ \phi\theta x^{\phi-1}e^{-\theta x^{\phi}} \qquad ,x\ >\ 0;\ \phi,\theta\ >\ 0$$

$$F\ (x) = \ 1\ -\ \left(1 - \ e^{-\theta x}\right)^{\alpha} \qquad ,x\ >\ 0;\ \alpha,\phi,\theta\ >\ 0, \text{(by using 1.1)}$$

while the corresponding pdf (f(x)) is given by

$$f\ (x)\ =\ \alpha\phi\theta x^{-\phi-1}e^{-\theta x^{\phi}}\left(1 - e^{-\theta x^{-\phi}}\right)^{\alpha-1}\ x\ >\ 0;\ \alpha,\phi,\theta\ >\ 0,$$

where α and ϕ are the shape parameters and θ is the scale parameters

Theorem

If $y = x^{-\phi}$ and X is distributed according to the EG type-2 distribution then, Y is distrib due to Gupta

$$f(x) = \frac{\alpha\phi\theta y^{1/\phi+1}e^{-y\theta}\left[1 - e^{-y\theta}\right]^{\alpha-1}}{\phi y^{1/\phi+1}}$$

Proof:

The tr ied by f(y) = Thus,

$|dx/d$

$$= \alpha\phi\theta y^{1/\phi+1}e^{-y\theta}\left[1 - e^{-y\theta}\right]^{\alpha-1}\ \ y\ >\ 0,\alpha,\theta >\ 0.$$

Reliability Function

The reliability function or the survival function of a random variable X is defined by

$(x) = P(X > x) = 1\text{-}F(x)$. It could be interpreted as the probability of a system not failing before some specified time t, Lee and Wang [7]. The EG type-2 distribution's reliability function is given by

$$R(x) = 1 - (1 - e^{-\theta x})^{\alpha} \quad ,x > 0;\ \alpha,\phi,\theta > 0.$$

Hazard Rate Function

The hazard rate function (h(x)) or the instantaneous failure rate of a random variable X is the probability that a system fails given that it has survived up to time t and is given by

h(x) = f(x)/R(x) [7]. Hence, we define the hazard rate function of the EG type-2 distribution is given by.

$$h(x) = \frac{\alpha\phi\theta x^{-\phi-1} e^{-\theta x^{\phi}} \left(1-e^{-\theta x^{-\phi}}\right)^{\alpha-1}}{1-\left(1-e^{-\theta x}\right)^{\alpha}} x > 0;\ \alpha,\phi,\theta > 0$$

3. RESULTS

3.1. APPLICATION

3.1.1. Background

To establish an EG type-2 distribution, the life time data was adopted from the published work of Cordido F, et al., 2010 [16]. They have performed the study with 48 premenopausal obese patients at a BMI of 36.0±6.4 kg/m2, aged 35–52 years to assess the association between GHRH-induced GH secretion in obese premenopausal women and cardiovascular risk markers or insulin resistance [16]. Please refer Cordido F, et al. 2010 for further clarification on experimental and study design. In the present study, we have acquired the following data (shown in Figure 1) to develop a stochastic model.

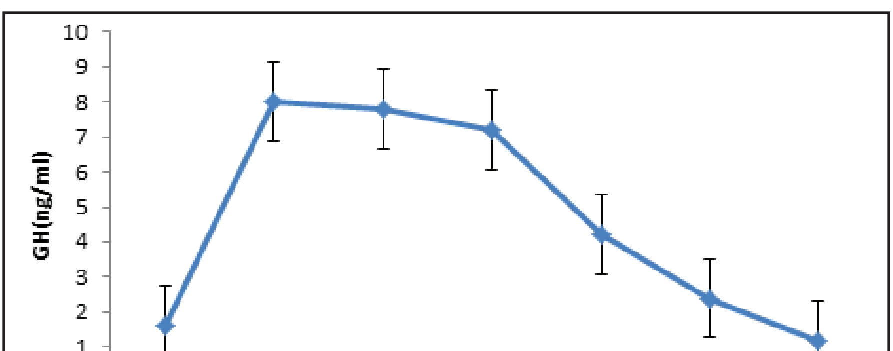

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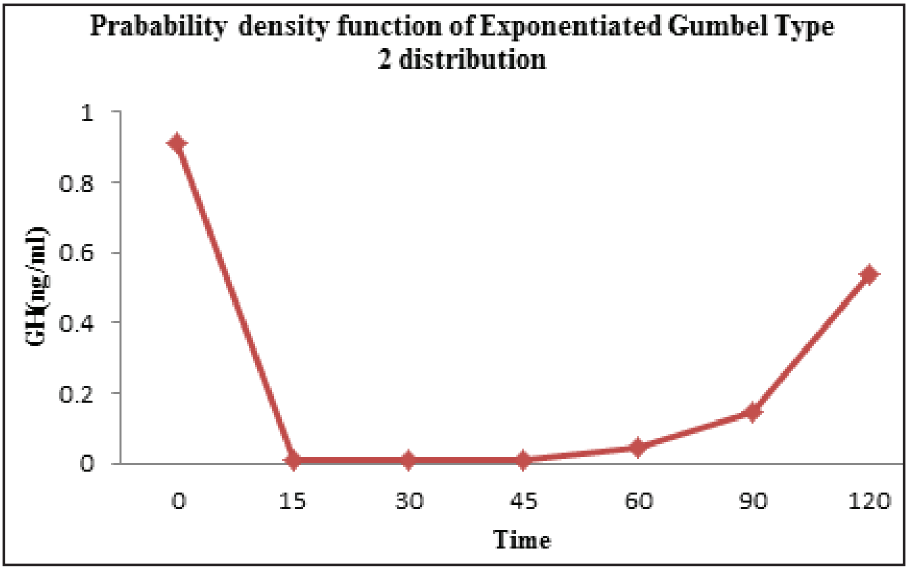
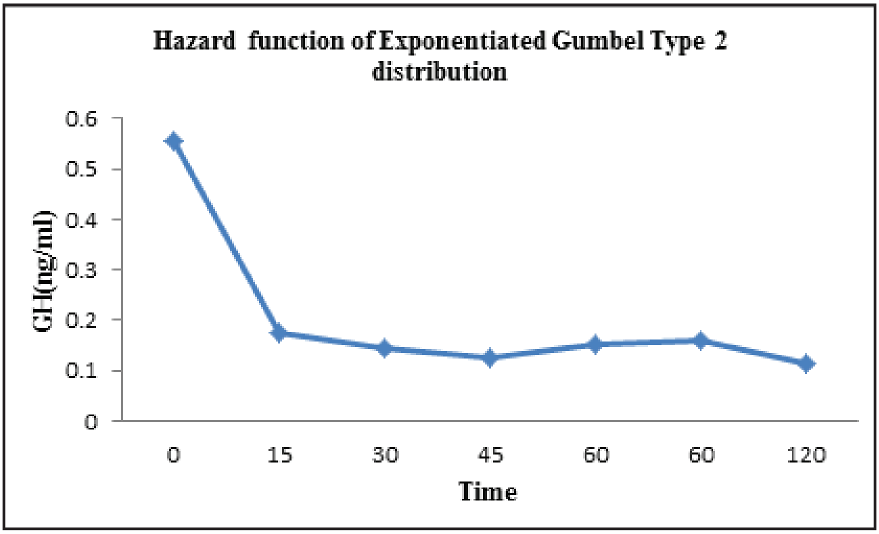
Figure 1: shows the level of GH secretion after stimulation with GHRH at the different time points in obese premenopausal women.

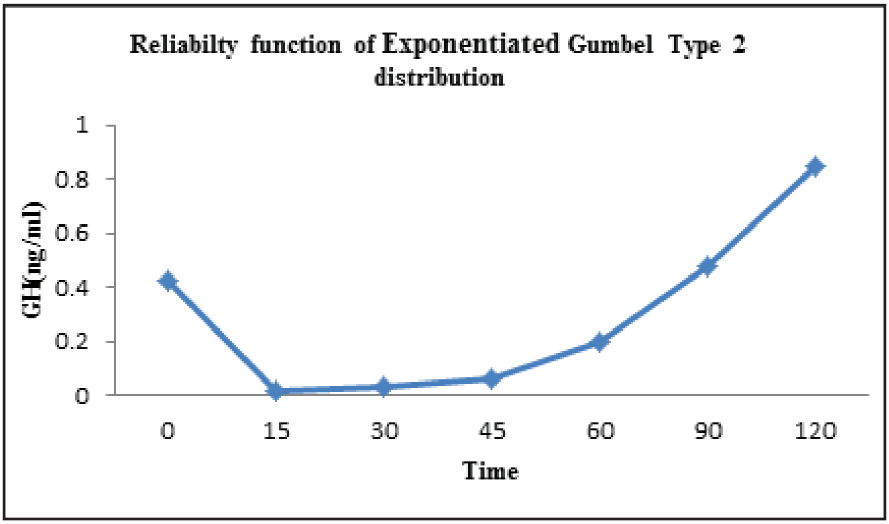
4. MATHEMATICAL RESULTS



Mathematical Fig. I: Shows the Exponentiated Gumbel type-2 probability density function (pdf) on the levels of GH secretion in obese premenopausal women after stimulation with GHRH



Mathematical Fig. II: Depicts the Exponentiated Gumbel type-2 hazard function (h(x)) on the levels of GH secretion in obese premenopausal women after stimulation with GHRH



Mathematical Fig. III: Illustrates the Exponentiated Gumbel type-2 hazard function (r(x)) on the levels of GH secretion in obese premenopausal women after stimulation with GHRH

5. DISCUSSION:

Statistical distributions are incredibly helpful in recognizing and predicting the real global phenomena. One among them is the extreme value distribution. It has been expansively used to model lifetime data and modelling natural phenomena [17]. In this manuscript, we employed the Exponentiated Gumbel type 2 distributions to analyze the levels of GH secretion in obese premenopausal women after stimulation with GHRH at the different times. The results showed that the probability density function plots in mathematical figure (I) represents an increased levels of GH secretion in obese premenopausal women after stimulation with GHRH at the different times as follows in minutes 0,15,30,45,60,90,120. In contrast, the hazard rate function of Exponentiated Gumbel type 2 distribution h(x) plots as in mathematical figure (II) showed the decreased levels of GH secretion in obese premenopausal women after stimulation with GHRH at the different times as follows in minutes 0, 15, 30, 45, 60, 90, 120. Interestingly, the reliability function of the Exponentiated Gumbel type 2 distribution r(x) plots [mathematical figure (III)] shows increased levels of GH secretion in obese premenopausal women after stimulation with GHRH at the different times as follows in minutes 0, 15, 30, 45, 60, 90, 120; suggesting that the degree to which the result of a measurement, calculation, or specification are depended on to be accurate and consistently well.

6. CONCLUSON:

In conclusion, we have explicated the mathematical expressions for some of its essential statistical properties such as the probability density function, the reliability and hazard rate functions. The obtained results reveals that levels of GH secretion in obese premenopausal women after stimulation with GHRH at the different times were increased in probability density function (In mathematical fig.I) as well as the reliability function (In mathematical fig.III) and decreased levels were observed in hazard function (In mathematical fig.II), which gives the better understanding of scientific lifetime data. This would be a novel approach to express the life time data in improved statistical distributions.

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