IJCTA, 6(2) December 2013, pp. 97-109

A Ten-Term Novel 4-D Hyperchaotic System with Three Quadratic Nonlinearities and its Control

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ABSTRACT

First, this paper introduces a ten-term novel 4-D hyperchaotic system and discusses its qualitative properties. The proposed system is a ten-term novel polynomial hyperchaotic system with three quadratic nonlinearities. The novel hyperchaotic system has three unstable equilibria. The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 2.2848$, $L_2 = 0.1437$, $L_3 = 0$ and $L_4 = -24.3318$. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 2.2848$ and Lyapunov dimension as $D_L = 3.0998$. Next, we derive a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. The adaptive control result has been established using adaptive control theory and Lyapunov stability theory. Numerical simulations with MATLAB have been shown to illustrate the phase portraits of the novel 4-D hyperchaotic system and the adaptive control results for the hyperchaotic system.

Keywords: Chaos, hyperchaos, hyperchaotic systems, adaptive control, Lyapunov stability theory.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1]. The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. A chaotic system is also defined as a dynamical system having at least one positive Lyapunov exponent. In the last four decades, many chaotic systems have been found such as Lorenz system [2], Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], Sundarapandian-Pehlivan system [12], etc.

A *hyperchaotic system* is a chaotic system having more than one Lyapunov exponent. For continuous-time dynamical systems, the minimal dimension for a hyperchaotic system is four. The first hyperchaotic system was found by Rössler [13]. This was followed by the finding of many hyperchaotic systems such as hyperchaotic Lorenz system [14], hyperchaotic Lü system [15], hyperchaotic Chen system [16], hyperchaotic Wang system [17], etc. Hyperchaotic systems have attractive features like high security, high capacity and high efficiency and they find miscellaneous applications in several areas like neural networks [18-20], oscillators [21-22], circuits [23-26], secure communication [27-28], encryption [29], synchronization [30-35], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [36-37]. Some popular methods for chaos control are active control [38-39], adaptive control [40-43], sliding mode control [44], etc.

In this paper, we have proposed a ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities. We establish that the novel hyperchaotic system has three unstable equilibria. The Lyapunov

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exponents of the novel hyperchaotic system are obtained as $L_1 = 2.2848$, $L_2 = 0.1437$, $L_3 = 0$ and $L_4 = -24.3318$. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 2.2848$ and Lyapunov dimension as $D_L = 3.0998$. Next, we derive a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. The adaptive control result has been established using adaptive control theory and Lyapunov stability theory.

The rest of this paper is organized as follows. Section 2 contains the description of the ten-term novel 4-D hyperchaotic system proposed in this paper. Section 3 contains the qualitative properties of the novel hyperchaotic system. This section details important properties such as symmetry, invariance, dissipativity, equilibrium points and their stability nature, Lyapunov exponents and Lyapunov dimension of the novel hyperchaotic system. Section 4 contains the adaptive control results for the novel hyperchaotic system with unknown parameters. MATLAB simulations have been provided to illustrate all the main results obtained in this paper.

2. A TEN-TERM NOVEL 4-D HYPERCHAOTIC SYSTEM

In this section, we describe a ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities.

The novel 4-D hyperchaotic system is modeled by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= c x_2 - x_1 x_3 + x_4 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \\ \dot{x}_4 &= -d(x_1 + x_2) \end{aligned} \tag{1}$$

where x_2, x_2, x_3, x_4 are the state variables and a, b, c, d are constant, positive, parameters of the system.

The system (1) exhibits a *strange hyperchaotic attractor* when the constant parameter values are chosen as

$$a = 39, \;\; b = 4, \;\; c = 21, \;\; d = 2 \tag{2}$$

For numerical simulations, we take the initial values as

$$x_1(0) = 1.5, \; x_3(0) = 0.4, \;\; x_3(0) = 1.8, \;\; x_4(0) = 2.5 \tag{3}$$

Figures 1-4 give the 3-D view of the strange hyperchaotic attractor in (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

3. PROPERTIES OF THE NOVEL HYPER CHAOTIC SYSTEM

(A) Symmetry

The novel 4-D hyperchaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3, x_4) \to (-x_1, -x_2, x_3, -x_4) \tag{3}$$

Since the transformation (3) persists for all values of the system parameters, the novel chaotic system (1) has rotation symmetry about the x_3– axis and that any non-trivial trajectory must have a twin trajectory.

(B) Invariance

The x_3–axis ($x_1 = 0$, $x_2 = 0$, $x_4 = 0$) is invariant for the system (1). Hence, all orbits of the system (1) starting on the x_3 – axis stay in the x_3– axis for all values of time.

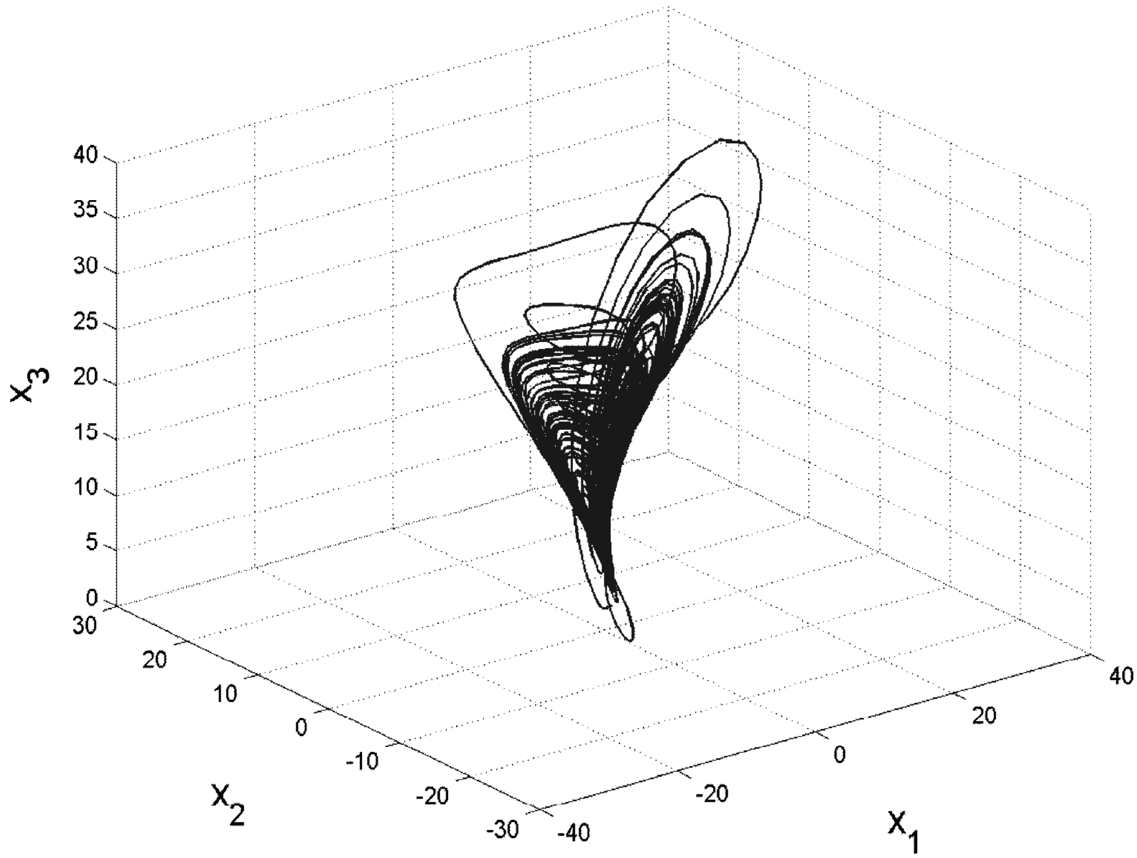

Figure 1: 3-D View of the Novel Chaotic System in (x_1, x_2, x_3) Space

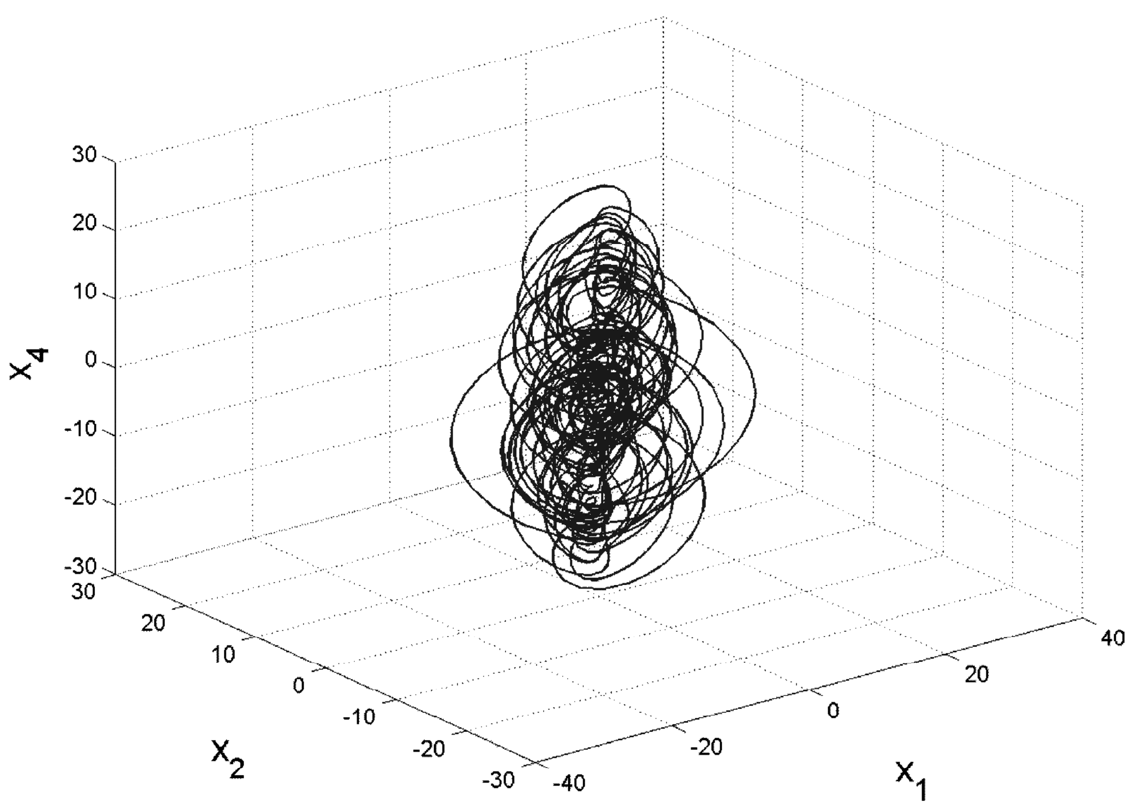

Figure 2: 3-D View of the Novel Chaotic System in (x_1, x_2, x_4) Space

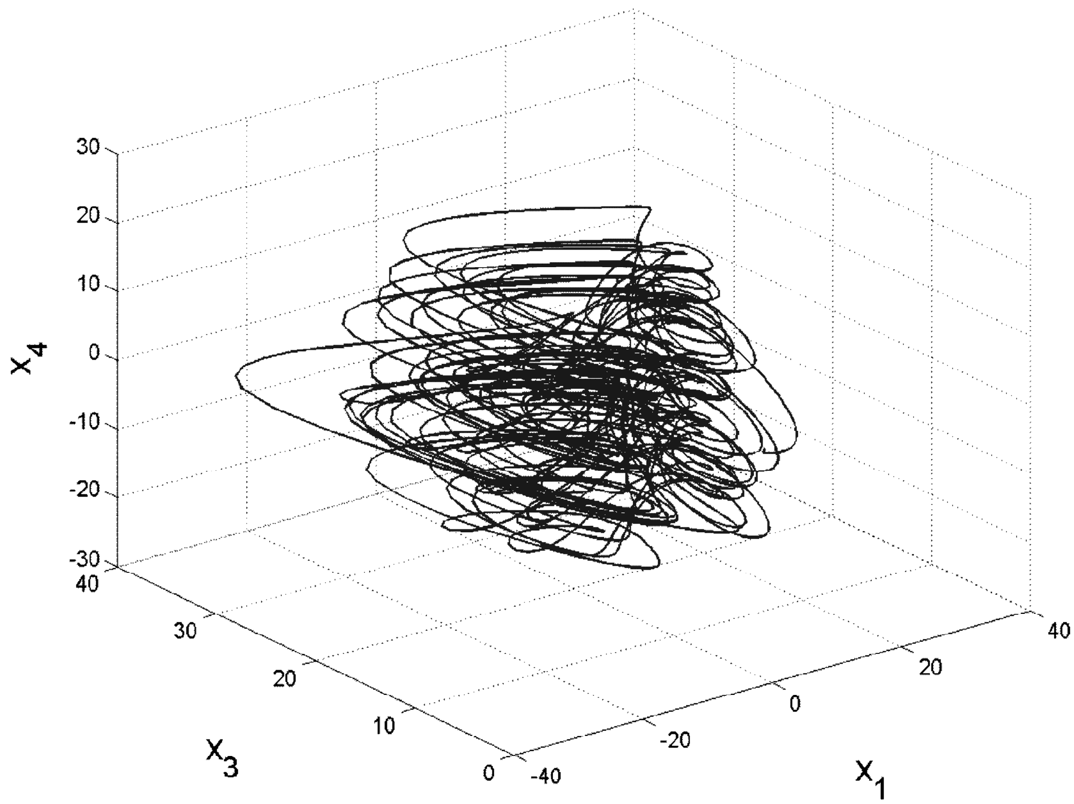

Figure 3: 3-D View of the Novel Chaotic System in (x_1, x_3, x_4) Space

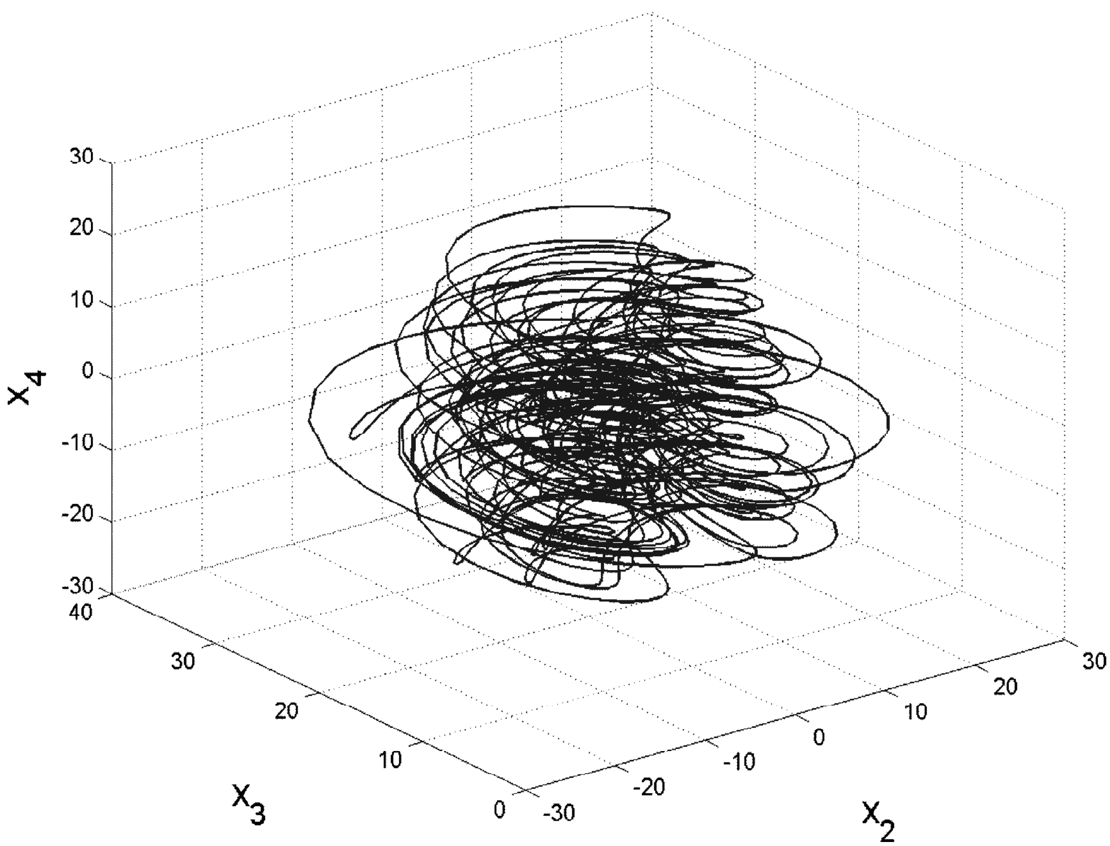

Figure 4: 3-D View of the Novel Chaotic System in (x_2, x_3, x_4) Space

(C) Dissipativity

We write the system (1) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_2 x_3 \\ f_2(x) &= c x_2 - x_1 x_3 + x_4 \\ f_3(x) &= x_1 x_2 - b x_3 \\ f_4(x) &= -d(x_1 + x_2) \end{aligned} \tag{5}$$

The divergence of the vector field f on R^4 is obtained as

$$\operatorname{div} f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -(a - c + b) = -\mu, \tag{6}$$

where

$$\mu = a - c + b \tag{7}$$

We take the parameter values as

$$a = 39, \ b = 4, \ c = 21, \ d = 2 \tag{8}$$

Then

$$\mu = a - c + b = 39 - 21 + 4 = 22 > 0. \tag{9}$$

Let Ω be any region in R^4 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f. Let $V(t)$ denote the hypervolume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV(t)}{dt} = \int\limits_{\Omega(t)} (\operatorname{div} f)\, dx_1 dx_2 dx_3 dx_4 = -\mu \int\limits_{\Omega(t)} dx_1 dx_2 dx_3 dx_4 = -\mu V(t) \tag{10}$$

Integrating the linear differential equation (10), we get the solution as

$$V(t) = V(0) \exp(-\mu t) \tag{11}$$

From Eq. (11), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel hyperchaotic system (1) is dissipative. Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

(D) Equilibrium Points

The equilibrium points of the novel hyperchaotic system (1) are obtained by solving the nonlinear equations

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_2 x_3 = 0 \\ f_2(x) &= c x_2 - x_1 x_3 + x_4 = 0 \\ f_3(x) &= x_1 x_2 - b x_3 = 0 \\ f_4(x) &= -d(x_1 + x_2) = 0 \end{aligned} \tag{12}$$

Solving the nonlinear system of equations (12), we obtain three equilibrium points as

$$\begin{aligned} &E_0 : (0,0,0,0) \\ &E_1 : (\sqrt{2ab}, -\sqrt{2ab}, -2a, \sqrt{2ab}(c-2a)) \\ &E_2 : (-\sqrt{2ab}, \sqrt{2ab}, -2a, -\sqrt{2ab}(c-2a)) \end{aligned} \tag{13}$$

We take the parameter values as in the hyperchaotic case, *viz.*

$$a = 39, \;\; b = 4, \;\; c = 21, \;\; d = 2 \tag{14}$$

Using the values (11), we obtain three equilibrium points of the novel chaotic system (1) as

$$\begin{aligned} &E_0 : (0,0,0,0) \\ &E_1 : (17.6635, -17.6635, -78, -1006.82) \\ &E_2 : (-17.6635, 17.6635, -78, 1006.82) \end{aligned} \tag{15}$$

The Jacobian matrix of the novel chaotic system (1) is obtained as

$$J = \begin{bmatrix} -a & a + x_3 & x_2 & 0 \\ -x_3 & c & x_1 & 1 \\ x_2 & x_1 & -b & 0 \\ -d & -d & 0 & 0 \end{bmatrix} \tag{16}$$

The Jacobian matrix at the equilibrium E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -39 & 39 & 0 & 0 \\ 0 & 21 & 0 & 1 \\ 0 & 0 & -4 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix} \tag{17}$$

The matrix J_0 has the eigenvalues

$$\lambda_1 = -4, \;\; \lambda_2 = -39.0333, \;\; \lambda_3 = 0.1918, \;\; \lambda_4 = 20.8415 \tag{18}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable.

The Jacobian matrix at the equilibrium E_1 is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} -39 & -39 & -17.6635 & 0 \\ 78 & 21 & 17.6635 & 1 \\ -17.6635 & 17.6635 & -4 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix} \tag{19}$$

The matrix J_1 has the eigenvalues

$$\lambda_1 = -10.0303,\ \lambda_2 = 0.0801,\ \lambda_{3,4} = -6.0249 \pm 38.9565\, i \tag{20}$$

This shows that the equilibrium E_1 is a saddle-focus, which is unstable.

The Jacobian matrix at the equilibrium E_2 is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -39 & -39 & 17.6635 & 0 \\ 78 & 21 & -17.6635 & 1 \\ 17.6635 & -17.6635 & -4 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix} \tag{21}$$

The matrix J_2 has the eigenvalues

$$\lambda_1 = -10.0303,\ \lambda_2 = 0.0801, \lambda_{3,4} = -6.0249 \pm 38.9565\, i \tag{22}$$

This shows that the equilibrium E_2 is a saddle-focus, which is unstable.

Hence, E_0, E_1, E_2 are all unstable equilibrium points, where E_0 is a saddle point and E_1, E_2 are saddle-focus points.

(E) Lyapunov Exponents

We take the parameter values of the system (1) as

$$a = 39,\ b = 4,\ c = 21,\ d = 2 \tag{23}$$

We take the initial state as

$$x_1(0) = 1.5, x_3(0) = 0.4,\ x_3(0) = 1.8,\ x_4(0) = 2.5 \tag{24}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 2.2848,\ L_2 = 0.1437,\ L_3 = 0,\ L_4 = -24.3318 \tag{25}$$

Eq. (25) shows that the system (1) is hyperchaotic, since it has two positive Lyapunov exponents. Since the sum of the Lyapunov exponents is negative, the system (1) is a dissipative hyperchaotic system.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 2.2848$.

The dynamics of the Lyapunov exponents is depicted in Figure 5.

(F) Lyapunov Dimension

The Lyapunov dimension of the hyperchaotic system (1) is determined as

$$D_L = j + \frac{\sum_{i=1}^{j} L_i}{|L_{j+1}|} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.0998, \tag{26}$$

which is fractional. Thus, the ten-term 4-D system (1) is a dissipative hyperchaotic system with fractional Lyapunov dimension.

4. ADAPTIVE CONTROL OF THE NOVEL HYPERCHAOTIC SYSTEM

In this section, we derive new results for the adaptive controller to stabilize the unstable novel chaotic system with unknown parameters for all initial conditions.

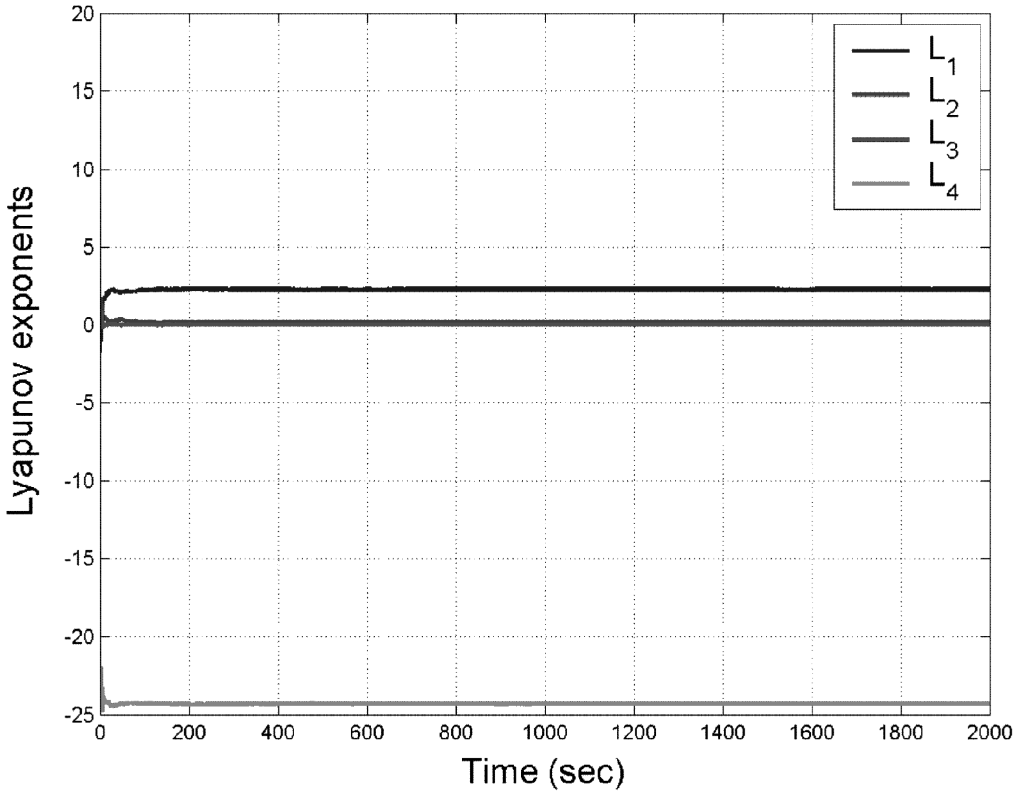

Figure 5: Dynamics of the Lyapunov Exponents

Thus, we consider the controlled novel 4-D hyperchaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{x}_2 &= c x_2 - x_1 x_3 + x_4 + u_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 + u_3 \\ \dot{x}_4 &= -d(x_1 + x_2) + u_4 \end{aligned} \tag{27}$$

where x_1, x_2, x_3, x_4 are state variables, a, b, c, d are constant, unknown, parameters of the system and u_1, u_2, u_3, u_4 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{aligned} u_1 &= -A(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2 &= -C(t) x_2 + x_1 x_3 - x_4 - k_2 x_2 \\ u_3 &= -x_1 x_2 + B(t) x_3 - k_3 x_3 \\ u_4 &= D(t)(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{28}$$

where $A(t)$, $B(t)$, $C(t)$, $D(t)$ are estimates for the unknown parameters a, b, c, d, respectively, and k_1, k_2, k_3 are positive gain constants.

The closed-loop system is obtained by substituting (28) into (27) as

$$\begin{aligned} \dot{x}_1 &= (a - A(t))(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= (c - C(t)) x_2 - k_2 x_2 \\ \dot{x}_3 &= -(b - B(t) x_3 - k_3 x_3 \\ \dot{x}_4 &= -(d - D(t))(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{29}$$

To simplify (29), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \\ e_d(t) &= d - D(t) \end{aligned} \tag{30}$$

Substituting (30) into (29), we obtain

$$\begin{aligned} \dot{x}_1 &= e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= e_c x_2 - k_2 x_2 \\ \dot{x}_3 &= -e_b x_3 - k_3 x_3 \\ \dot{x}_4 &= -e_d(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{31}$$

Differentiating the parameter estimation error (30) with respect to t, we get

$$\begin{aligned} \dot{e}_a(t) &= -\dot{A}(t) \\ \dot{e}_b(t) &= -\dot{B}(t) \\ \dot{e}_c(t) &= -\dot{C}(t) \\ \dot{e}_d(t) &= -\dot{D}(t) \end{aligned} \tag{32}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, x_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{33}$$

which is positive definite on R^8.

Differentiating V along the trajectories of (31) and (32), we obtain

$$\begin{aligned} \dot{V} = &-k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a\left[x_1(x_2 - x_1) - \dot{A}\right] + e_b\left[-x_3^2 - \dot{B}\right] \\ &+ e_c\left[x_2^2 - \dot{C}\right] + e_d\left[-x_4(x_1 + x_2) - \dot{D}\right] \end{aligned} \tag{34}$$

In view of (34), we define an update law for the parameter estimates as

$$\begin{aligned} \dot{A} &= x_1(x_2 - x_1) + k_5 e_a \\ \dot{B} &= -x_3^2 + k_6 e_b \\ \dot{C} &= x_2^2 + k_7 e_c \\ \dot{D} &= -x_4(x_1 + x_2) + k_8 e_d \end{aligned} \tag{35}$$

where k_5, k_6, k_7, k_8 are positive gain constants.

Theorem 1. The novel 4-D hyperchaotic system (27) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (28) and the parameter update law (35), where k_i, (i = 1, 2,..., 8) are positive constants. All the parameter estimation errors e_a, e_b, e_c, e_d globally and exponentially converge to zero with time.

Proof. The result is proved using Lyapunov stability theory [45].

We consider the quadratic Lyapunov function V defined by (33), which is a positive definite function on R^8.

Substituting the parameter update law (35) into (34), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{36}$$

which is a negative definite function on R^8.

Thus, by Lyapunov stability theory [45], all the states x_1, x_2, x_3, x_4 and the parameter estimation errors e_a, e_b, e_c, e_d globally and exponentially converge to zero with time.

This completes the proof.

Numerical Results

For the novel chaotic system (27), the parameter values are taken as in the chaotic case, *viz.*

$$a = 39, \; b = 4, \; c = 21, \; d = 2 \tag{37}$$

We take the feedback gains as

$$k_i = 5 \text{ for } i = 1, 2, \ldots, 8.$$

The initial values of the chaotic system (27) are taken as

$$x_1(0) = 2.5, \; x_2(0) = -1.8, \; x_3(0) = -3.5, \; x_4(0) = 5.6 \tag{38}$$

The initial values of the parameter estimates are taken as

$$A(0) = 14, \; B(0) = 27, \; C(0) = 18, \; D(0) = 30 \tag{39}$$

Figure 6 depicts the time-history of the controlled novel hyperchaotic system.

Figure 7 depicts the time-history of the parameter estimates $A(t)$, $B(t)$, $C(t)$, $D(t)$.

Figure 8 depicts the time-history of the parameter estimation errors $e_a(t)$, $e_b(t)$, $e_c(t)$, $e_d(t)$.

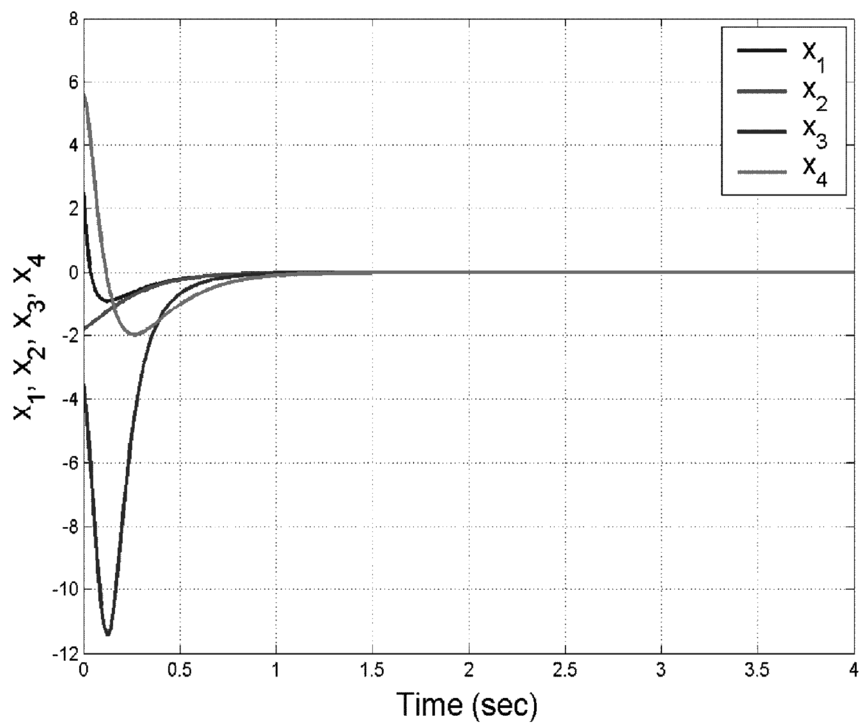

Figure 6: Time-History of the Controlled Novel Hyperchaotic System

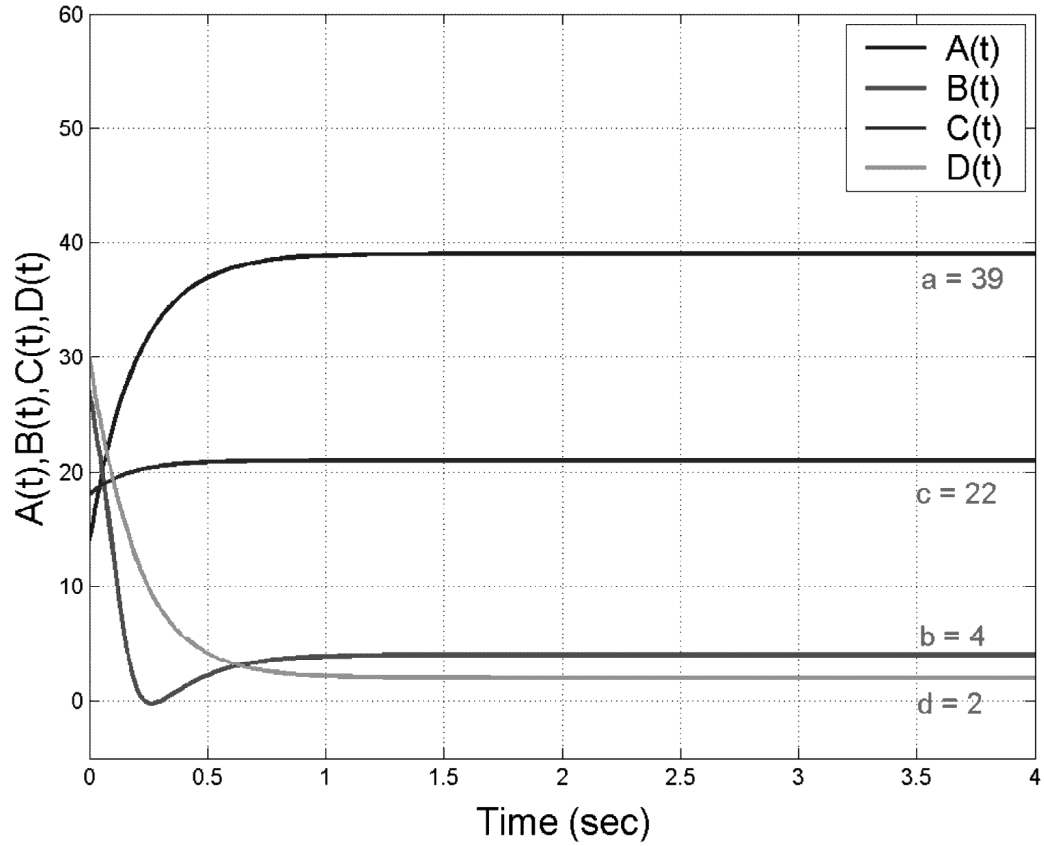

Figure 7: Time-History of the Parameter Estimates *A*(*t*), *B*(*t*), *C*(*t*), *D*(*t*)

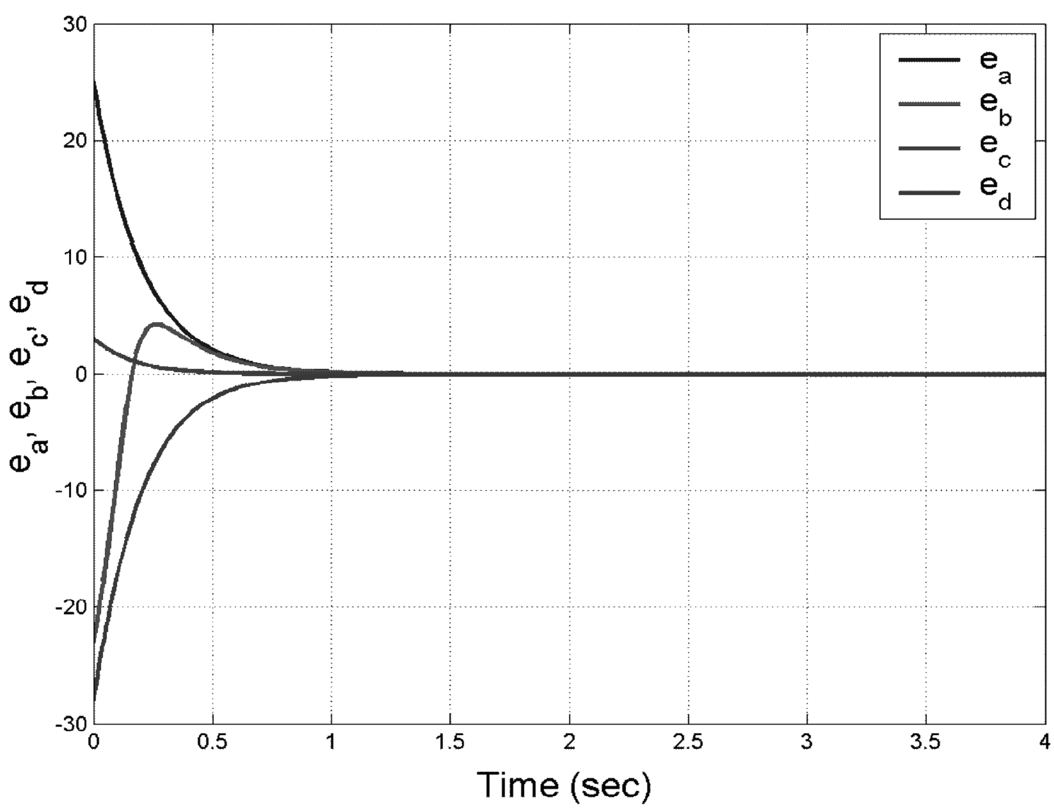

Figure 8: Time History of the Parameter Estimation Errors e_a, e_b, e_c, e_d

5. CONCLUSIONS

In this paper, we have introduced introduces a ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities. We have given a detailed qualitative analysis of the proposed system in this paper. The novel hyperchaotic system has three unstable equilibria. The novel hyperchaotic system has the Lyapunov exponents given by $L_1 = 2.2848$, $L_2 = 0.1437$, $L_3 = 0$ and $L_4 = -24.3318$. Since the sum of the Lyapunov exponents is negative, the novel hyperchaotic system is a dissipative system. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 2.2848$ and Lyapunov dimension as $D_L = 3.0998$. In this paper, we have derived a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. MATLAB simulations have been shown to illustrate the phase portraits of the novel 4-D hyperchaotic system and the adaptive control results for the novel hyperchaotic system.

REFERENCES

[1] K. T. Alligood, T. Sauer and J. A. Yorke, *Chaos: An Introduction to Dynamical Systems*, Springer-Verlag: New York, USA, 1997.

[2] E.N. Lorenz, "Deterministic nonperiodic flow," *Journal of Atmospheric Sciences*, **20**, 130-141, 1963.

[3] O. E. Rössler, "An Equation for Continuous Chaos," *Physics Letters*, **57A**, 397-398, 1976.

[4] T. Shimizu and N. Morioka, "On the Bifurcation of a Symmetric Limit Cycle to an Assymetric One in a Simple Model," *Physics Letters A*, **76**, 201-204, 1980.

[5] R. Shaw, "Strange Attractors, Chaotic Behavior and Information Flow," *Z. Naturforsch*, **36A**, 80-112, 1981.

[6] G. Chen and T. Ueta, "Yet another Chaotic Oscillator," *International Journal of Bifurcation and Chaos*, **9**, 1465-1466, 1999.

[7] J. Lü and G. Chen, "A New Chaotic Attractor Coined," *International Journal of Bifurcation and Chaos*, **12**, 659-661, 2002.

[8] H. K. Chen and C. I. Lee, "Anti-control of Chaos in Rigid Body Motion," *Chaos, Solitons and Fractals*, **21**, 957-965, 2004.

[9] G. Cai and Z. Tan, "Chaos Synchronization of a New Chaotic System via Nonlinear Control," *Journal of Uncertain Systems*, **1**, 235-240, 2007.

[10] G. Tigan and D. Opris, "Analyis of a 3D Chaotic System," *Chaos, Solitons and Fractals*, **36**, 1315-1319, 2008.

[11] D. Li, "A Three-scroll Chaotic Attractor," *Physics Letters A*, **372**, 387-393, 2008.

[12] V. Sundarapandian and I. Pehlivan, "Analysis, Control, Synchronization and Circuit Design of a Novel Chaotic System," *Mathematics and Computer Modelling*, **55**, 1904-1915, 2012.

[13] O. E. Rössler, "An Equation for Hyperchaos," *Physics Letters A*, **71**, 155-157, 1979.

[14] Q. Jia, "Hyperchaos Generated from the Lorenz Chaotic System and its Control," *Physics Letters A*, **366**, 217-222, 2007.

[15] A. Chen, J. Lu, J. Lü and S. Yu, "Generating Hyperchaotic Lü Attractor via State Feedback Control," *Physica A*, **364**, 103-110, 2006.

[16] X. Li, "Modified Projective Synchronization of a New Hyperchaotic System via Nonlinear Control," *Commun. Theor. Physics*, **52**, 274-278, 2009.

[17] J. Wang and Z. Chen, "A Novel Hyperchaotic System and its Complex Dynamics," *International Journal of Bifurcation and Chaos*, **18**, 3309-3324, 2008.

[18] Q. Li, X.S. Yang and F. Yang, "Hyperchaos in Hopfield-type Neural Networks," *Neurocomputing*, **67**, 275-280, 2004.

[19] Y. Huang and X.S. Yang, "Hyperchaos and Bifurcation in a New Class of Four-dimensional Hopfield Neural Networks," *Neurocomputing*, **69**, 1787-1795, 2006.

[20] P. C. Rech, "Chaos and Hyperchaos in a Hopfield Neural Network," *Neurocomputing*, **74**, 3361-3364, 2011.

[21] K. Grygiel and P. Szlachetka, "Chaos and Hyperchaos in Coupled Kerr Oscillators," *Optics Communications*, **177**, 425-431, 2000.

[22] Y. L. Zou, J. Zhu, G. Chen, X. S. Luo, "Synchronization of Hyperchaotic Oscillators via Single Unidirectional Chaotic Coupling," *Chaos, Solitons and Fractals*, **25**, 1245-1253, 2005.

[23] A. S. Elwakil and M. P. Kennedy, "Inductorless Hyperchaos Generator," *Microeletronics Journal*, **30**, 739-743, 1999.

[24] N. Yujun, W. Xingyuan, W. Mingjun and Z. Huaguang, "A New Hyperchaotic System and its Circuit Implementation," *Communications in Nonlinear Science and Numerical Simulation*, **15**, 3518-3524, 2010.

[25] K. Thamilmaran, M. Lakshmanan and A. Venkatesan, "Hyperchaos in a Modified Canonical Chua's Circuit," *International Journal of Bifurcation and Chaos*, **14**, 221-243, 2004.

[26] T. Banerjee, D. Biswas and B.C. Sarkar, "Design of Chaotic and Hyperchaotic Time-delayed Electronic Circuit," *Bonfring International Journal of Power Systems and Integrated Circuits*, **2**, 13-17, 2012.

[27] C. Li, X. Liao and K. Wong, "Lag Synchronization of Hyperchaos with Application to Secure Communications," *Chaos, Solitons and Fractals*, **23**, 183-193, 2005.

[28] X. Wu, H. Wang and H. Lu, "Modified Generalized Projective Synchronization of a New fractional-order Hyperchaotic System and its Application to Secure Communication," *Nonlinear Analysis: Real World Applications*, **13**, 1441-1450, 2012.

[29] Q. Zhang, L. Guo and X. Wei, "A Novel Image Fusion Encryption Algorithm based on DNA Sequence Operation and Hyper-chaotic System," *Optik-International Journal for Light and Electron Optics*, **124**, 3596-3600, 2013.

[30] M. T. Yassen, "On hyperchaos Synchronization of a Hyperchaotic Lü System," *Nonlinear Analysis: Theory, Methods and Applications*, **68**, 3592-3600, 2008.

[31] V. Sundarapandian and R. Karthikeyan, "Global Chaos Synchronization of Hyperchaotic Liu and Lorenz Systems by Active Nonlinear Control," *International Journal of Control Theory and Applications*, **3**, 79-91, 2010.

[32] V. Sundarapandian, "Adaptive Stabilization and Synchronization of Hyperchaotic Qi System," *Computer Science Engineering: An International Journal*, **1**, 14-25, 2011.

[33] V. Sundarapandian and S. Sivaperumal, "Global Chaos Synchronization of Hyperchaotic Qi Systems by Sliding Mode Control," *International Journal on Computer Science and Engineering*, **3**, 2430-2437, 2011.

[34] P. Sarasu and V. Sundarapandian, "The Generalized Projective Synchronization of Hyperchaotic Lorenz and Hyperchaotic Qi systems via active Control," *International Journal of Soft Computing*, **6**, 216-223, 2011.

[35] V. Sundarapandian and R. Karthikeyan, "Global Chaos Synchronization of Hyperchaotic Pang and Hyperchaotic Wang Systems via Adaptive Control," *International Journal of Soft Computing*, **7**, 28-37, 2012.

[36] E. Ott, C. Grebogi and J.A. Yorke, "Controlling Chaos," *Physical Review Letters*, **64**, 1196-1199, 1990.

[37] J. Wang, T. Zhang and Y. Che, "Chaos Control and Synchronization of two Neurons Exposed to ELF External Electric Field," *Chaos, Solitons and Fractals*, **34**, 839-850, 2007.

[38] V. Sundarapandian, "Output Regulation of the Tigan System," *International Journal on Computer Science and Engineering*, **3**, 2127-2135, 2011.

[39] V. Sundarapandian, "Output Regulation of the Sprott-G chaotic System by State Feedback Control," *International Journal of Instrumentation and Control Systems*, **1**, 20-30, 2011.

[40] V. Sundarapandian, "Adaptive Control and Synchronization of Uncertain Liu-Chen-Liu System," *International Journal of Computer Information Systems*, **3**, 1-6, 2011.

[41] V. Sundarapandian, "Adaptive Control and Synchronization of the Shaw Chaotic System," *International Journal in Foundations of Computer Science and Technology*, **1**, 1-11, 2011.

[42] V. Sundarapandian, "Adaptive Control and Synchronization of Uncertain Liu-Liu-Liu-Su Chaotic System," *International Journal of Control Theory and Applications*, **4**, 99-109, 2011.

[43] V. Sundarapandian, "Adaptive Control and Synchronization Design for the Lu-Xiao Chaotic System," *Springer-Verlag Lecture Notes in Electrical Engineering*, **131**, 319-327, 2013.

[44] V. Sundarapandian, "Sliding Mode Control based Global Chaos Control of Liu-Liu-Liu-Su Chaotic System," *International Journal of Control Theory and Applications*, **5**, 15-20, 2012.

[45] W. Hahn, *The Stability of Motion*, Springer-Verlag, New York, 1967.