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A MULTIPLE OBJECTIVE MODEL FOR NURSE SCHEDULING

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ABSTRACT

Nurse-scheduling is one of the most important phenomena from the hospital-management point of view. Here a computerized nurse-scheduling model is developed. The model is an alternative to the current manual-made schedules which is an intricate and time-consuming phenomena. Even then it leaves the users (nurses) in a dissatisfying state. The developed model accounts both for hospital-objectives and nurses' preferences. Hospital objectives includes ensuring a continuous service with appropriate skills and the staff-requirement avoiding additional costs for unnecessary overtime. Nurses-preferences include unbiased consideration, avoiding consecutive shifts, one shift in a day and more day-shifts than night-shifts. The paper presents an application of 0-1 goal programming for nurse-scheduling with multiple conflicting objectives of a study for one week distribution. The results of the study indicate that this model can be used in hospitals in today's competing decision-environment.

Keywords: *Nurse scheduling, 0-1 goal programming, Hospital objectives, Appropriate staffing size, Allocation decisions.*

INTORDUCTION

The present work is an attempt to develop a systematic procedure for allocating nurses to work shifts and workdays in a way so as to ensure a continuous and appropriate service of patient care in such a way that specific work requirements are satisfied while using minimum staffing to avoid wasted manpower. Here some goals are also under consideration such as employee requests versus the need to balance workload, work pattern, work stretch and personal requests for vacations. Nurse scheduling is difficult and time consuming task. The schedule should determine the day to day shift assignments of each nurse for a specified horizon of time in such a way that satisfies the given requirements. The scheduling should also be under fair distribution to everyone and to fulfill various goals.

The system will also rely on fairness bases among nurses and will consider nurses preferences to maximize their satisfaction. This will help them to provide a

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proper quality of service. Here a number of priority levels are considered in developing the scheduling system.

Modeling nurse scheduling is not a new idea. Until 1960s, scheduling tools considered of graphical devices such as Gantt chart. Scheduling systems began to be based on heuristic models in hospital units such as Smith (1979) and Isken and Hancock (1991). Ozkarahan (1991) has described nurse scheduling based on linear programming for health system. Other optimization techniques have been used in nurse scheduling for the non-cyclical type which include assignment problem of jobs described by Caron and Hansen (1999). Tucker and Smith *et al.* (1999) have described 12hours-shift systems for a health care unit, which is also helpful to retain some instructions for shifts schedule. Hameier (1991) has used linear programming concept to get optimal nurse scheduling and Huarng (1999) has described shift rotation scheduling with the help of goal programming.

To develop the mathematical model, first nursing policies were decided. These policies were based mainly on current hospital practices. In fact, human beings has various physical limitations and the lack of ergonomic consideration causes frustration and reduces productivity quantitatively and qualitatively.

The problem consists of scheduling a number of nurses for a single week period in a particular unit while satisfying minimum staffing requirements alongwith other constraints. The nurse-scheduling model will also attempt to satisfy several goals including reducing overstaffing and hence overtime costs as well as incorporating nurses' preferences and establishing fairness bases among nurses.

MATHEMATICAL MODEL

First, current nursing practices prevailing in the hospitals were investigated. A hospital has several departments like medicine, surgery, pediatric etc. Most of the hospitals have specialized nursing-staff in each department and they work only in that department, while in some hospitals nursing-staff is interchanged among the various departments. Each department (or unit) is covered by the nursing-staff in three shifts. First shift starts at 6 A.M. and ends at 2.00 P.M., second shift form 2.00 P.M. to 10 P.M. and third shift (also known as night shift) from 10.00 P.M. to 6.00 A.M. Herein, weekly schedules for a unit are considered and each nurse is required to work at the most 6 days a week, but not less than 5 days a week (proportionality factor). It ensures each nurse at least a day-off in each week. Discussion with the nurses also demonstrate that generally the nurses prefer that number of night-shifts should be less than total number of day-shifts (satisfaction factor). Further, no nurse is allowed to work in consecutive shifts (healthiness factor). Requirement of nurses in each shift and in each day is to be met (optimality and completeness factor).

VARIABLES

The following variables have been used in the model

n = number of days in a schedule

m = number of nurses available for the unit of interest

i = index for days, i = 1, 2, 3 ————, n

k = index for nurses, k = 1, 2, 3 ————, m

$$X_{ik} = \begin{cases} 1, & \text{if nurse k is assigned first shift for day i} \\ 0, & \text{Otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1, & \text{if nurse k is assigned first shift for day i} \\ 0, & \text{Otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1, & \text{if nurse k is assigned first shift for day i} \\ 0, & \text{Otherwise} \end{cases}$$

The parameters used in the goal programming model are defined as:

P_i : represents the staff requirement for first shift of day i, i = 1, 2, 3 ———, n.

Q_i : represents the staff requirement for second shift of day i, i = 1, 2, 3 —, n.

R_i : represents the staff requirement for third shift of day i, i = 1, 2, 3 ———, n.

GOALS AND PRIORITIES

In order to incorporate the soft constraints in the scheduling model, we will include the goals, which are consistent respectively with the above constraints. There are three goals which have important weights to each goal reflecting the relative importance of that goal compared to the others. Penalty levels for violating the corresponding goals express these importance weights. These levels are described as:

P_1: to minimize the deviations between the sum of actual days on and the maximum required days on.

P_2: to avoid assigning a nurse to work night shift and the first shift of the following day.

P_3: keep number of more day-duties in comparison to night duties.

CONSTRAINTS

Hard Constraints

1. Daily staff requirements for the three shifts was described through three constraints as

$$\sum_{k=1}^{m} X_{ik} \geq P_i \; ; \; i = 1, 2, ..., n \tag{1}$$

$$\sum_{k=1}^{m} Y_{ik} \geq Q_i \; ; \; i = 1, 2, ..., n \tag{2}$$

$$\sum_{k=1}^{m} Z_{ik} \geq R_i \; ; \; i = 1, 2, ..., n \tag{3}$$

2. It was decided to assign at the most on shift per day.

$$X_{ik} + Y_{ik} + Z_{ik} = 1 \text{ for all } i = 1, \text{—}, n \text{ and } k = 1, \text{—}, m \tag{4}$$

Soft Constraints

To incorporate the soft constraints in the schedule, following goals were identified which are consistent with the above hard constraints.

Goal 1: The following constraints represent maximum six working days per week schedule so that the nursing-staff gets at least a day-off in a week.

$$\sum_{i=1}^{n} (X_{ik} + Y_{ik} + Z_{ik}) + (d_{1k}^{-} - d_{1k}^{+}) \leq 6; \; k = 1, ..., m \tag{5}$$

Goal 2: It avoids assigning a nurse to work a third shift in a day and the first shift of the following day so that a nurse does not get two consecutive shifts in a day.

$$Z_{ik} + X_{(i+1)k} + (d_{2ik}^{-} - d_{2ik}^{+}) = 1; \; i = 1, ..., (n-1) \; \& \; k = 1, ...m \tag{6}$$

Goal 3: It attempts to have in schedule more day shifts (first and second shifts are considered as day shifts) than night shifts (third shift is taken as night shift).

$$\sum_{i=1}^{n} (X_{ik} + Y_{ik} - Z_{ik}) + (d_{3k}^{-} - d_{3k}^{+}) = 1; \quad k = 1, ..., m \tag{7}$$

OBJECTIVE FUNCTION

Importance weights are assigned to each goal reflecting the relative importance of each goal compared to others. It was determined on the basis of discussion with nursing-staff and described by priorities. The expression for the O.F. is given by

$$\text{Min } z = P_1 \sum_{k=1}^{m} d_{1k}^{+} + P_2 \left[\sum_{i=1}^{n-1} \sum_{k=1}^{m} d_{2ik}^{+} \right] + P_3 \sum_{k=1}^{m} d_{3k}^{+} \tag{8}$$

MODEL-IMPLEMENTATION

The linear goal-programming model consists of minimizing the above objective function under hard constraints and soft constraints . The model has a total (5mn + 2m) decision variables (3mn fixed variables and 2mn + 2m deviation variables). The total number of model-constraints are 2mn + m + 3n [(mn + 3n) are hard constraints and m (n + 1) goal-constraints]. For the purpose of implementation of model, a unit (medicine unit) of the hospital is selected. In this unit, there are in total nine nurses. Table 1 shows the requirement of nurses in each shift for a week. Head nurse prepares the schedule manually. It is a tedious and time-consuming job. It is the source of grievances by the duty-staff. The problem where nine nurses are to be scheduled for a week consists of 333 decision variables and 156 constraints in total. The size of the problem in the current application is computationally large from the PC- point of view and therefore some heuristic must be used for a satisfactory solution on PC.

Sub Grouping

Huarng [1997, 1999] proposes the approach of subgrouping by splitting nurses and workloads into several subgroups so that each subgroup will be of manageable size. The subgroups should be selected in such a way not to violate the hard constraints. The duration of single week for scheduling the nurses has been chosen by this heuristic only. However, there is no systematic way for subgrouping and the approach is model-dependent. Larger the number of subgroups, smaller is the computational time (CPU time) and computer-storage. It can be considered main advantage of larger number of subgroups. But as the number of subgroups increase, the deviation from optimality also tends to increase.

CASE-STUDY

There are nine nurses in the unit in total for which the schedule is to be prepared. The problem was solved using a computer-program in FORTRAN 95 version by linear goal programming technique. The problem could not be tackled as a single problem due to enormous computational storage. Therefore, It was decided to distribute the entire problem into two subgroups. After careful considerations regarding the constraints and computer storage, the 5-4 size of subgroups was finally decided. The reason is obvious, since size of subgroup upto five is more suitable in comparison to subgroups of sizes more than five. Therefore, the subgrouping 5-4 is most appropriate.

RESULTS AND DISCUSSION

The total staff of nine nurses was divided into two subgroups consisting of five and four nurses respectively. The requirement of nurses in the first, second and third shifts of each day was 3,2 and 2 respectively as decided by the incharge. The requirement of nurses in first shift as decided by the incharge was greater than second and third shifts, since first shift has additional activities like visit by doctors and preparing medicine-schedule for the whole day. Therefore values of P_i, Q_i and R_i are taken as 3,2,2 $\forall$ i = 1,2,3, -----,7. Since total nurses was divided into two subgroups, so values of P_i, Q_i and R_i are also to be distributed into two subgroups. First, subgroup of five nurses was considered and values of P_i, Q_i and R_i for this subgroup were taken as 2,1,1 respectively. This subproblem consisted of 136 constraints and 105 variables. Subproblem was executed and the results are reported in table 2. Next subgroup of four nurses was considered and values of P_i, Q_i and R_i are taken as 1 each $\forall$ i = 1,2,3,——,7. This subproblem consists of 89 constraints and 84 variables. This subproblem was also executed and results are reported in table 3. Finally, both the results were combined and final schedule is exhibited in table 4. This table demonstrated certain notable features. First of all, requirement of nurses in each shift is fulfilled (hard constraint 1) and each nurse has at the most one duty per day (hard constraint 2). All the soft (goal) constraints are also satisfied and all the priorities are completely achieved. Each nurse get at least a day-off (first goal). No nurse is assigned two consecutive shifts (second goal) and number of day-shifts is always greater than number of night-shifts for each nurse. The computerized schedule was shown to the incharge and the individual nurses. It was found satisfactory to all. In the next three weeks of the month, the components (nurses) in the subgroups may be changed and the schedule for each week be prepared. It will provide more unbiased attitude to the scheduling and finally monthly-schedule of the nurses may be prepared.

Table 1

	Number of nurses	*$\forall i$ = 1, 2, 3, -----, 7*		
		P_i	Q_i	R_i
Subgroup 1(A-E)	5	2	1	1
Subgroup 2(F-I)	4	1	1	1
Total	9	3	2	2

Table 2

	1			2			3			4			5			6			7			Total
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	Total D+N =
A	–	y	–	y	–	–	–	–	y	–	y	–	–	–	y	–	–	–	–	–	–	3+2=5
B	–	–	y	–	y	–	–	y	–	y	–	–	–	–	–	y	–	–	–	y	–	5+1=6
C	y	–	–	y	–	–	y	–	–	–	–	–	y	–	–	–	–	y	–	–	y	4+2=6
D	y	–	–	–	–	y	–	–	–	y	–	–	y	–	–	–	y	–	y	–	–	5+1=6
E	–	–	–	–	–	–	y	–	–	–	–	y	–	y	–	y	–	–	y	–	–	4+1=5
Total	2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	21+7=28

Table 3

	1			2			3			4			5			6			7			Total
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	Total D+N =
F	–	–	–	y	–	–	–	–	y	–	–	–	–	–	y	–	y	–	–	y	–	3+2=5
G	y	–	–	–	–	y	–	y	–	–	y	–	–	–	–	–	–	y	–	–	–	3+2=5
H	–	y	–	–	y	–	y	–	–	–	–	y	–	y	–	–	–	–	–	–	y	4+2=6
I	–	–	y	–	–	–	–	–	–	y	–	–	y	–	–	y	–	–	y	–	–	4+1=5
Total	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	14+7=21

Table 3

	1			2			3			4			5			6			7			Total
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	Total D+N =
A	–	y	–	y	–	–	–	–	y	–	y	–	–	–	y	–	–	–	–	–	–	3+2=5
B	–	–	y	–	y	–	–	y	–	y	–	–	–	–	–	y	–	–	–	y	–	5+1=6
C	y	–	–	y	–	–	y	–	–	–	–	–	y	–	–	–	–	y	–	–	y	4+2=6
D	y	–	–	–	–	y	–	–	–	y	–	–	y	–	–	–	y	–	y	–	–	5+1=6
E	–	–	–	–	–	–	y	–	–	–	–	y	–	y	–	y	–	–	y	–	–	4+1=5
F	–	–	–	y	–	–	–	–	y	–	–	–	–	–	y	–	y	–	–	y	–	3+2=5
G	y	–	–	–	–	y	–	y	–	–	y	–	–	–	–	–	–	y	–	–	–	3+2=5
H	–	y	–	–	y	–	y	–	–	–	–	y	–	y	–	–	–	–	–	–	y	4+2=6
I	–	–	y	–	–	–	–	–	–	y	–	–	y	–	–	y	–	–	y	–	–	4+1=5
Total	3	2	2	3	2	2	3	2	2	3	2	2	3	2	2	3	2	2	3	2	2	35+14=49

CONCLUSION

In the present study, a 0-1 linear goal-programming model is developed for nurse-scheduling in a hospital. Currently, the incharge (head nurse) prepares a manual schedule through a trial-and-error approach. This approach is not only tedious and time-consuming, but also inefficient and dissatisfying to various nurses. Moreover these manual schedules do not satisfy a number of important criteria for efficient scheduling. These include balanced schedules, fairness considerations and nurses preferences in addition to staffing requirements. The developed model provides important improvements in this regard besides the fact that it offers a practical computerized tool. Satisfying all the criteria simultaneously is not possible, so constraints are classified into two categories-first hard constraints which have to be satisfied compulsorily and soft constraints which must be satisfied according to their priorities.

For measuring a schedule quality, Oldenkamp and Simon (1995) specify the following four factors:

1. Optimality: represents the degree in which nursing expertise is distributed over the different shifts.
2. Completeness: represents the degree in which the quantitative demands per shift are met.
3. Proportionality: represents the degree in which each nurse has been approximately given same number of duties.
4. Healthiness: represents the degree in which it has been taken care of that no nurse should be assigned consecutive duties and she should be assigned more day-duties than night duties.

The developed model performs quite well based on the above criteria. The model has been found not only to satisfy hospital's objectives, but also to large extent nurses grievances and avoids the biasedness factor often imposed on the incharge. The solution has been obtained through subgrouping due to excessive computational time and computer storage. Subgroup sizes upto 5 are found most appropriate. For the next week schedules, the distribution of subgroups may be changed to give schedule more logical basis. Keeping in view of all these factors, monthly schedule of the nurses of different units of the hospital may be efficiently and computationally prepared.

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