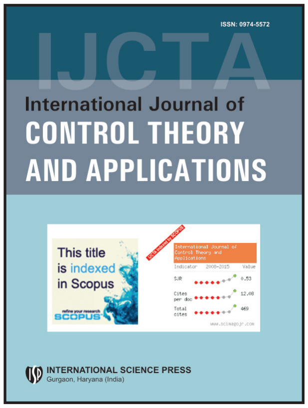


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# NSM PSpice Solutions of Free Convective Flow from Non-Isothermal Vertical Cone with MHD Effects

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***Abstract:*** A study of effects of unsteady incompressible flow past a vertical cone with variable surface temperature $T'_w(x) = T'_\infty + ax^n$ varying as a power function of distance from the apex ($x = 0$) and magnetic field applied normal to the surface. The dimensionless coupled partial differential boundary layer equations are solved numerically using the Network Simulation Method (NSM), a robust numerical technique which demonstrates high efficiency and accuracy. The present results are compared with available results and are found to be in good agreement. The velocity and temperature fields have been studied for various combinations of physical parameters (Prandtl number Pr, Exponent in power law variation in surface temperature $n$ and magnetic field parameter M) which are presented and analyzed graphically.

***Keywords:*** Free convection, MHD flow, Network simulation method, Unsteady, Variable surface temperature.

## 1. INTRODUCTION

Natural convection is induced by the result of temperature difference which causes a buoyancy force when a heated surface is in contact with fluid. Natural convection flows under the influence of the gravitational force which frequently occurs in nature as well as in science and engineering application and hence they have been investigated extensively.

From a technological point of view, the study of convection heat transfer from a cone has a wide range of practical applications. Heat transfer problems mostly deal with the design of spacecraft, nuclear reactors, solar power collectors, steam generators, power transformers, etc. From 1953, many investigations (Merk and Prins [9-10], Hering and Grosh [7], Hering [6], Hossain and Paul [8].) have developed similarity/nonsimilarity solutions for axisymmetrical problems for natural convection flows over a vertical cone in steady state. Bapuji et. al., [1-3] numerically studied the transient natural convection from a vertical cone with isothermal, non-isothermal surface temperature or non-uniform surface heat flux using an implicit finite-difference method.

The MHD flow and heat transfer situation occurs in many geothermal, geophysical, technological and engineering applications such as nuclear reactors, migration of moisture through air contained in fibrous insulation, grain storage, nuclear waste disposal, dispersion of chemical pollutants through water, saturated soil and others and hence is of considerable interest. Geothermal gases are electrically conducting and are affected by the presence of magnetic field. Vajravelu and Nayfeh [11] studied hydro magnetic convection from a cone and a wedge with variable surface temperature and internal heat generation or absorption. Our aim is to solve unsteady laminar free convection flow past a non-isothermal vertical cone in the presence of magnetic field using NSM.

In NSM, Ordinary differential equations are generated from Partial differential equations which define the mathematical model of the physical process and that ordinary differential equation is the basis to create electrical network model. Here the network model is composed of very few electrical devices connected in series, in which the boundary conditions are added to form the whole model. Hence, NSM is based on the classical thermoelectric analogy between thermal and electrical variables. NSM incorporating the transient solver PSPICE is used. In Beg et. al., [4], Beg et. al., [5], Zueco, et. al., [12-13], Zueco [14], Zueco[15], Zueco[16]. NSM technique is used to solve the finite-difference differential equations resulting from dimensionless continuity, momentum balance and energy equation.

## 2. MATHEMATICAL ANALYSIS

A problem of an axisymmetrical, unsteady, laminar free convection flow past a vertical cone of a viscous incompressible electrically conducting fluid with variable surface temperature under the influence of transversely applied magnetic field is formulated mathematically in this section. The following assumptions are made for the magnetic field: (i) the magnetic field is constant and is applied in a direction perpendicular to the cone surface; (ii) the magnetic Reynolds number is small so that the induced magnetic field is neglected and, therefore, does not distort the magnetic field; (iii) the coefficient of electrical conductivity is a constant throughout fluid; (iv) the joule heating of the fluid (magnetic dissipation) and viscous dissipation are neglected; (v) the hall effect of magneto hydrodynamics is avoided; (vi) the system is considered as axisymmetrical; and (vii) the effect of pressure gradient is assumed negligible.

The coordinate system is chosen (as shown in Figure 1) such that x measures the distance along the surface of the cone from the apex ($x = 0$) and $y$ measures the distance normally outward. Here $\phi$ is the semi-vertical angle of the cone and $r(x)$ is the local radius of the cone. Initially ($t' \leq 0$), it is also assumed that the cone surface and the surrounding fluid, which is at rest, have the same temperature $T'_{\infty}$. Then, at time $t' > 0$, the temperature of the cone surface is suddenly raised to $T'_w(x) = T'_{\infty} + ax^n$ and is maintained at this value, where a is a positive constant and n is the exponent in power law variation in surface temperature. The governing boundary layer equations of continuity, momentum, and energy under Boussines approximation are as follows:

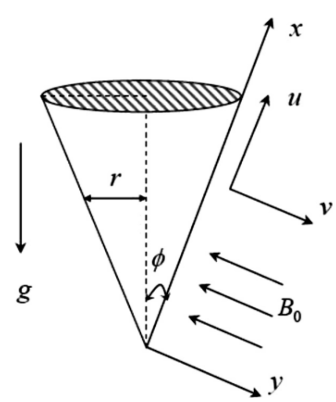


**Figure 1: Physical Model and co-ordinate system**

Equation of continuity:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

Equation of momentum:

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(\mathrm{T}' - \mathrm{T}'_\infty)\cos\phi + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mathrm{B}_o^2 u}{\rho} \tag{2}$$

Equation of energy:

$$\frac{\partial \mathrm{T}'}{\partial t'} + u\frac{\partial \mathrm{T}'}{\partial x} + v\frac{\partial \mathrm{T}'}{\partial y} = \alpha\frac{\partial^2 \mathrm{T}'}{\partial y^2} \tag{3}$$

where, $u$ and $v$ are the velocity components along the $x$- and $y$-axis, $g$ is the acceleration due to gravity, $\alpha$ is the thermal diffusivity, $\beta$ is the coefficient of thermal expansion, $\mathrm{B}_o$ is the magnetic field strength, $\rho$ is the density, and $\sigma$ is the electrical conductivity of the fluid.

The initial and boundary conditions are

$$\begin{aligned} &i' \le 0;\ u = 0,\ v = 0,\ \mathrm{T}' = \mathrm{T}'_\infty\ \forall x \text{ and } y \\ &i' > 0;\ u = 0,\ v = 0,\ \mathrm{T}'_w(x) = \mathrm{T}'_\infty + ax^n \text{ at } y = 0 \forall x \text{ and } y \\ &u = 0,\ \mathrm{T}' = \mathrm{T}'_\infty \text{ at } x = 0 \\ &u \to 0,\ \mathrm{T}' \to \mathrm{T}'_\infty \text{ at } y \to \infty \end{aligned} \tag{4}$$

Further, we introduce the following non dimensional variables:

$$\mathrm{X} = \frac{x}{\mathrm{L}},\ \mathrm{Y} = \frac{y}{\mathrm{L}}\mathrm{Gr}^{\frac{1}{4}},\ \mathrm{U} = u\left(\frac{\mathrm{L}}{\nu}\mathrm{Gr}_\mathrm{L}^{-\frac{1}{2}}\right)$$

$$\mathrm{V} = v\left(\frac{\mathrm{L}}{\nu}\mathrm{Gr}_\mathrm{L}^{-\frac{1}{4}}\right),\ t = t'\left(\frac{\nu}{\mathrm{L}^2}\mathrm{Gr}_\mathrm{L}^{\frac{1}{2}}\right) \tag{4}$$

$$\mathrm{T} = \frac{\mathrm{T}' - \mathrm{T}'_\infty}{\mathrm{T}'_w(\mathrm{L}) - \mathrm{T}'_\infty},\ \mathrm{R} = \frac{r}{\mathrm{L}},\ \mathrm{M} = \frac{\sigma \mathrm{B}_0^2 \mathrm{L}^2}{\mu}\mathrm{Gr}_\mathrm{L}^{-\frac{1}{2}}$$

where, M is the magnetic field parameter, L is the length of slant height, $\mu$ is the dynamic viscosity, $\nu$ is the kinematic viscosity, $\mathrm{Gr_L} = g\beta(\mathrm{T}'_w(\mathrm{L}) - \mathrm{T}'_\infty)\mathrm{L}^3 \cos\phi/\nu^2$ is the Grashof number based on L, and $r = x \sin\phi$.

Equations (1)-(3) can then be written in the following non dimensional form:

$$\frac{\partial(\mathrm{RU})}{\partial \mathrm{X}} + \frac{\partial(\mathrm{RV})}{\partial \mathrm{Y}} = 0 \tag{5}$$

$$\frac{\partial \mathrm{U}}{\partial t} + \mathrm{U}\frac{\partial \mathrm{U}}{\partial \mathrm{X}} + \mathrm{V}\frac{\partial \mathrm{U}}{\partial \mathrm{Y}} = \mathrm{T} + \frac{\partial^2 \mathrm{U}}{\partial \mathrm{Y}^2} - \mathrm{MU} \tag{7}$$

$$\frac{\partial \mathrm{T}}{\partial t} + \mathrm{U}\frac{\partial \mathrm{T}}{\partial \mathrm{X}} + \mathrm{V}\frac{\partial \mathrm{T}}{\partial \mathrm{Y}} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{Y}^2} \tag{8}$$

where, $\mathrm{Pr} = \nu/\alpha$ is the prandtl number.

The corresponding non dimensional initial and boundary conditions are

$$i \le 0;\ U = 0,\ V = 0,\ T = 0 \text{ for all X and Y}$$
$$i > 0;\ U = 0,\ V = 0,\ T = X^n \text{ at } Y = 0 \qquad (9)$$
$$U = 0,\ T = 0 \text{ at } X = 0$$
$$U \to 0,\ T \to 0 \text{ at } Y \to \infty$$

## 3. NUMERICAL PROCEDURE

The governing partial differential equations (6)–(8) are unsteady, coupled and non-linear with initial and derivative boundary conditions (9). They are solved numerically by Network Simulation Method (NSM) described in detail by Beg et. al., [4-5] and Zueco et. al., [12-16].

Also network models designed (combinations of resistors $R_u$, $R_T$, current control generators $G_u$, $G_T$ and capacitors $C_u$, $C_T$) as shown in Figure 1(a) and 1(b) is explained by Beg et. al., [4-5] and Zueco et. al., [12-16]. Finally it is worth to satisfy Kirchhoff's laws.

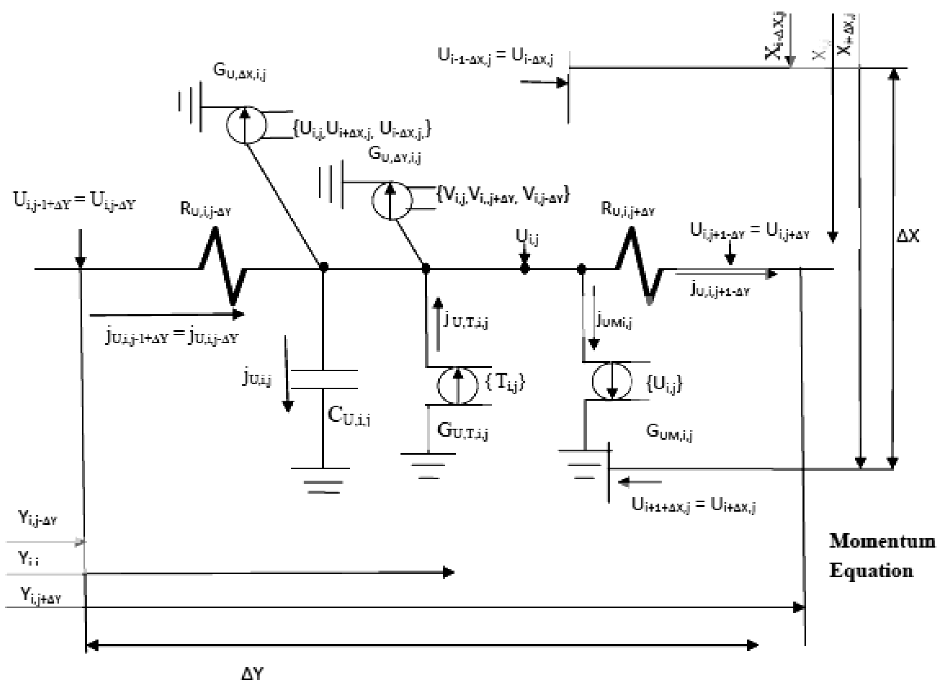


**Figure 1: (a) Network model of the control volume – Momentum equation**

## 4. RESULT AND DISCUSSION

Figures 2-5 present transient velocity and temperature profiles at X = 1.0 with various parameters Pr, *n* andmagnetic parameter M. The value *t* with a star symbol denotes the steady state time. In Figure 2 and 3 transient velocity and temperature profiles are plotted for different values of Pr and M.

Application of a magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that acts in the direction, opposite to that of the flow. This force is called the Lorentz force. This resistive force tends to slow down the motion of the fluid along the cone and causes decrease in its temperature and increase in velocity as M decreases, which is clear from Figure 2 and 3.

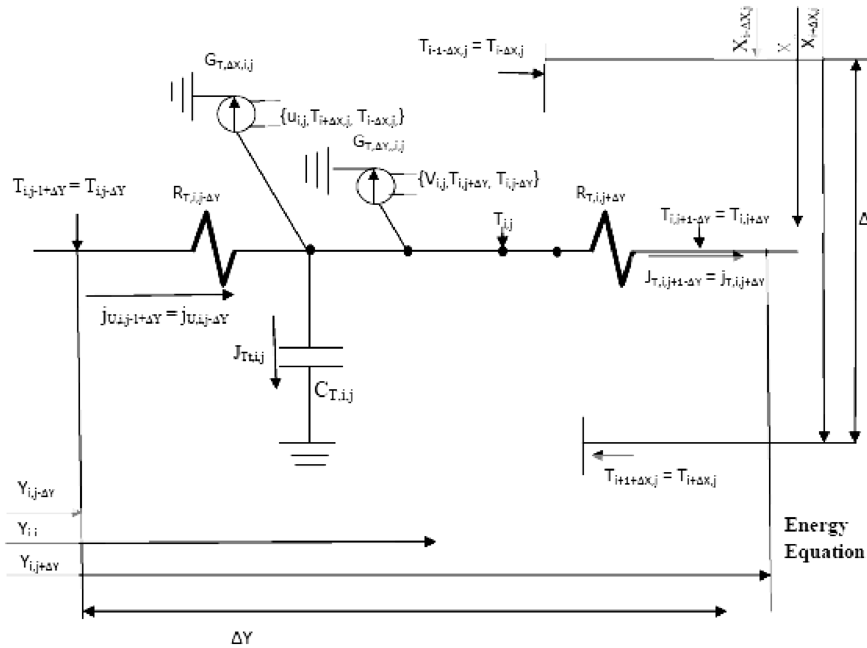


**Figure 1: (b) Network model of the control volume – Energy equation**

Also, from Figures. 2 and 3, it is clear that whenever Pr values increase, the momentum and thermal boundary layers become thin.

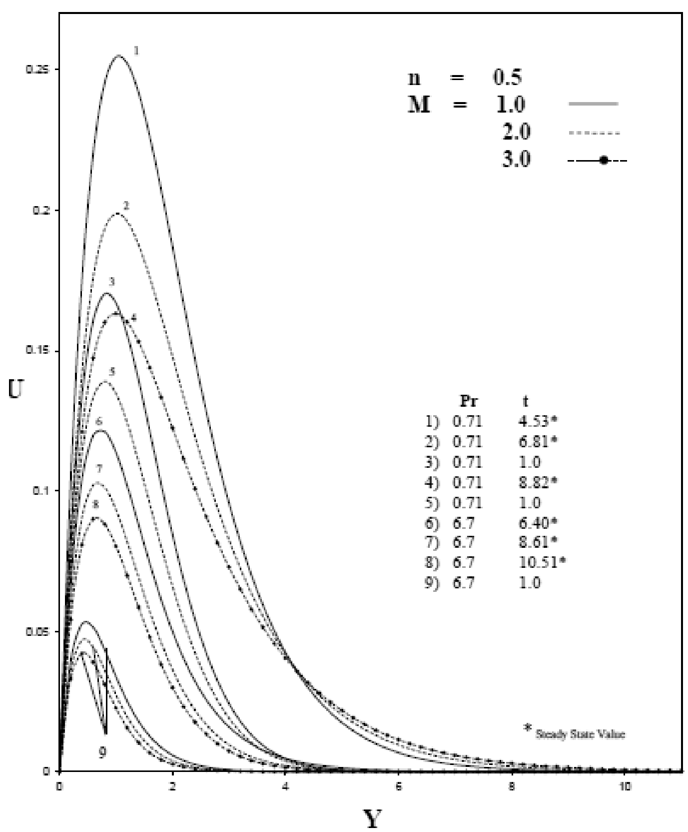


**Figure 2: Transient velocity profiles at X = 1.0 for various values of Pr and M**

The viscous force reduces and thermal diffusivity increases with decreasing Pr, which causes increase in the velocity and temperature as expected. It is also noticed that the time taken to reach steady state increases with increasing Pr. Transient velocity and temperature profiles for various values of n and M are shown in Figure 4 and 5.

It is noticed that as n decreases, velocity and temperature increase and time taken to reach steady state decreases. Momentum and thermal boundary layers become thick when the velocity of *n* decreases.

Finally, from Figures 2-5, it is concluded that time to reach steady state, momentum and thermal boundary Layer thickness decrease with decreasing M.

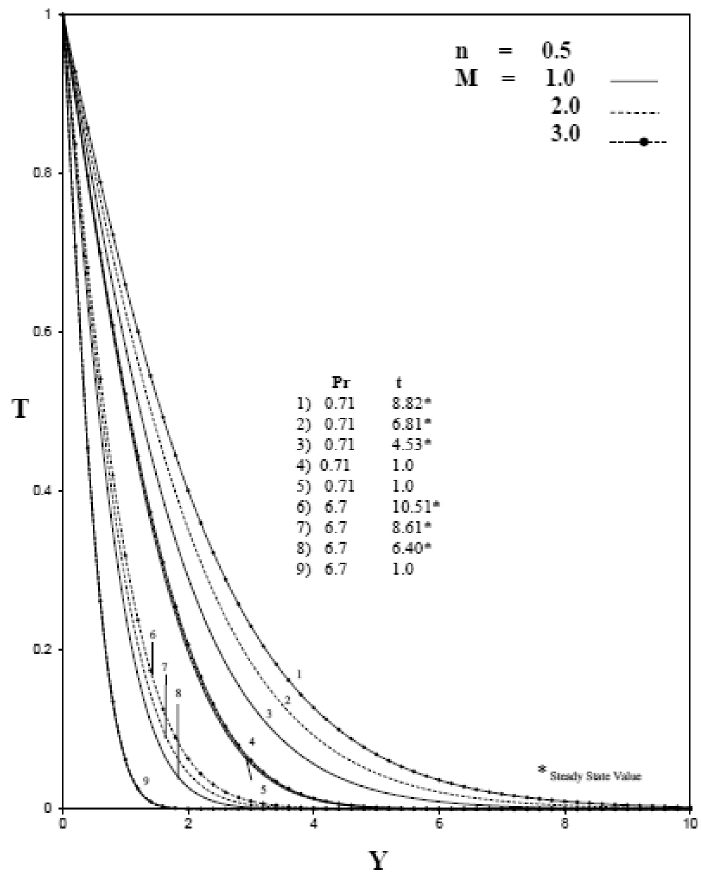


**Figure 3: Transient temperature profiles at X = 1.0 for various values of Pr and M**

## 5. CONCLUSION

This article studies about the unsteady laminar free convection flow past a non isothermal vertical cone in the presence of magnetic field using NSM. The dimensionless governing boundary layer equations are solved using NSM technique. The following conclusions are drawn

- When the parameters Pr, *n*, and M are increased, then the velocity U decreases.
- Surface temperature T increases for the higher values of M and the lower values of Pr, *n*.
- Momentum boundary layers become thin when M is decreased or values of Pr, *n* are increased.
- The thermal boundary layer becomes thick when M is increased or values of Pr, *n* are reduced.

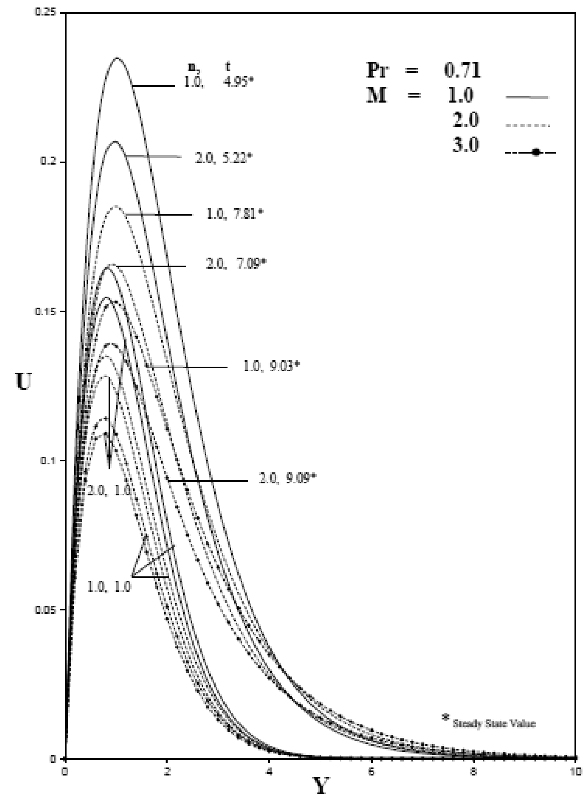


**Figure 4: Transient velocity profiles at X = 1.0 for various values of *n* and M**

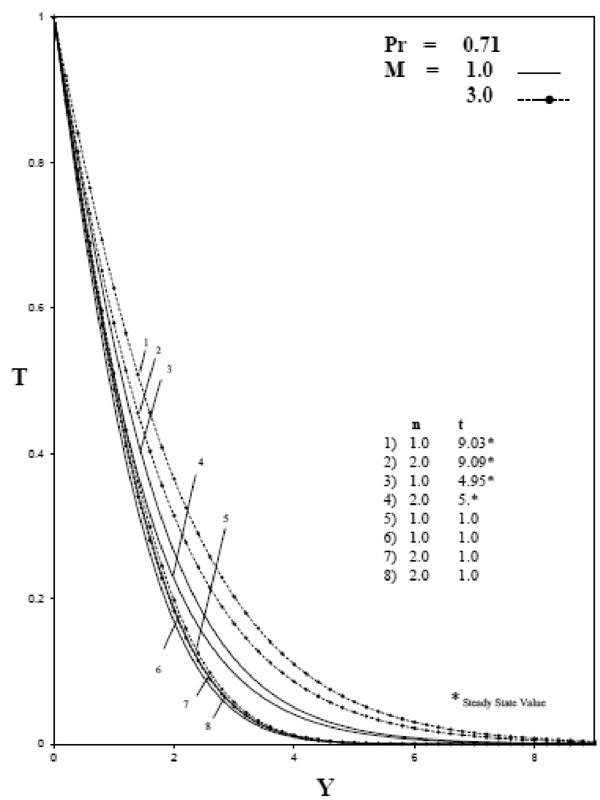


**Figure 5: Transient temperature profiles at X = 1.0 for various values of *n* and M**

**Nomenclature:**

| | |
|---|---|
| $a$ | Constant |
| $B_0$ | magnetic field strength |
| $Gr_L$ | Grash of number |
| $g$ | acceleration due to gravity |
| L | length of slant height |
| M | magnetic field parameter |
| $n$ | exponent in power law variation in surface temperature |
| Pr | Prandtl number |
| R | dimensionless local radius of the cone |
| $r$ | local radius of the cone |
| $T'$ | Temperature |
| T | Dimensionless temperature |
| $t'$ | Time |
| $t$ | dimensionless time |
| U | dimensionless velocity in X – direction |
| $u$ | velocity component in $x$ – direction |
| V | dimensionless velocity in Y – direction |
| $v$ | velocity component in $y$ – direction |
| X | dimensionless spatial coordinate |
| $x$ | spatial coordinate along cone generator |
| Y | dimensionless spatial coordinate along the normal to the cone generator |
| $y$ | spatial coordinate along the normal to the cone generator |

**Greek Letters:**

| | |
|---|---|
| $\alpha$ | Thermal diffusivity |
| $\beta$ | Volumetric thermal expansion |
| $\mu$ | Dynamic viscosity |
| $\nu$ | Kinematic viscosity |
| $\rho$ | Density |
| $\sigma$ | Electrical conductivity of the fluid |
| $\phi$ | Semi-vertical angle of the cone |

**Subscripts:**

| | |
|---|---|
| $w$ | Condition on the wall |
| $\infty$ | Free stream condition |

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