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DEFORMATION OF AN INFINITE POROELASTIC MEDIUM HAVING DOUBLE POROSITY WITH GENERALIZED THERMOELATICITY IN THE PRESENCE OF A SPHERICAL CAVITY

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Abstract: A problem of deformation of an infinite thermoelastic body with double porosity, having a spherical cavity subjected to harmonic source has been considered using the Lord and Shulman theory of thermoelasticity (1967). The expressions for radial stress, hoop stress, equilibrated stresses and temperature distribution has been obtained by using Laplace transform technique. Further numerical inversion technique is used to obtain the resulting components in the physical domain and the results are depicted graphically to show the effect of porosity and thermal relaxation time.

Keywords: thermoelastic; double porosity; spherical cavity; harmonic source; stress and temperature distribution.

INTRODUCTION

Thermoelasticity deals with the dynamical system whose interaction with the surrounding include not only mechanical work and external work but the exchange of heat also. Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of thermal signals inherent in the conventional coupled theory of thermoelasticity. The first generalization was proposed by Lord and Shulman (1967), which involves one thermal relaxation time parameter.

A porous medium is regarded as a material whose solid portion is continuously connected throughout the whole volume to form a solid matrix with voids through which the liquid or gas may flow. Mathematical theory of poroelasticity deals with the mechanical behaviour of saturated porous medium. With the development of science and technology, the dynamic analysis of porous media plays an important role in various areas such as geophysics, soil mechanics, civil engineering, petroleum engineering, environmental engineering, earthquakes and geomechanics etc. Biot

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(1941) proposed a general theory of three dimensional deformation of fluid saturated porous salts.

One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et. al. (1960), where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers. Also, in the study of many important problems concerning the civil engineering, there are new possibilities with the double porosity model. Wilson and Aifanits (1984a) presented the theory of consolidation with the double porosity. Khaled et. al. (1984) employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Wilson and Aifantis (1984a). Wilson and Aifantis (1984b) discussed the propagation of acoustics waves in a fluid saturated porous medium. Beskos and Aifantis (1986) presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan (1996) studied the unified theory of flow and deformation in double porous media. Khalili and Selvadurai (2003) presented a fully coupled constitutive model for thermo-hydro–mechanical analysis in elastic media with double porosity structure.

Some authors worked on the problems of thermoelasticity in a double porous medium such as Svanadze (2005 and 2010), Straughan (2013) solved various problems on elastic solids and thermoelastic solids with double porosity. Iesan and Quintanilla (2014) used the Nunziato-Cowin theory of materials with voids to derive a theory of thermoelastic solids, which have a double porosity structure.

Youseff (2005a) investigated the problem of an infinite body with a cylindrical cavity and variable material properties in generalized thermoelasticity. Youseff (2005b) studied the problem of an infinite material with a spherical cavity and variable thermal conductivity subjected to ramp-type heating. Allam et. al. (2010) considered the model of generalized thermoelasticity proposed by Green and Naghdi, to study the electromagneto–thermoelastic interactions in an infinite perfectly conducting body with a spherical cavity. Abd-Alla and Abo-Dahab (2012) investigated the effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity. Scarpetta and Svanadze (2014) obtained the fundamental solutions in the theory of thermoelasticity for solids with double porosity. Kumar et. al. (2016) studied the reflection of plane waves in thermoelastic medium with double porosity.

In this paper, a problem of deformation of an infinite thermoelastic body with double porosity having a spherical cavity subjected to harmonic source has been considered using the Lord and Shulman theory of thermoelasticity. Transformed technique has been used to find the components of stress and thermal temperature distribution in the transformed domain. A numerical inversion technique is used to obtain the components in the physical domain, and the results are depicted graphically to show the effect of porosity and relaxation time.

Basic Equations

The constitutive relations and field equations for homogeneous isotropic thermoelastic material with double porosity structure in the absence of body forces, extrinsic equilibrated body forces and heat sources are taken, following the Iesan and Quintanilla (2014) and the theory given by Lord and Shulman (1967), as

Constitutive Relations:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i} \tag{2}$$

$$\chi_i = b_1\varphi_{,i} + \gamma\psi_{,i} \tag{3}$$

Equation of motion:

$$\mu\nabla^2 u_i + (\lambda+\mu)u_{j,ji} + b\varphi_{,i} + d\psi_{,i} - \beta T_{,i} = \rho\ddot{u}_i, \tag{4}$$

Equilibrated stress equations of motion:

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - bu_{r,r} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\ddot{\varphi}, \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - du_{r,r} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\ddot{\psi}, \tag{6}$$

Equation of heat conduction:

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\beta T_0\dot{u}_{j,j} + \gamma_1 T_0\dot{\varphi} + \gamma_2 T_0\dot{\psi} + \rho C^*\dot{T}\right) = K^*\nabla^2 T \tag{7}$$

where λ and μ are Lame's constants, ρ is the mass density;

$\beta = (3\lambda + 2\mu)\alpha_t$; α_t is the linear thermal expansion; C^* is the specific heat at constant strain, u_i is the displacement components; t_{ij} is the stress tensor; κ_1 and κ_2 are coefficients of equilibrated inertia; σ_i is the components of the equilibrated stress vector associated to pores; χ_i is the components of the equilibrated stress vector associated to fissures; φ is the volume fraction field corresponding to pores and ψ is the volume fraction field corresponding to fissures; K^* is the coefficient of thermal conductivity, τ_0 is the thermal relaxation time, κ_1 and κ_2 are coefficients of equilibrated inertia and $b, d, b_1, \gamma, \gamma_1, \gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; T is the temperature change measured form the absolute temperature $T_0\left(T_0 \neq 0\right)$; a superposed dot represents differentiation with respect to time variable t.

Formulation of the problem

A homogeneous, isotropic thermoelastic infinite body having double porosity structure with a spherical cavity of radius 'a' has been considered. The spherical cavity is subjected to a harmonic source. Accordingly, the problem considered is a spherically symmetric problem.

Thus, spherical polar coordinates (r, ϑ, ϕ) are taken to represent a point of the body at time t, also the origin of the coordinate system is taken at the centre of the spherical cavity. As such all the variables considered will be functions of the radial distance r and the time t.

Due to spherical symmetry, the displacement components are of the form

$$u_r = u(r,t), \quad u_\vartheta = u_\phi = 0 \tag{8}$$

The components of stress for a spherical symmetric system become

$$t_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e + b\varphi + d\psi - \beta T \tag{9}$$

$$t_{\vartheta\vartheta} = t_{\phi\phi} = 2\mu\frac{u}{r} + \lambda e + b\varphi + d\psi - \beta T \tag{10}$$

$$t_{r\vartheta} = t_{r\phi} = t_{\vartheta\phi} = 0 \tag{11}$$

$$\sigma_r = \alpha\frac{\partial\varphi}{\partial r} + b_1\frac{\partial\psi}{\partial r} \tag{12}$$

$$\chi_r = b_1\frac{\partial\varphi}{\partial r} + \gamma\frac{\partial\psi}{\partial r} \tag{13}$$

where

$$\begin{gathered} e = e_{rr} + e_{\vartheta\vartheta} + e_{\phi\phi} = \frac{\partial u}{\partial r} + \frac{2u}{r}, \\ e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\vartheta\vartheta} = e_{\phi\phi} = \frac{u}{r}, \quad e_{r\vartheta} = e_{r\phi} = e_{\vartheta\phi} = 0 \end{gathered} \tag{14}$$

Introducing the non-dimensional quantities as

$$r' = \frac{\omega_1}{c_1} r, \quad u' = \frac{\omega_1}{c_1} u \ , t'_{ij} = \frac{t_{ij}}{\beta T_0}, \varphi' = \frac{\kappa_1 \omega_1{}^2}{\alpha_1} \varphi, \ \psi' = \frac{\kappa_1 \omega_1{}^2}{\alpha_1},$$

$$T' = \frac{T}{T_0}, t' = \omega_1 t, \sigma'_i = \left(\frac{c_1}{\alpha \omega_1}\right) \sigma_i, \chi'_i = \left(\frac{c_1}{\alpha \omega_1}\right) \chi_i, \tau'_0 = \omega_1 \tau_0 \tag{15}$$

where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}, \omega_1 = \dfrac{\rho C^* c_1^2}{K^*}$

Using the dimensionless quantities given by (15) on Eqs. (4)-(7) and Eq.(14) yields (dropping primes for convenience)

$$\frac{\partial e}{\partial r} + a_1 \frac{\partial \varphi}{\partial r} + a_2 \frac{\partial \psi}{\partial r} - a_3 \frac{\partial T}{\partial r} = \frac{\partial^2 u}{\partial t^2} \tag{16}$$

$$a_4 \nabla^2 \varphi + a_5 \nabla^2 \psi - a_6 e - a_7 \varphi - a_8 \psi + a_9 T = \frac{\partial^2 \varphi}{\partial t^2} \tag{17}$$

$$a_{10} \nabla^2 \varphi + a_{11} \nabla^2 \psi - a_{12} e - a_{13} \varphi - a_{14} \psi + a_{15} T = \frac{\partial^2 \psi}{\partial t^2} \tag{18}$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\left(a_{16} \frac{\partial e}{\partial t} + a_{17} \frac{\partial \varphi}{\partial t} + a_{18} \frac{\partial \psi}{\partial t} + \frac{\partial T}{\partial t}\right) = \nabla^2 T \tag{19}$$

where

$$a_1 = \frac{b\alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_2 = \frac{d\alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_3 = \frac{\beta T_0}{\rho c_1^2}, a_4 = \frac{\alpha}{\kappa_1 c_1^2}, a_5 = \frac{b_1}{\kappa_1 c_1^2}, a_6 = \frac{b}{\alpha_1}, a_7 = \frac{\alpha_1}{\kappa_1 \omega_1^2},$$
$$a_8 = \frac{\alpha_3}{\kappa_1 \omega_1^2}, a_9 = \frac{\gamma_1 T_0}{\alpha_1}, a_{10} = \frac{b_1}{\kappa_2 c_1^2}, a_{11} = \frac{\gamma}{\kappa_2 c_1^2}, a_{12} = \frac{d\kappa_1}{\kappa_2 \alpha_1}, a_{13} = \frac{\alpha_3}{\kappa_2 \omega_1^2}, a_{14} = \frac{\alpha_2}{\kappa_2 \omega_1^2},$$
$$a_{15} = \frac{\gamma_2 T_0 \kappa_1}{\alpha_1 \kappa_2}, a_{16} = \frac{\beta c_1^2}{\rho C^*}, a_{17} = \frac{\gamma_1 \alpha_1 c_1^2}{K^* \kappa_1 \omega_1^3}, a_{18} = \frac{\gamma_2 \alpha_1 c_1^2}{K^* \kappa_1 \omega_1^3}$$

Initial conditions

The initial conditions of the problem, i.e., at t = 0, we have

$$\begin{aligned} &u = 0 = \frac{\partial u}{\partial t}, \quad \varphi = 0 = \frac{\partial \varphi}{\partial t}, \\ &\psi = 0 = \frac{\partial \psi}{\partial t}, \quad T = 0 = \frac{\partial T}{\partial t} \quad \text{at } t = 0. \end{aligned} \tag{20}$$

Solution of the problem

To solve the problem, we define the Laplace transform for the function f(t) as:

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t) e^{-st} dt \tag{21}$$

Applying the laplace transformation as defined above on the Eqs. (16)-(19) under the initial conditions (20), we obtain

$$\left(\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4\right)\left(\bar{e}, \bar{\varphi}, \bar{\psi}, \bar{T}\right) = 0 \tag{22}$$

where

B_1, B_2, B_3 and B_4 are given in appendix I.

The above system of equations can be factorized as

$$\left\{\left(\nabla^2 - \lambda_1^2\right)\left(\nabla^2 - \lambda_2^2\right)\left(\nabla^2 - \lambda_3^2\right)\left(\nabla^2 - \lambda_4^2\right)\right\}\left(\bar{e}, \bar{\varphi}, \bar{\psi}, \bar{T}\right) = 0 \tag{23}$$

where λ_1^2, λ_2^2, λ_3^2, λ_4^2 are the roots of the following characteristic equation

$$\lambda^8 + B_1\lambda^6 + B_2\lambda^4 + B_3\lambda^2 + B_4 = 0 \tag{24}$$

$$\bar{e}(r,s) = \frac{1}{r}\left(C_1 \exp(-\lambda_1 r) + C_2 \exp(-\lambda_2 r) + C_3 \exp(-\lambda_3 r) + C_4 \exp(-\lambda_4 r)\right) \tag{25}$$

$$\bar{\varphi}(r,s) = \frac{1}{r}\left(g_{11}C_1 \exp(-\lambda_1 r) + g_{12}C_2 \exp(-\lambda_2 r) + g_{13}C_3 \exp(-\lambda_3 r) + g_{14}C_4 \exp(-\lambda_4 r)\right) \tag{26}$$

$$\bar{\psi}(r,s) = \frac{1}{r}\left(g_{21}C_1 \exp(-\lambda_1 r) + g_{22}C_2 \exp(-\lambda_2 r) + g_{23}C_3 \exp(-\lambda_3 r) + g_{24}C_4 \exp(-\lambda_4 r)\right) \tag{27}$$

$$\bar{T}(r,s) = \frac{1}{r}\left(g_{31}C_1 \exp(-\lambda_1 r) + g_{32}C_2 \exp(-\lambda_2 r) + g_{33}C_3 \exp(-\lambda_3 r) + g_{34}C_4 \exp(-\lambda_4 r)\right) \tag{28}$$

where g_{1i}, g_{2i}, g_{3i}, $(i = 1,2,3,4)$ are given in appendix II.

On solving Eq. (14) with the aid of Eq.(25) and assuming that $u(r,t)$ vanishes at infinity, we obtain

$$\bar{u}(r,s) = -\frac{1}{r^2}\begin{bmatrix} \dfrac{(\lambda_1 r+1)}{\lambda_1^2} C_1 \exp(-\lambda_1 r) + \dfrac{(\lambda_2 r+1)}{\lambda_2^2} C_2 \exp(-\lambda_2 r) \\ + \dfrac{(\lambda_3 r+1)}{\lambda_3^2} C_3 \exp(-\lambda_3 r) + \dfrac{(\lambda_4 r+1)}{\lambda_4^2} C_4 \exp(-\lambda_4 r) \end{bmatrix} \tag{29}$$

On using Eqs. (25)-(28) in Eqs. (9), (12), (13) and with the help of Eqs. (15) and (21), we obtain the corresponding expressions for radial stress and equilibrated stresses as

$$\bar{t}_{rr}(r,s)=\sum_{i=1}^{4}\left(p_1\left(\frac{\lambda_i^2+2(\lambda_i+1)}{\lambda_i^2}\right)+p_2+p_3g_{1i}+p_4g_{2i}-g_{3i}\right)C_i(s)\exp(-\lambda_i r) \tag{30}$$

$$\bar{\sigma}_r(r,s)=\sum_{i=1}^{4}\lambda_i(p_5g_{1i}+p_6g_{2i})C_i(s)\exp(-\lambda_i r) \tag{31}$$

$$\bar{\chi}_r(r,s)=\sum_{i=1}^{4}\lambda_i(p_6g_{1i}+p_7g_{2i})C_i(s)\exp(-\lambda_i r) \tag{32}$$

where

$$p_1=\frac{2\mu}{\beta T_0},\ \ p_2=\frac{\lambda}{\beta T_0},\ \ p_3=\frac{b\alpha_1}{\beta T_0\kappa_1\omega_1^2},\ \ p_4=\frac{d\alpha_1}{\beta T_0\kappa_1\omega_1^2},$$
$$p_5=\frac{\alpha_1}{\kappa_1\omega_1^2},\ \ p_6=\frac{b_1\alpha_1}{\alpha\kappa_1\omega_1^2},\ \ p_7=\frac{\gamma\alpha_1}{\alpha\kappa_1\omega_1^2}$$

Boundary conditions

Since, a harmonic source is applied at the surface of the cavity, i.e. at r = a, so the boundary conditions at r = a are given by

i. $t_{rr}(a,t)=\cos at$

ii. $\sigma_r(a,t)=0$ (33)

iii. $\chi_r(a,t)=0$

iv. $T(a,t)=0$

In Laplace transform domain, the boundary conditions become

i. $\bar{t}_{rr}(a,s)=\dfrac{s}{a^2+s^2}=F_1(\text{say}),$

ii. $\bar{\sigma}_r(a,s)=0$

iii. $\bar{\chi}_r(a,s)=0$

iv. $\bar{T}(a,s)=0$

Substituting the values of $\bar{t}_{rr},\bar{\sigma}_r,\bar{\chi}_r$ and $\bar{T}$ from Eqs. (28), (30)-(32) in the boundary conditions (34) yield the corresponding expressions for radial stress, equilibrated stresses and temperature distribution as

$$\bar{t}_{rr}(r,s)=\frac{1}{\Gamma}\left(H_{11}\Gamma_1\exp(-\lambda_1 r)+H_{12}\Gamma_2\exp(-\lambda_2 r)+H_{13}\Gamma_3\exp(-\lambda_3 r)+H_{14}\Gamma_4\exp(-\lambda_4 r)\right) \tag{35}$$

$$\bar{\sigma}_r(r,s)=\frac{1}{\Gamma}\left(H_{21}\Gamma_1\exp(-\lambda_1 r)+H_{22}\Gamma_2\exp(-\lambda_2 r)+H_{23}\Gamma_3\exp(-\lambda_3 r)+H_{24}\Gamma_4\exp(-\lambda_4 r)\right) \tag{36}$$

$$\bar{\chi}_r(r,s)=\frac{1}{\Gamma}\left(H_{31}\Gamma_1\exp(-\lambda_1 r)+H_{32}\Gamma_2\exp(-\lambda_2 r)+H_{33}\Gamma_3\exp(-\lambda_3 r)+H_{34}\Gamma_4\exp(-\lambda_4 r)\right) \tag{37}$$

$$\bar{T}(r,s)=\frac{1}{\Gamma}\left(H_{41}\Gamma_1\exp(-\lambda_1 r)+H_{42}\Gamma_2\exp(-\lambda_2 r)+H_{43}\Gamma_3\exp(-\lambda_3 r)+H_{44}\Gamma_4\exp(-\lambda_4 r)\right) \tag{38}$$

where

$$\Gamma=\begin{vmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{vmatrix} \tag{39}$$

$\Gamma_i\left(i=1,2,3,4\right)$ are obtained by replacing i^{th} column of (39) with $\left[F_1\ 0\ 0\ F_2\right]^T$ where $H_{1i},H_{2i},H_{3i},H_{4i};\left(i=1,2,3,4\right)$ are given in the appendix II.

This completes the solution of the problem in Laplace domain.

Special cases

Case 5.1: If we take $b_1=\alpha_3=\gamma=\alpha_2=\gamma_2=d\rightarrow 0$ in the governing equations then the problem is reduced to an infinite thermoelastic single porous body with a spherical cavity. Accordingly expressions (35)-(38) gives the transformed stress and temperature distributions for an infinite thermoelastic single porous body with a spherical cavity.

Case 5.2: Taking $\tau_0=0$, in the governing equations and the Eqs.(35)-(38) yield the corresponding expressions for an infinite thermoelastic double porous body with a spherical cavity in the context of coupled theory of thermoelasticity.

Inversion of the Laplace transform

In order to invert the Laplace transform, we adopt a numerical inversion method based on a Fourier series expansion {Honig and Hirdes(1984)}.

By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t)=\frac{e^{ct}}{t_1}\left[\frac{1}{2}\bar{f}(\mathrm{c})+\mathrm{Re}\sum_{k=1}^{N}\bar{f}\left(c+\frac{ik\pi}{t_1}\right)\exp\left(\frac{ik\pi t}{t_1}\right)\right],\quad 0<t_1<2t$$

where N is sufficiently large integer representing the number of terms in the truncated Fourier series, chosen such that

$$f(t) = \exp(\mathrm{ct})\,\mathrm{Re}\left[\bar{f}\left(c + \frac{iN\pi}{t_1}\right)\exp\left(\frac{iN\pi t}{t_1}\right)\right] \le \varepsilon_1$$

where ε_1 is a prescribed small positive number that corresponds to the degree of accuracy required. The parameter c is a positive free parameter that must be greater than the real part of all the singularities of $\bar{f}(s)$.The optimal choice of c was obtained to the criterion described in expansion {Honig and Hirdes (1984)}.

Two methods are used to reduce the total error. First, Korrecktur method is used to reduce the discretization error. Next, the e-algorithm is used to reduce the truncation error and hence to accelerate convergence. It should be noted that a good choice of the free parameters N and ct is not only important for the accuracy of the results but also for the application of Korrecktur method and the methods for the acceleration of convergence.

Numerical results and discussion

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh (2005) as,

$$\lambda = 7.76\times10^{10}\,Nm^{-2}, C^* = 3.831\times10^3\,m^2s^{-2}K^{-1}, \mu = 3.86\times10^{10}\,Nm^{-2},$$
$$K^* = 3.86\times10^3\,Ns^{-1}K^{-1}, T_0 = 293\,K, \alpha_t = 1.78\times10^{-5}\,K^{-1}\ , \rho = 8.954\times10^3\,Kgm^{-3}$$

The double porous parameters are taken as,

$$\alpha_2 = 2.4\times10^{10}\,Nm^{-2}, \alpha_3 = 2.5\times10^{10}\,Nm^{-2}, \gamma = 1.1\times10^{-5}\,N, \alpha = 1.3\times10^{-5}\,N$$
$$\gamma_1 = 0.16\times10^5\,Nm^{-2}, b_1 = 0.12\times10^{-5}\,N, d = 0.1\times10^{10}\,Nm^{-2}$$
$$\gamma_2 = 0.219\times10^5\,Nm^{-2}, \kappa_1 = 0.1456\times10^{-12}\,Nm^{-2}s^2, b = 0.9\times10^{10}\,Nm^{-2}$$
$$\alpha_1 = 2.3\times10^{10}\,Nm^{-2},\ \kappa_2 = 0.1546\times10^{-12}\,Nm^{-2}s^2$$

The software MATLAB has been used to find the values of radial stress t_{rr}, hoop stress $t_{\vartheta\vartheta}$, equilibrated stresses σ_r, χ_r and temperature distribution T.The variations of these values with respect to radial distance r are shown in figures (1)-(9). In figs.1-5, effect of relaxtion time is shown graphically. In these figures, solid lines correspond to Lord-Shulman (LS) theory of thermoelasticity and small dashed line correspond to coupled theory (CT) of thermoelasticity. Also, the effect of porosity is depicted graphically in figs.6-9. In figs. (6)-(9), solid lines correspond to thermal double porous material (TDP) and small dashed line correspond to thermal primary porous material (TPP).

Effect of relaxation time

From figs.1 and 2, it is noticed that the trend and behavior of variation of t_{rr} and $t_{\vartheta\vartheta}$ are similar for both LS and CT theories of thermoelasticity. The value of these stresses decreases for the region $r \le 2$ and increases further with increase in r. It is found that the relaxation time decreases the values of stresses for LS theory in comparison to CT theory. Figs. 3 and 4 show that the values of σ_r and χ_r increase for $1 < r \le 2$, decrease for $2 < r \le 3.5$ and become almost stationary as $r > 3.5$. The behavior of variation is same for both LS and CT theories of thermoelasticity. It is clear from the figures that for σ_r, the values are more for CT theory as compared to the values of LS theory while incase of χ_r, an opposite behavior is noticed. Fig.5 depicts that, the value of T increases for the region and then decreases onwards. The values are more for LS theory due to the effect of relaxation time.

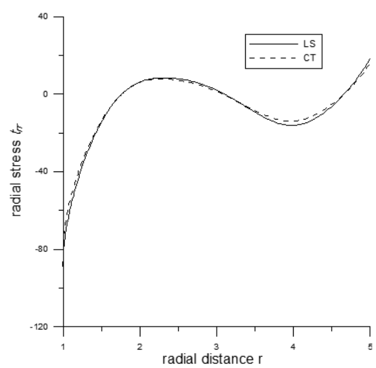

Fig.1 Variation of radial stress t_{rr} radial distance *r*

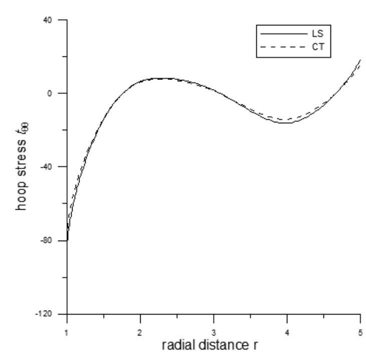

Fig.2 Variation of hoop stress $t_{\vartheta\vartheta}$ w.r.t. w.r.t. radial distance *r*

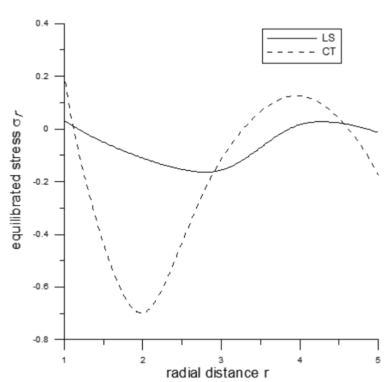

Fig.3 Variation of equilibrated stress σ_r w.r.t. radial distance *r*

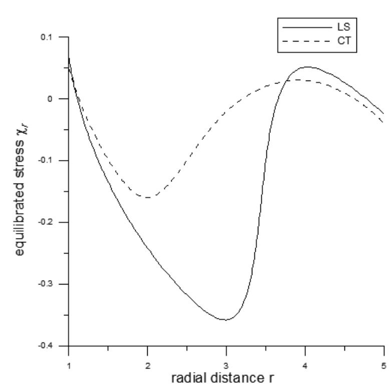

Fig.4 Variation of equilibrated stress χ_r w.r.t. radial distance *r*

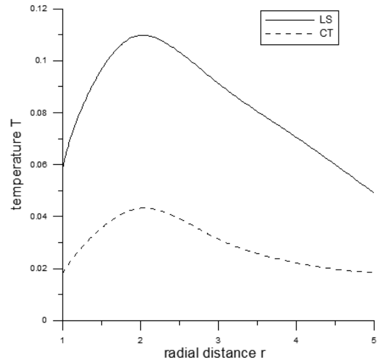

Fig.5 Variation of temperature distribution T w.r.t. radial distance r

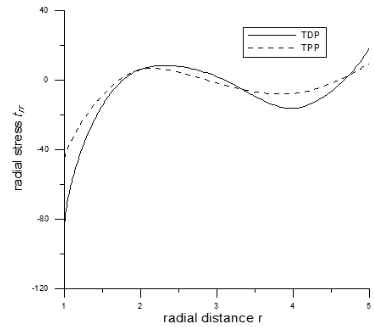

Fig.6 Variation of radial stress t_{rr} w.r.t. radial distance r

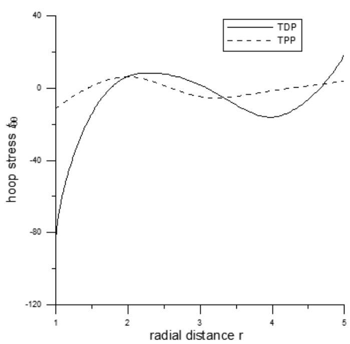

Fig.7 Variation of hoop stress $t_{\vartheta\vartheta}$ w.r.t. radial distance r

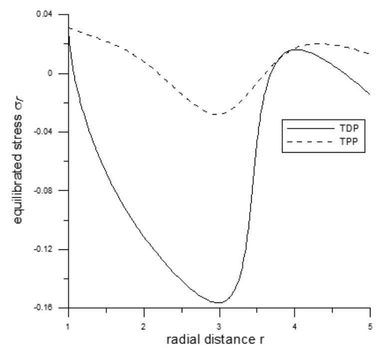

Fig.8 Variation of equilibrated stress χ_r w.r.t. radial distance r

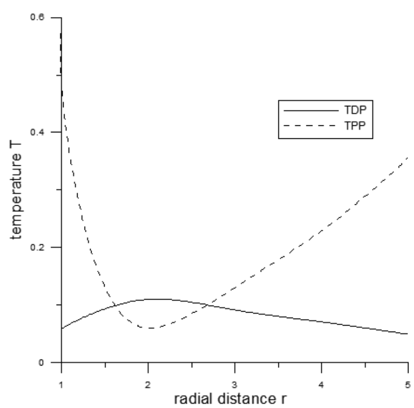

Fig.9 Variation of temperature distribution T w.r.t radial distance r

Effect of porosity

Figs.6 and 7, depict that the trend and behavior of variation of and are similar for both TDP and TPP. The value of and decrease for and increases further as increases. The values of these stresses are more for TPP as compared to the values for TDP due to effect of porosity. Fig. 8 represents that the value of is higher near the boundary surface of the cavity and decreases with increase in The value of TSP remain higher than that of TDP. From fig. 9, it is clear that the trend and behavior of variation is opposite for TDP and TPP. For TPP, it decreases for and then increases onwards while for TDP it initially increases for and then decreases slowly and steadily.

Appendix-I

$$a_{19} = s(1+\tau_0 s), a_{20} = s(1+\tau_0 s)a_{16}, a_{21} = s(1+\tau_0 s)a_{17}, a_{22} = s(1+\tau_0 s)a_{18},$$
$$n_1 = -\left(a_7 + s^2\right), n_2 = -\left(a_{14} + s^2\right), r_1 = a_5 a_{10} - a_4 a_{11},$$
$$r_2 = a_4(a_{11}a_{19} - n_2) - a_{11}n_1 - a_7 a_{10} - a_5(a_{10}a_{19} + a_{13}),$$
$$r_3 = n_1(a_{11}a_{19} - n_2) + a_4(n_2 a_{19} - a_{15}a_{22}) + a_5(a_{13}a_{19} + a_{15}a_{21}) + a_7(a_{10}a_{19} + a_{13}) + a_9(a_{10}a_{22} - a_{11}a_{21}),$$
$$r_4 = n_1(n_2 a_{19} - a_{15}a_{22}) - a_8(a_{13}a_{23} + a_{15}a_{21}) - a_9(a_{13}a_{22} + n_2 a_{21}), r_5 = a_6 a_{11} - a_5 a_{12},$$
$$r_6 = -a_6(a_{11}a_{19} - n_2) + a_7 a_{12} + a_5(a_{19}a_{12} + a_{15}a_{20}) - a_9 a_{11}a_{20},$$

$$r_7 = -a_6(n_2 a_{19} - a_{15}a_{22}) - a_7(a_{12}a_{19} + a_{15}a_{20}) - a_8(a_{12}a_{22} + n_2 a_{20}),$$
$$r_8 = a_6 a_{10} - a_4 a_{12}, r_9 = -a_6(a_{13} + a_{10}a_{19}) - n_1 a_{12} + a_4(a_{12}a_{19} + a_{15}a_{20}),$$
$$r_{10} = a_9(a_{13}a_{20} - a_{12}a_{21}) + n_1(a_{12}a_{19} + a_{15}a_{20}) + a_6\left(a_{13}a_{19} + a_{15}a_{21}\right),$$
$$r_{11} = a_{20}(a_4 a_{11} - a_5 a_{10}), r_{12} = a_6(a_{11}a_{21} - a_{10}a_{22}) + a_{20}(n_1 a_{11} + a_7 a_{10})$$
$$+ a_4(a_{12}a_{22} + n_2 a_{20}) + a_5(a_{13}a_{20} - a_{12}a_{21}),$$
$$r_{13} = a_7(a_{12}a_{21} - a_{13}a_{20}) + a_6(a_{13}a_{22} + n_2 a_{21}) + n_1(a_{12}a_{22} + n_2 a_{20}),$$

$$B_1 = \left(r_2 - s^2 r_1\right)/r_1, B_2 = \left(r_3 - s^2 r_2 - a_1 r_5 + a_2 r_8 + a_3 r_{11}\right)/r_1,$$
$$B_3 = \left(r_4 - s^2 r_3 - a_1 r_6 + a_2 r_9 + a_3 r_{12}\right)/r_1, B_4 = \left(-s^2 r_4 - a_1 r_7 + a_2 r_{10} + a_3 r_{13}\right)/r_1$$

Appendix-II

$$g_{1i} = -\left\{r_5\lambda_i^4 + r_6\lambda_i^2 + r_7\right\}/\left\{r_1\lambda_i^6 + r_2\lambda_i^4 + r_3\lambda_i^2 + r_4\right\}, \quad g_{2i} = \left\{r_8\lambda_i^4 + r_9\lambda_i^2 + r_{10}\right\}/\left\{r_1\lambda_i^6 + r_2\lambda_i^4 + r_3\lambda_i^2 + r_4\right\},$$
$$g_{3i} = -\left\{r_{11}\lambda_i^4 + r_{12}\lambda_i^2 + r_{13}\right\}/\left\{r_1\lambda_i^6 + r_2\lambda_i^4 + r_3\lambda_i^2 + r_4\right\}; \quad i = 1,2,3,4$$

$$H_{11} = p_1\left(\frac{\lambda_1^2 + 2(\lambda_1 + 1)}{\lambda_1^2}\right) + p_2 + p_3 g_{11} + p_4 g_{21} - g_{31},$$
$$H_{12} = p_1\left(\frac{\lambda_2^2 + 2(\lambda_2 + 1)}{\lambda_2^2}\right) + p_2 + p_3 g_{12} + p_4 g_{22} - g_{32},$$
$$H_{13} = p_1\left(\frac{\lambda_3^2 + 2(\lambda_3 + 1)}{\lambda_3^2}\right) + p_2 + p_3 g_{13} + p_4 g_{23} - g_{33},$$
$$H_{14} = p_1\left(\frac{\lambda_4^2 + 2(\lambda_4 + 1)}{\lambda_4^2}\right) + p_2 + p_3 g_{14} + p_4 g_{24} - g_{34},$$

$$H_{21} = \lambda_1(p_5g_{11} + p_6g_{21}), H_{22} = \lambda_2(p_5g_{12} + p_6g_{22}), H_{23} = \lambda_3(p_5g_{13} + p_6g_{23}),$$

$$H_{24} = \lambda_4(p_5g_{14} + p_6g_{24}), H_{31} = \lambda_1(p_6g_{11} + p_7g_{21}), \mathrm{H}_{32} = \lambda_2(p_6g_{12} + p_7g_{22}),$$

$$H_{33} = \lambda_3(p_6g_{13} + p_7g_{23}), H_{34} = \lambda_4(p_6g_{14} + p_7g_{24}), H_{41} = g_{31}, H_{42} = g_{32},$$

$$H_{43} = g_{33}, H_{44} = g_{34}$$

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