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A Mixed Model Reduction Method for the Discrete Time High-Order Systems

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Abstract : There are several methods available for the reduction of high order linear continuous time systems. But few methods are available for the reduction of high order discrete time systems. In this paper, a new mixed procedure to reduce high order discrete systems is proposed. The proposed method of model order reduction is based on generating the reduced order denominator by the Modified Factor Division Method [2] and numerator using Simplified Routh Approximation Method [1]. The method is illustrated through typical numerical example.

1. INTRODUCTION

The low order models are significantly used in the control system stability analysis and controller design.. Many methods available for the reduction of high order continuous time systems but only few are available for the reduction of discrete time systems. The familiar methods available in literature for the reduction of high order discrete time systems are viz., Stability Equation method of R. Prasad et.al [8]., Biased Continued Fraction Expansion Method suggested by Hwang et.al.[10] and Routh Approximation Method given by Farsi et.al.[4] Routh approximation method suggested by Warwick et.al [4], Stability Equation method of Chen et.al [5], Biased continued fraction method proposed by Chee-Fai Young et.al [7] etc. This paper is to proposes new method to reduce high order discrete systems which overcomes limitations with Factor Division Algorithm [3]. The proposed method is a mixed method which derives its denominator using Factor Division Method [2] and Numerator using SRAM [1]. This method gives the same results with much simpler computations than the Factor Division Algorithm [3]. The proposed method guarantees the stability of the high order discrete time system in the reduced order models. The proposed method is digital computer oriented. The effectiveness and computational simplicity of the proposed method is illustrated through a typical numerical example

2. THE PROPOSED METHOD

Consider original discrete time system defined by

$$\mathrm{G}(z) = \frac{\mathrm{B}_0 + \mathrm{B}_1 z + \ldots\ldots + \mathrm{B}_{n-1} z^{n-1}}{\mathrm{A}_0 + \mathrm{A}_1 z + \ldots\ldots + \mathrm{A}_n z^n}$$

By applying the bilinear transformation $z = p + 1$, the system is transformed into p-domain. The transfer function in p-domain is defined as:

$$\mathrm{G}(p) = \frac{\mathrm{B}_0 + \mathrm{B}_1 p + \ldots\ldots + \mathrm{B}_{n-1} p^{n-1}}{\mathrm{A}_0 + \mathrm{A}_1 p + \ldots\ldots + \mathrm{A}_n p^n} \tag{1}$$

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The k^{th} order reduced model is to be found such that

$$G_k(p) = \frac{\bar{N}(p)}{\bar{D}(p)}$$

Denominator $\overline{\mathbf{D}}(p)$

$\overline{D}(p)$ is the reduced stable denominator which may be found by any of the many techniques available. Here in this work, Modified Factor Division [2] is used to obtain the reduced order denominator $\overline{D}(p)$.

Let the original denominator be given as:

$$\overline{D} = a_n p^n + a_{n-1} p^{n-1} + \ldots\ldots + a_0$$

Then the reduced order denominator is given by Q(p),

Where $$Q(p) = \frac{K(p)}{p+h}$$; $p + h$ is the unwanted pole factor.

Determination of '*h*'

The selection of 'w' is largely arbitrary, but for more reliable results, an approximation is given as $\alpha \le h \le \beta$.

Where $$\alpha = \frac{a_{n-1}}{n.a_n}$$

$$\beta = \begin{cases} \dfrac{a_{n-1} + \sqrt{a^2_{n-1} - 4a_n a_{n-2}}}{2a_n} \\ \dfrac{a_{n-1}}{2a_n} \end{cases}$$

A better approximation is obtained by taking

$$h = \frac{1}{2}(\alpha + \beta)$$

Determination of K(P)

Let $$AD(p) - BD(-p) = 0$$

$$B = \frac{AD(p)}{D(-p)}$$

Then denominator of eq.(1) *i.e.*, K(p) is given as :

$$K(p) = a_n p^n \{A - (-1)^n B\} + a_{n-1} p^{n-1} \{A - (-1)^{n-1} B\} + \ldots + a_0(A - B)$$

Where, $$A = D(h) \; ; \; B = D(-h)$$

By evaluating the eq(1) iteratively the required reduced order denominator is obtained, *i.e.*, the reduced order denominator of k^{th} order is obtained by successively applying the above procedure (k–1) times.

Determination of numerator $\overline{\mathbf{N}}(p)$

The reduced numerator $\overline{N}(p)$ is obtained by using SRAM [1].

Let the k^{th} reduced order transfer function in p-domain be of the form

$$G_k(p) = \frac{N_{k(t,m)}(p)}{\bar{D}(p)} = \frac{b_0 + b_1 p + \ldots\ldots + b_{k-1} p^{k-1}}{a_0 + a_1 p + \ldots\ldots + a_k p^k}$$

Where $\overline{D}(p)$ is the reduced order denominator obtained by Factor Division Method [2].

The reduced order numerator $N_{k(t,m)}(p) = N_{kt}(p) + N_{km}(p)$; $k = t + m$

Where t = No. time moments to be retained

m = No. of Markov parameters to be retained

Then
$$N_{kt}(p) = T_1 + T_2 + \ldots\ldots + T_t\, p^{k-m+1}$$
$$N_{km}(p) = M_m p^{k-m} + \ldots\ldots + M_2\, p^{k-2} + M_1 p^{k-1}$$

Where
$$T_1 = \frac{a_0}{A_0}.B_0$$
$$T_2 = \frac{a_0}{A_0}.B_1$$
$$\vdots$$
$$\vdots$$
$$T_t = \frac{a_0}{A_0}.B_{t-1} \tag{3}$$

And
$$M_1 = \frac{1}{A_n}(B_{n-1}a_k)$$
$$M_m = \frac{1}{A_n}\left(\sum_{i=1}^{m} B_{n-i}a_{k-(m-i)} - \sum_{j=0}^{m-1} M_j A_{n-(m-j)}\right) \text{ with } M_0 = 0 \tag{4}$$

By applying the inverse bilinear transformation $p = z - 1$, the reduced model in z-domain is obtained.

This proposed method doesn't require the computation of α and β tables as in the Factor division method [3] and relatively very few steps of evaluation are involved which give exactly the same results as in the factor division method. This shows the simplicity of the proposed method over the former method.

3. EXAMPLE

Consider the system given by:

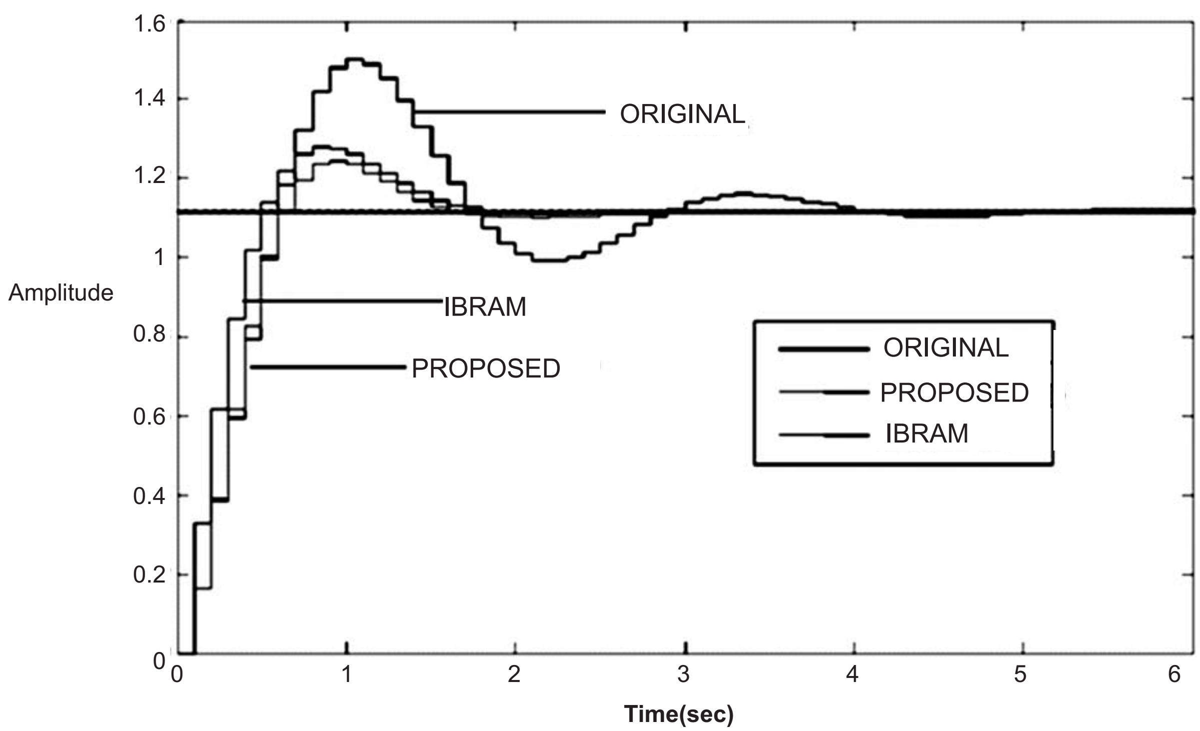

Figure 1

$$G(z) = \frac{0.1625z^7 + 0.125z^6 - 0.0025z^5 - 0.00525z^4 + 0.00262z^3 - 0.0000875z^2 + 0.003z - 0.000412}{z^8 + 0.6208z^7 - 0.416z^6 + 0.07613z^5 - 0.05915z^4 + 0.1906z^3 + 0.09736z^2 - 0.01635z + 0.002226}$$

Reduced order transfer function of 2nd order :

$$R_2(z) = \frac{0.1625\, z - 0.02353}{z^2 - 1.51\, z + 0.6344} \quad \text{(Proposed method)}$$

Reduced order model by IBRAM [11]:

$$R^*_2(z) = \frac{0.4094\, z - 0.2947}{1.236\, z^2 - 1.948\, z + 0.8158} \quad \text{(IBRAM)}$$

Fig 1 shows comparison of the step responses of the original system G(z) and its reduced 2nd order models obtained by the proposed method and IBRAM [11].

4. CONCLUSION

A new mixed procedure to reduce high order discrete systems is proposed.. The proposed method is a mixed method using Factor Division Method and SRAM. The proposed method is computationally simple. The effectiveness and computational simplicity of the proposed method is illustrated through typical numerical example available in literature.

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