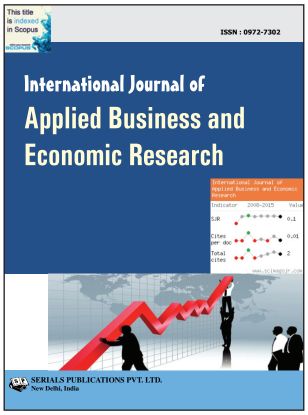

International Journal of Applied Business and Economic Research

ISSN : 0972-7302

available at http: www.serialsjournals.com

Volume 15 • Number 22 • 2017

Vendor Managed Inventory Model with Stochastic Demand and Variable Lead Time

Debashish Kumar[1], D.A. Khan[2] and T. Mandal[3]

[1] *Information Technology, XLRI Jamshedpur, Jharkhand, India. Email: dkumar@xlri.ac.in*
[2] *Department of Computer Applications, NIT Jamshedpur, Jharkhand, India*
[3] *Department of Mathematics, NIT Jamshedpur, Jharkhand, India*

ABSTRACT

Vendor managed inventory (VMI) is an integrated approach for vendor-retailor coordination, where a vendor places order on behalf of its retailer(s). This paper discusses how a vendor manages its multiple retailers operating under a VMI contract that specifies upper stock limits at the retailer's premises under stochastic demand by considering different lead time for different retailers. It is assumed that the vendor replenishes all the retailers at the same time and incurs a penalty cost for exceeding the upper stock limits. In this paper, a mixed-integer non-linear programming (MINLP) model is developed to address the replenishment problem among multiple retailers with stochastic demand that minimizes the joint relevant inventory costs. A heuristic solution along with a numerical illustration is also provided to demonstrate the proposed MINLP and the validity of the model has been tested through simulation.

Keywords: Supply chain, inventory management, vendor managed inventory, stochastic demand, lead time, transportation cost.

1. INTRODUCTION AND BACKGROUND

Research and applications in supply chain management have looked for ways to increase coordination and integration among supply chain members to achieve higher service levels for end customers at a lower total supply chain cost. VMI is one of the coordination mechanism that has been gaining a lot of attention recently and many successful business have demonstrated the benefit of VMI, e.g. Wal-Mart, JC Penny, HP, Shell, Campbell Soup, Barilla, Johnson & Johnson, Kodak Canada Inc and White Bread Beer (Centinkaya and Lee, 2000; Dong and XU, 2002; Kuk, 2004; Smaros et. al., 2003).

In VMI, the retailer must provide all relevant information like sales, inventory level etc. and the vendor determine the timing and replenishment quantity of the product for each retailer based on those information. In other word, the stock remains under the ownership of the vendor who incur the resulting inventory holding and transportation cost. VMI contract has some advantages for both parties and also the customer service levels may increase in terms of reliability of product availability (Pasandideh, 2011). In order to reap maximum benefit, the vendor always tends to move much of its inventory to the retailer premises by shipping large quantities, whereas each retailer restricts the vendor to keep inventory level below an agreed maximum level. Thus it is quite common in VMI contracts that the retailer is protected from over-supplies from the vendor by means of a mutually agreed-upper stock limit at the retailer and the vendor is penalized for items exceeding these limits (Shah and Goh, 2006; Danese, 2006; Chen et. al., 2006). And thus retailer relieved of keeping track of its inventory and placing orders with the vendor from time to time, thereby eliminating its order costs.

In this paper, we consider a single-vendor multi-retailer supply chain operating with stochastic demand under VMI model and consider transportation cost also as an important parameter and study the effect of it on total system cost. The retailer face normal distribution of demand, where the lead time is deterministic but different for different retailers and thus the overall behavior of the demand is stochastic (Mateen et. al., 2015).

The paper proceeds as follows. The next section presents a review of relevant literature. The problem statement, associated notations and mathematical models developed for evaluating VMI systems in the subsequent section. In order to demonstrate the application of the proposed methodology, we provide a numerical example and solving it with one of the heuristic search algorithm, genetic algorithm (GA) and also solved the same using LINGO optimizer. A sensitivity analysis is conducted followed by some concluding remarks and recommendations for future research.

2. LITERATURE REVIEW

Since last few decade a large number of noticeable studies emerged related to buyer-vendor coordination. Most of these studies focused on the model either to minimize the total relevant costs for both the vendor and the buyer (e.g. Goyal, 1976; Goyal, 1977; Banerjee, 1986; Goyal, 1988; Goyal and Gupta, 1989) or to minimize the vendor total annual cost subject to the maximum cost that the buyer may be prepare to incur (e.g. Lu, 1998).

Many VMI models extended joint economic lot sizing (JELS) models by relaxing assumptions, incorporating new variables, and improving solutions and computational efficiency. Early studies on JELS have mostly used economic order quantity (EOQ) models to minimize a total cost function that adds up the costs of both buyer as well as vendor (Banerjee, 1986; Goyal, 1988; Hill, 1997; Viswanathan, 1998). Most of the VMI models published so far are based on deterministic demand, while only few researchers considered stochastic nature kind of demand. When the demand is assumed to be stochastic, lead time becomes an important issue and its control leads to many benefits. Many researchers looked at the problem of lead time optimization and established a linear relationship between lead time and lot size followed the papers by Kim and Benton, 1995; Hariga, 1999; Ben-Daya and Raouf, 1994 and Ben-Daya and Hariga, 2004.

VMI has been defined as a collaboration strategy to optimize the availability of products at minimal costs, where the retailer hands over the operational control of inventory within a mutually agreed framework.

VMI seeks to improve the aggregate performance of a supply chain and has gained prominence in practice with the increasing collaboration and integration that is taking place in supply chain. Various studies showed that supply chain members can reap substantial benefits from VMI implementation. Such benefits include but are not the limited to reduced lead-time and stock-outs, improved control of bullwhip effect, increased service level and reduced in costs (Angulo et. al., 2004; Kulp et. al., 2004).

Many researchers have tried to mathematically model different aspects of VMI systems. Centinkaya and Lee, 2000 presented an analytical model for coordinating inventory and transportation decisions in VMI systems. Viswanathan and Piplani, 2001 proposed a strategy where the vendor specifies common replenishment period in a single-vendor multi-retailer supply chain. Dong and Xu, 2002 presented an analytical model to evaluate the short-term and long-term impacts of VMI on supply chain profitability. Choi et. al., 2004 presented an analytical model to measure the service level under VMI. Jasemi, 2006 developed supply chain model with a single supplier and n buyers and compare the performance of one of the traditional type VMI system and consider a pricing system for profit sharing between parties.

Darwish and Odah, 2010 proposed a single-vendor multi-retailer VMI model in which the vendor incurs a penalty cost for items exceeding the bounds that are agreed upon in a contractual agreement between vendor and retailers. Pasandideh et. al., 2011 developed an one-supplier one-retailer multi-product VMI model in which shortages are backordered, the supplier's warehouse space has limited capacity and there is an upper bound on the number of orders, where supplier order to the retailer according to the well known (R,Q) policy.

Mateen et. al., 2015 developed an approximate model in which the vendor replenishes all the retailers at the same time under stochastic demand and in case of a shortage at the vendor, the available stock is allocated to the retailers on the basis of equal stock out probability. Mateen and Chatterjee, 2015 discussed analytical models for various approaches through which a single-vendor multiple-retailer system may be coordinated through VMI. Here authors developed four VMI models with different operating assumptions like vendor sends shipment to each retailer cyclically, vendor replenishes all the retailers at the same time with uniform batch sizes, vendor synchronizes the system such that these deliveries reach the retailers only when their existing stock gets exhausted and in the last model the vendor delivered the product to the retailer in increasing sub-batch sizes.

In this paper, we consider a practical situation when a VMI partnership is implemented in a two-stage supply chain with limited retailer's storage capacities and the lead time is deterministic but different for different retailers and thus the overall behavior of the demand is stochastic. Our study focus on the synchronization of ordering cycle by minimizing the total inventory costs over the entire supply chain by introducing the transportation cost as an important parameter and study the effect of it on total system cost.

3. PROBLEM STATEMENT

Consider a system where a vendor (may be a manufacturer) supply a single product to meet the demand for multiple retailers under a VMI agreement. It is assumed that the vendor orders the product from an external source with unlimited capacity and responsible for initiating the orders and setting the replenishment quantities on behalf of the retailers, where the vendor orders stock corresponding to 'n' times the common retailer replenishment cycle. Under a VMI agreement, the retailer pays for the item as soon as he receives it, thus, the retailer is the owner of the inventory and incurs holding costs. All the remaining costs, viz. vendor holding costs, retailer order costs, setup costs, penalty costs and transportation cost are born by the vendor. VMI program usually include a contractual agreement between vendor and retailers which involves a bound Uj on the inventory level of retailer j such that the vendor is penalized for items exceeding this bound (Fry et. al., 2001; Shah and Goh, 2006).

The objective of this study is to determine the optimal replenishment policy with stochastic demand situations under VMI agreement that is economically beneficial to the entire supply chain. It is shown in Figure 1 that the variation over time of the stock at each echelon is saw-toothed in shape, which simplifies the formulation of the total holding costs per unit of time.

The assumptions and notations used in the paper are listed below.

A. Assumptions

- There is no procurement lead time for the vendor.
- Vendor orders the item from an external source having unlimited supply.
- Cost of holding one unit of item per unit of time at the vendor facility is less than that of each one of the retailers.
- Each retailer specifies an upper stock level under VMI, and the vendor is financially penalized whenever that level exceeded.
- It is also assumed that the standard deviation of the demand is low compared to the mean demand.
- It is assume that there is no shortages in both echelon.
- Since there is no shortages, both vendor and retailers safety factor assume to be unity.
- The retailer lead time is deterministic and is different for different retailers.

B. Notations

Let *r* be the number of retailers, and *j* be the index for retailers, $j = 1, 2, \ldots, r$. Then,

A_v Vendor ordering cost per order ($/order)

A_j Cost charged to the j^{th} retailer for receiving its ordered shipment and cost incurred by the vendor for initiating and releasing an order to the j^{th} retailer ($/order)

D_v Mean demand rate for the vendor (units/order)

D_j Demand rate per unit of time for retailer *j* (units/order)

h_v Vendor holding cost per unit per unit of time ($/unit/order)

h_{cj} Capital cost per unit per unit of time of j^{th} retailer ($/unit/order)

h_{sj} Storage cost per unit per unit of time of j^{th} retailer ($/unit/order)

h_{rj} Storage cost per unit per unit of time at the rented facility to store the over-stock quantity of the j^{th} retailer ($/unit/order)

h_j Retailer holding cost per unit per unit of time (= $h_{cj} + h_{sj}$) ($/unit/order)

l_i lead time of the j^{th} retailer (years)

σ_j Standard deviation of the demand rate for the j^{th} retailer (unit/order)

π_j Penalty cost per unit for the over-stock quantity at the j^{th} retailer (= $h_{rj} - h_{sj}$) ($/unit)

U_j Upper stock limit at the j^{th} retailer facility (units)

T Common replenishment cycle (years) [a decision variable]

n Number of deliveries made during the vendor's order cycle [a decision variable]

S_v Order up to level for the vendor (units)

S_j Order up to level for the j^{th} retailer (units)

TR_j Transportation cost from the vendor to the j^{th} retailer ($/unit/order)

z_j Over-stock quantity at the j^{th} retailer (units)

C. Mathematical Model

Vendor annual order costs $= \dfrac{A_v}{nT}$

Retailer annual order cost $= \sum_{j=1}^{r} \dfrac{A_j}{T}$

Retailer transportation cost $= \sum_{j=1}^{r} \dfrac{TR_j}{T}$

Considering retailer replenishment interval as well as the lead times, the order up to level for jth retailer would be:

$$S_j = D_j(T + l_j) + \sigma_j \sqrt{T + l_j}$$

From the previous equation, we can write

Average annual holding cost for the j^{th} retailer $= \left[\dfrac{1}{2} D_j(T + l_j) + \sigma_j \sqrt{T + l_j}\right](h_j - h_v)$

Average annual holding cost for the retailers $= \sum_{j=0}^{r} \left[\dfrac{1}{2} D_j(T + l_j) + \sigma_j \sqrt{T + l_j}\right](h_j - h_v)$

The order up to level at the vendor is calculated keeping in mind that the orders are for nT period and that the variances for all the retailers over the period would have to be considered (Mateen et. al., 2015).

Then the standard deviation of the demand at the vendor $= \sqrt{nT \sum_{j=0}^{r} \sigma_j^2}$

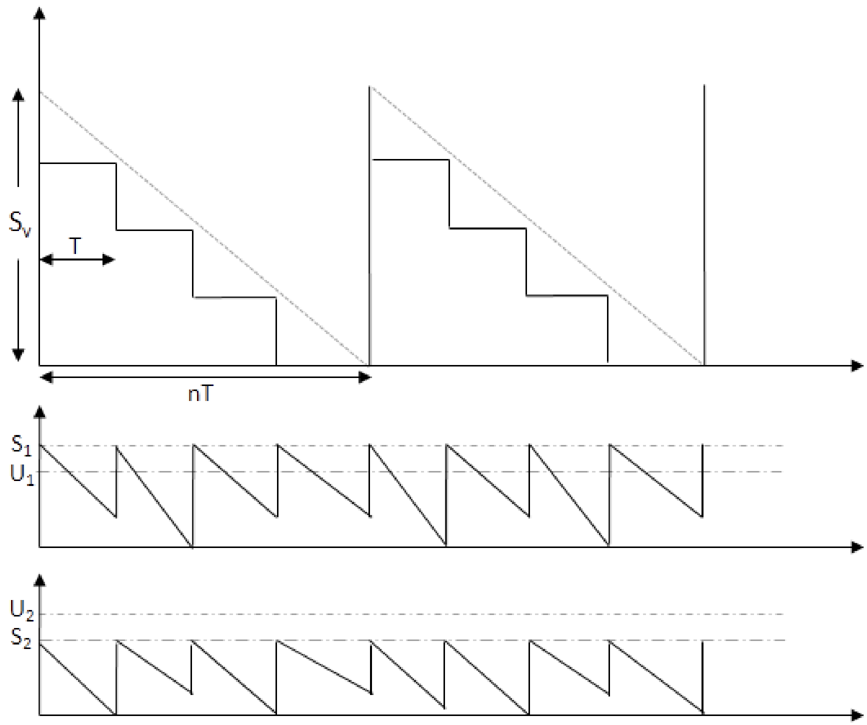

Figure 1: Demand variation over time of the inventory levels for the vendor and two retailers

Then the order up to level at the vendor:

$$S_v = DnT + \sqrt{nT \sum_{j=0}^{r} \sigma_j^2}$$

From the previous equation, we can write

$$\text{Average holding cost for the vendor} = \left[\frac{1}{2} DnT + \sqrt{nT \sum_{j=0}^{r} \sigma_j^2} \right] h_v$$

Referring to Figure 1, which shows the variation over time of the vendor inventory levels and all retailers, the over stock at the j^{th} retailer, z_j can be written as:

$$z_j = S_j - U_j = D_j(T + l_j) + \sigma_j \sqrt{T + l_j} - U_j, \quad j \in R$$

$$z_j = 0, \quad j \in \bar{R}$$

where, R is the set of retailers with over stock and $\bar{R}$ is the complement set of R.

The above equations can be rewrite as:

$$D_j(T_j + l_j) + \sigma_j \sqrt{T_j + l_j} - U_j \le z_j \quad \text{for} \quad j = 1, 2, \ldots, r$$

$$z_j \ge 0 \quad \text{for} \quad j = 1, 2, \ldots, r$$

Since the penalty cost is charged per unit of time and based on the average over-stock, the total over-stock penalty costs per unit of time is

$$\sum_{j=0}^{r} \pi_j \frac{z_j^2}{2TD_j}$$

The optimization problem with stochastic demand can then be stated as

$$\text{Min TC} = \frac{A_v}{nT} + \sum_{j=0}^{r} \frac{A_j + TR_j}{T} + \left[\frac{1}{2} DnT + \sqrt{nT \sum_{j=0}^{r} \sigma_j^2}\right] h_v$$

$$+ \sum_{j=0}^{r} \left[\frac{1}{2} D_j (T + l_j) + \sigma_j \sqrt{T + l_j}\right](h_j - h_v) + \sum_{j=0}^{r} \pi_j \frac{z_j^2}{2TD_j}$$

s.t.

$$D_j(T_j + l_j) + \sigma_j \sqrt{T_j + l_j} - U_j \le z_j \quad \text{for} \quad j = 1, 2, \ldots, r$$

$$z_j \ge 0 \quad \text{for} \quad j = 1, 2, \ldots, r \qquad (1)$$

$$n = \{1, 2, \ldots\} \quad \text{for} \quad j = 1, 2, \ldots, r$$

$$T \ge 0$$

The goal is to determine the common replenishment cycle (T) and total number of deliveries (n) in a vendor cycle so that the total cost (TC) under the VMI policy given in (1) is minimized and all the constraints are fulfilled. The difficulty originates from the nonlinearity of the objective function and constraint, and integer restriction of n in the mixed-integer nonlinear programming (MINLP) formulation.

In the next section a heuristic search solution, Genetic Algorithm (GA) is elaborate to efficiently solve the problem.

4. A SOLUTION PROCEDURE

The model that was formulated in the previous section belongs to MINLP problem and we have used Genetic Algorithm (GA) approach in order to obtain good solutions (Park, 2001; Pasandideh et. al., 2008, 2011). Genetic Algorithms are adaptive heuristic search algorithm for solving optimization problem based on the principle of survival of the fittest in biological evaluation and genetic. The basic idea of the GA approach is to code the decision variables of the problem as a finite length array (called chromosome) and calculate the fitness value (objective function) of each string.

GA start with an initial set of random solutions, called a population, and then produce parents to reproduce. Each individual in the population is called a chromosome that is represented as a candidate solution to a problem and consists of a number of genes. The chromosome must go through a successive set of solution called generation and the fitness function is used to evaluate all individuals. A simple GA that renders good solutions is composed of three operators: selection, crossover, and mutation. These operators are used to create new chromosomes called offspring. The population of solution become better in each successful generation until satisfying solution is obtained. A GA terminated either after a predefined number of generations or when no further improvement in the solution.

In this paper, the decision variables of the proposed model are common replenishment cycle (T), number of deliveries in each vendor order cycle (n) and over-stock quantity (z_j) in each retailer with respect to order up to level (S_j) to respective retailer. The chromosome is modeled by a $1 \times (n + 2)$ matrix and the general form of chromosome is considered is represented as $[n \; T \; z_1 \; \ldots \; z_n]$.

The developed GA is coded in MATLAB 2017a software and solved on Intel Core(TM) i5 2.5 GHz with 4 GB RAM CPU. Since the state of the random number generators change in each run, we get different results in each run. The problem has run more than 100 times with different value of crossover rate (Pc) ranging from 0.65 to 0.8 and population size (N) ranging from 100 to 200, and feasible fitness value along with corresponding solution is recorded. Because of integer restriction in number of shipment (n), GA not permit different value of mutation rate (Pm). The best results are summarized in Table 1. Based on the results of Table 1, the best fitness value is 2006.933 with $n = 7$, T = 0.12781 and z_j = (27.317, 69.963, 129.815, 181.338).

Table 1
The experimental results of the GA

n	T	z_j	*Fitness value*	n	T	z_j	*Fitness value*
9	0.10308	13.978, 43.785, 89.874, 233.867	2034.226	8	0.11268	19.13, 53.966, 105.415, 264.794	2015.637
7	0.12150	23.854, 63.404, 119.652, 293.117	2009.849	10	0.08889	10.134, 28.695, 66.813, 188.013	2087.175
7	0.12978	29.373, 72.037, 132.984, 319.652	2007.900	8	0.11506	26.946, 56.488, 109.268, 272.449	2017.156
7	0.12928	28.186, 71.51, 132.183, 318.054	2007.157	9	0.10377	14.396, 44.523, 91.001, 236.106	2032.795
8	0.11910	22.575, 60.774, 115.792, 285.427	2011.246	7	0.13965	33.538, 82.442, 148.848, 351.231	2016.187
8	0.10706	16.119, 48.015, 96.334, 246.715	2026.073	8	0.11839	22.188, 60.004, 114.633, 283.123	2011.356
7	0.13831	33.88, 81.034, 146.7, 346.956	2015.053	7	0.12780	27.22, 69.951, 131.711, 313.323	2008.164
7	0.12233	24.298, 64.177, 120.995, 295.789	2009.000	6	0.14702	37.46, 90.199, 160.672, 374.762	2020.087
7	0.13443	30.756, 76.943, 140.465, 334.542	2009.941	9	0.10434	14.673, 45.128, 91.924, 237.943	2031.679
8	0.11787	21.909, 59.455, 113.803, 281.457	2011.509	6	0.15166	40.749, 95.097, 168.134, 389.583	2026.831
7	0.12141	23.808, 63.204, 119.513, 292.84	2009.818	8	0.10742	17.196, 48.397, 96.927, 247.867	2025.628
7	0.13771	33.688, 80.393, 145.724, 345.013	2014.349	8	0.11146	20.122, 52.677, 103.449, 260.875	2018.185
7	0.13642	31.818, 79.044, 143.664, 340.913	2011.956	6	0.14819	38.096, 91.423, 162.532, 378.48	2021.415
7	0.13476	30.932, 77.29, 141.053, 335.597	2010.282	5	0.16576	47.434, 109.883, 190.641, 434.442	2053.865
7	0.12857	27.629, 70.77, 131.043, 315.781	2006.931	7	0.12510	25.775, 67.094, 125.447, 304.65	2007.351
7	0.13513	31.129, 77.679, 141.585, 336.774	2010.593	8	0.10801	16.632, 49.024, 97.863, 249.761	2023.847
7	0.13682	34.721, 79.466, 144.31, 342.197	2014.383	6	0.14941	38.734, 92.713, 164.496, 382.388	2022.934
7	0.14045	33.974, 83.293, 150.157, 353.803	2017.457	7	0.13498	31.051, 77.525, 141.357, 336.31	2010.454
8	0.11323	19.426, 55.734, 106.306, 266.558	2016.108	6	0.14004	33.744, 82.85, 149.468, 352.467	2015.169
9	0.10156	13.836, 42.179, 87.409, 228.978	2038.106	6	0.14540	36.6, 88.488, 158.059, 369.572	2018.477
7	0.12203	25.833, 63.858, 120.512, 294.828	2010.291	6	0.15030	39.814, 93.641, 165.91, 385.203	2024.603
10	0.09245	8.261, 32.5, 72.617, 199.544	2069.813	7	0.12781	27.317, 69.963, 129.815, 313.338	2006.933
7	0.13060	28.732, 72.899, 134.301, 322.269	2007.467	10	0.08685	5.309, 26.519, 63.497, 181.383	2097.554
8	0.11562	20.716, 57.079, 110.162, 274.23	2012.699	7	0.13114	29.005, 73.486, 135.184, 324.025	2007.708
7	0.13206	29.492, 74.44, 137.824, 326.945	2008.995	7	0.13724	32.287, 79.911, 144.982, 343.53	2012.956
7	0.13939	38.056, 82.169, 148.444, 350.402	2019.392				

5. AN ILLUSTRATIVE EXAMPLE

The MINLP for the replenishment problem among multiple retailers with stochastic demand developed in the preceding section will now be illustrated by a numerical example to show the effectiveness of the model. We follow the illustrative example proposed in Mateen et. al., 2015 and introducing the transportation cost of each retailers.

We consider the situation with a vendor serving 4 retailers and incurring ordering cost (A_v) and holding cost (h_v) of 500 and 0.2 respectively. The vendor's mean demand (D_v) is 6000 and assume there is no shortages at the vendor. The retailer parameters are given in Table 2.

Table 2
Retailers general data

Retailer	D_j	σ_j	A_j	h_{ci}	h_{sj}	h_{rj}	h_j	l_j	U_j	π_j	TR_j
1	500	25	20	0.3	0.3	1.8	0.6	0.008219	50	1.5	2
2	1000	40	10	0.3	0.2	2.2	0.5	0.002740	75	2	6
3	1500	80	15	0.3	0.1	1.1	0.4	0.005479	100	1	4
4	3000	150	10	0.3	0.1	1.1	0.4	0.008219	150	1	2

The proposed MINLP model is programmed in LINGO 16.0 software, a comprehensive tool designed to make building and solving mathematical optimization model, and solved on an Intel Core(TM) i5 2.5 GHz with 4 GB RAM CPU. The best objective value found after 149 solver iterations with the help of inbuilt specialized branch-and-bound solver which takes 0.18 seconds of CPU time are summarize in Table 3.

Table 3
Optimal solution of proposed MINLP

n	*T*	S_v	S_j	*System penalty cost*	*TC*
7	0.12770	5530.078	77.175, 144.884, 228.959, 463.049	218.246	1938.235

6. SENSITIVITY ANALYSIS

In this section we perform sensitivity analysis in order to study the effect of the vendor ordering cost (A_v), vendor holding cost (h_v), retailer over-stock limit (U_j), penalty cost (π_j) and transportation cost (TR_j) on the system cost. The sensitivity analysis is shown by increasing or decreasing cost parameters, taking one at a time and keeping the others at their own values. The figures corresponding to 0% change are the base cases, as taken in the numerical example.

A. Effect of Vendor Order Cost (A_v)

To study the effect of the vendor ordering cost (A_v) on the system total cost (TC), we obtained optimal solutions for selected values of A_v, ranging from 250 to 750. The results are summarized in Table 4. TC is directly related to the vendor order cost, resulting in higher total system cost as A_v increases. As the vendor ordering cost increases, the vendor tends to replenish the retailers more frequently, resulting in decrease in vendor lot sizes and thus decrease in system penalty cost.

Effect of Vendor Holding Cost (h_v)

It is shown in Table 5 that the total cost is moderately sensitive to change in vendor holding cost (h_v). This means that when h_v increases, number of shipments n decreases, resulting in higher total system cost. In other word, as h_v increases, vendor delivers orders less frequently and hence the length of the replenishment cycle as well as system penalty cost increases.

Table 4
Effect of vendor order cost

A_v	n	T	*System Penalty Cost*	*TC*	*Change (%)*
250	5	0.12694	215.558	1680.177	–16.261
300	5	0.13184	232.717	1757.466	–12.409
350	6	0.12624	213.127	1824.639	–9.061
400	6	0.13006	226.486	1889.667	–5.820
500	7	0.12770	218.246	2006.452	0.000
600	8	0.12511	209.212	2112.802	5.300
650	8	0.12765	218.058	2162.256	7.765
700	8	0.13015	226.792	2210.743	10.182
750	9	0.12465	207.612	2256.600	12.467

Table 5
Effect of vendor holding cost

h_v	n	T	*System Penalty Cost*	*TC*	*Change (%)*
0.05	15	0.12067	193.920	1511.660	–24.660
0.1	10	0.12453	207.211	1722.420	–14.156
0.125	9	0.12492	208.570	1804.872	–10.047
0.15	8	0.12702	215.846	1878.796	–6.362
0.2	7	0.12770	218.246	2006.452	0.000
0.25	6	0.13164	232.018	2116.526	5.486
0.275	6	0.12901	222.810	2166.067	7.955
0.3	6	0.12654	214.187	2214.640	10.376
0.35	5	0.13506	244.098	2299.453	14.603

C. Effect of Standard Deviation of Demand (σ_j) and Lead Rime (l_j) of Retailer

Table 6 shows the effect of the standard deviation of demand and lead time of retailer 4. The system cost (TC) increases with increase in demand uncertainty as signified by the increase in σ_j. The order up to level (S_4) and over-stock quantity (z_4) are directly proportional to any changes in σ_4, whereas the replenishment interval went down with an increase in σ_4. The TC is slightly sensitive to change in lead time (l_j), while protection would have to be provided for slightly longer duration leading to an increase in total cost (TC).

Table 6
Effect of standard deviation of demand and lead time of retailer 4

σ_4	z_4	*System Penalty cost*	*TC*	*Change (%)*	l_4	z_4	*System Penalty cost*	*TC*	*Change (%)*
50	281.922	196.084	1957.066	–2.461	0	287.877	198.873	1983.007	–1.168
100	297.600	206.854	1980.528	–1.292	0.002740	296.238	205.097	1990.630	–0.789
125	305.347	212.462	1993.258	–0.658	0.005479	304.627	211.550	1998.443	–0.399
150	313.058	218.246	2006.452	0.000	0.008219	313.058	218.246	2006.452	0.000
175	320.714	224.185	2020.023	0.676	0.010959	321.501	225.155	2014.655	0.409
200	328.346	230.316	2033.916	1.369	0.013699	329.985	232.306	2023.052	0.827
250	343.528	243.131	2062.529	2.795	0.016438	338.496	239.689	2031.640	1.255

Table 7
Effect of upper stock and penalty cost limit of retailer 4

U_4	z_4	*System Penalty cost*	*TC*	*Change (%)*	π_4	z_4	*System Penalty cost*	*TC*	*Change (%)*
50	410.927	310.634	2101.189	4.722	0.25	379.670	171.216	1900.087	–5.301
100	361.414	260.541	2050.555	2.198	0.50	366.847	200.388	1939.693	–3.327
125	337.089	238.423	2027.691	1.059	0.75	322.451	195.351	1973.889	–1.623
150	313.058	218.246	2006.452	0.000	1.00	313.058	218.246	2006.452	0.000
175	289.292	199.956	1986.828	–0.978	1.25	304.280	239.595	2037.870	1.566
200	265.815	183.551	1968.804	–1.876	1.50	273.017	229.375	2066.512	2.993
250	219.690	156.214	1937.480	–3.438	1.75	266.220	246.716	2093.039	4.315

D. Effect of Over-stock Limit (U_j) and Penalty Cost (π_j)

It is shown in Table 7 that an increase in over-stock limit led to a decrease in system total cost, while reverse held true for penalty cost. Increasing the over-stock limit of a retailer would attract the vendor to send more stock to respective retailer. It means that high values of U_j give the vendor more freedom to ship higher batch size without incurring any penalty. On the other hand when penalty cost is very high, the over-stock limit acts as a capacity constraint for the vendor and it is very costly for vendor to violate such a limit. And thus an increase in penalty cost led to an increase in system cost.

E. Effect of retailer holding cost (h_j) and Transportation cost (TR_j)

The system total cost increases with an increase in h_j as well as TR_j and the reverse held true for both as shown in Table 8. Since the transportation cost is bear by vendor only, in case of higher transportation cost, vendor tries to ship more quantities in each delivery to compensate the loss due to high transportation cost, which causes increase in retailer order up to level (S_j) and hence over-stock quantity (z_j). Total cost (TC) is slightly sensitive to change in TR_j, while moderately sensitive to change in h_j.

Table 8
Effect of holding cost and transportation cost of retailer 4

h_4	z_4	System Penalty cost	TC	Change (%)	TR_4	z_4	System Penalty cost	TC	Change (%)
0.25	318.906	224.630	1967.344	–1.949	0.5	310.978	215.983	1994.676	–0.587
0.30	316.927	222.466	1980.432	–1.297	1.0	311.669	216.735	1998.608	–0.391
0.35	314.974	220.339	1993.468	–0.647	1.5	312.359	217.486	2002.533	–0.195
0.40	313.058	218.246	2006.452	0.000	2.0	313.058	218.246	2006.452	0.000
0.45	311.149	216.169	2019.386	0.645	2.5	313.737	218.985	2010.364	0.195
0.50	309.274	214.132	2032.269	1.287	3.0	314.424	219.168	2014.270	0.390
0.55	307.424	212.126	2045.102	1.926	3.5	315.109	220.482	2018.169	0.584

7. CONCLUSIONS

In this paper, we considered a supply chain where a vendor manages its multiple retailer's inventory under a VMI contract. The vendor incurs penalty costs if retailer's inventory exceeded pre-set upper stock limit. All retailers are replenished in equal interval of time, while the demand varies due to different lead time of each retailers. Importantly, we also consider transportation costs explicitly in our analysis, which is different for different retailer due to their location from vendor warehouse, and show that they can significantly affect the performance outcomes. Under these conditions, we formulated the problem as a mixed-integer non-linear programming (MINLP) model and employed a genetic algorithm to solve it. At the end, a numerical example was presented and solve it using LINGO programming to demonstrate the application and the performance of the developed model.

A sensitivity analysis is conducted to study the effects of different vendor and retailer parameters. The results shows that as the vendor ordering cost increases, the vendor tends to replenish the retailers more frequently, however as the vendor holding cost increases, the vendor replenishment frequency reduce due to less in vendor stock level. But in both cases the system total cost increases. The retailer parameters also have significant impact as discuss in previous section and can be tune to minimize overall system cost.

This study can be extended in many directions. An obvious extension is to consider the case when lead time varies with time or randomness is introduced into the demand pattern or unequal shipment sizes. We consider transportation cost explicitly, however by adopting a new transportation policy under VMI contract, the overall system cost can be minimize. The model can further extended to some more practical situations, such as VMI model for single-vendor multi-retailer multi-product, multi-vendor multi-retailer single-product/multi-product case. We will consider these problems in the near future.

Acknowledgment

The author would like to thank to Prof. Mateen A., Chaterjee A.K., Mitra S., Darwish M.A., Odah O.M. and the anonymous referees for their constructive suggestions, which improved the presentation of this paper.

References

Angulo A., Nachtmann H. and Waller M.A. (2004), "Supply chain information sharing in a vendor managed inventory partnership." Journal of Business Logistics, 25 (1), 101-120.

Banerjee A. (1986), "A joint economic lot size model for purchaser and vendor." Decision Sciences, 17, 292-311.

Ben-Daya M. and Hariga M. (2004), "Integrated single vendor single buyer model with stochastic demand and variable lead time." International Journal of Production Economics, 92, 75-80.

Ben-Daya M. and Raouf A. (1994), "Inventory models involving lead time as a decision variable." Journal of the Operational Research Society, 45 579-582.

Cetinkaya S. and Lee C.Y. (2000), "Stock replenishment and shipment scheduling for vendor-managed inventory systems." Management Sciences, 46, 2017-232.

Chen X., Chen J. and Chen Y.F. (2006), "A coordination mechanism for a supply chain with demand information updating." International Journal of Production Economics, 103(1), 347-361.

Choi K.S., Dai J.G. and Song J.S. (2004), "On measuring supplier performance under vendor-managed-inventory program in capacitated supply chains." Manufacturing & Service Operations Management, 6 (1), 53-72.

Danese P. (2006), "The extended VMI for coordinating the whole supply network." Journal of Manufacturing Technology Management, 17(7), 888-907.

Darwish M.A. and Odah O.M. (2010), "Vendor-managed inventory model for single-vendor single-retailer supply chains." European Journal of Operation Research, 204(3), 473-484.

Dong Y. and Xu K. (2002), "A supply chain model of vendor managed inventory." Transportation Research Part E, 38, 75-95.

Fry M.J., Kapuscinski R. and Olsen T.L. (2001), "Production and delivery under a (z, Z)-type vendor-managed inventory contract." Manufacturing and Service Operations Management, 3(2), 151-173.

Goyal S.K. (1976), "An integrated inventory model for a single supplier single customer problem." International Journal of Production Research, 14, 107-111.

Goyal S.K. (1977), "Determination of optimum production quality for a two-stage production system." Operation Research Quaterly, 28, 865-870.

Goyal S.K. (1988), "A joint economic lot size model for purchaser and vendor: A comment." Decision Sciences, 19, 236-241.

Goyal S.K. and Gupta Y.P. (1989), "Integrated inventory models: The buyer-vendor coordination." European Journal of Operational Research, 97, 493-499.

Hariga M. (1999), "A stochastic inventory model with lead time and lot size interaction." Production Planning and Control, 10, 434-438.

Hill Roger M. (1997), "The single-vendor single-buyer integrated production inventory model with a generalized policy." Europen Journal of Operational Research, 97, 493-499.

Jasemi M. (2006), "A Vendor-Managed Inventory." Master of Science Thesis, Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran.

Kim J.S. and Benton W.C. (1995), "Lot size dependent lead times in a (Q,r) inventory system." International Journal of Production Research, 33, 41-58.

Kuk G. (2004), "Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes." Information & Management, 41 (5), 645-654.

Kulp S.C., Lee H.L. and Ofek E. (2004), "Manufacturer benefits from information integration with retail customers." Management Science, 50 (4), 431-444.

Lu L. (1995), "A one-vendor multi-buyer integrated inventory model." European Journal of Operational Research, 81, 312-323.

Mateen A. and Chatterjee A.K. (2015), "Vendor managed inventory for single-vendor multi-retailer supply chains." Decision Support Systems, 70, 31-41.

Mateen A., Chatterjee A.K. and Mitra S. (2015), "VMI for single-vendor multi-retailer supply chains under stochastic demand." Computer & Industrial Engineering, 79, 95-102.

Park Y. (2001), "A hybrid genetic algorithm for the vehicle scheduling problem with due times and time deadlines." International Journal of Production Economics, 73, 175-188.

Pasandideh S.H.R. and Niaki S.T.A. (2008), "Agenetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space." Applied Mathematics and Computation, 195, 506-514.

Pasandideh S.H.R., Niaki S.T.A. and Nia A.R. (2011), "A genetic algorithm for vendor managed inventory control system of multi-product multi-costraint economic order quantity model." Expert Systems with Applications, 38, 2708-2716.

Pasandideh S.H.R., Niaki S.T.A. and Nia A.R. (2011), "A genetic algorithm for vendor managed inventory control system of multi-product multi-costraint economic order quantity model." Expert Systems with Applications, 38, 2708-2716.

Shah J. and Goh M. (2006), "Setting operating policies for supply hubs." International Journal of Production Economics, 100 (2), 239-252.

Smaros J., Lehtonen J., Appelqvist P. and Holmstrom J. (2003), "The impact of increasing demand visibility on production and inventory control efficiency." International Journal of Physical Distribution & Logistic Management, 33 (4), 336-352.

Viswanathan S. (1998), "Optimal strategy for the integrated vendor-buyer inventory model." European Journal of Operational Research, 105, 38-42.

Viswanathan S. and Piplani R. (2001), "Coordinating supply chain inventories through common replenishment epochs." European Journal of Operation Research, 129(2), 277-286.