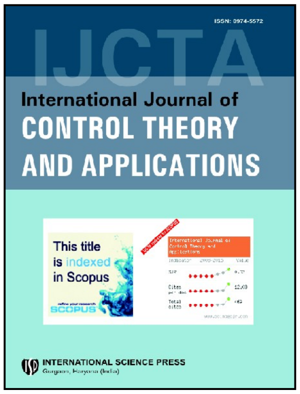

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A Novel Approach to Image Denoising by using Adaptive Parameter in WNNM Method

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Abstract: Applications likes computer vision and machine learning has depends on image enhancement. In such area, modified nuclear norm minimization (MNNM) gives all equal singular values due to this method can't solved efficiently and not fit in real scenarios. Nevertheless, so far, the optimal solution of MNNM problem is not completely yet due its non-convexity in general cases. Modified weighted nuclear norm minimization (MWNNM) is a natural extension and generalization of MNNM. Assign weights to singular values differently to make modified WNNM more flexible in application such as image enhancement. In this paper, we mainly concentrated on noise variance, number of iterations, peak signal to noise ratio (PSNR) and averaged PSNR by taking two different images like house and monarch.

Keywords: Image Denoising, NNM, MWNNM, Image difference(Subtraction).

1. INTRODUCTION

As a classical technique for low rank matrix approximation, recently applications like vision of computer and machine intelligence has depends on method modified nuclear norm minimization (MNNM) [1,3]. Standard representation in nuclear norm with respect to data matrix $X \in R^{m \times n}$ equivalent to addition of all singular values i.e., $\|X\|_* = \sum_i |\sigma_i(X)|$, where $\sigma_i(X)$ is the ith singular value of **X**. Nuclear norm is the compact curved recreation for a given matrix based on rank penalty. Let be the given data matrix is $Y \in R^{m \times n}$ The standard NNM problem aims to find an approximation matrix **X** of **Y** by minimizing the following energy function

$$\min_X \|Y - X\|_F^2 + \lambda \|X\|_* \qquad (1)$$

Where λ is a positive regularization parameter, it has been shown that the above NNM problem has a l where closed-form solution [2]

$$X^* = U\delta_\lambda\left(\sum\ \right)V^T \tag{2}$$

Where $Y = U\Sigma V^T$ is the SVD of Y and

$$\delta_\lambda(\Sigma\ ) = \max\left(0, \Sigma - \frac{\lambda}{2}\right) \tag{3}$$

Albeit easy to solve, the NNM model has some limitations. The nuclear norm to deal with all equal singular values and it know attention on prior knowledge. For example, like many vision applications in that compared to smaller values larger singular values are more important since they associated with main components of data or image. In real scenarios, we should assign weights to singular values different to make WNNM more flexible.

To revamp the pliability of modified NNM, researchers have proposed new method that is modified weighted nuclear norm minimization (MWNNM) [3,4] problem. The matrix 5ØnÜ of modified WNNM is stated by

$$\|X\|_{w,*} = \sum_i \left|w_i\sigma_i(X)\right| \tag{4}$$

Where $$\sigma_1(X) \geq \sigma_2(X) \geq \sigma_3(X) \geq ......\sigma_n(X) \tag{5}$$

$w = [w_1, w_2, .., w_n]$ and $w_i \geq 0$ is, the weight appoints to (X). Modified Weighted nuclear norm does not belong to a real norm since it does not always satisfy the triangle in equality. The WNNM problem is then formulated as [4]

$$\min_X \|Y - X\|_F^2 + \lambda\|X\|_{w,*} \tag{6}$$

Modified WNNM complication is furthermore arduous to solve efficiently than the modified nuclear norm minimization issue, due to its non-convexity in general cases of weights [3] and [5] have independently discussed the solution of modified WNNM and successfully applied modified WNNM to image denoising and reconstruction.

The organization paper as follows: Section-I explains introduction of the paper, low rank minimization with weighted nuclear norm will describes in Section-II, in Section-III explains about modified weighted nuclear norm minimization (MWNNM), In section –IV discuss the results and conclusion in final Section [5,6].

2. LOW RANK MINIMIZATION WITH WEIGHTED NUCLEAR NORM

We first give the following Lemma 1 [7], which builds the important relationship between the nuclear norm and the trace of a matrix.

Lemma 1[3]**.** For any $A \in R^{m\times n}$ and a positive diagonal matrix $W \in R^{m\times n}$ accompanied by non-ascending diagonal elements, let SVD of A is $A = X\Phi Y^T$, and

$$\sum_i \sigma_i(A)\sigma_i(W) = \max_{U^TU=I, V^TV=I} tr\left[WU^TAV\right] \tag{7}$$

Where identity matrix is $\boldsymbol{I}$ and ith singular values of matrices are $\sigma_i(A)$ and $\sigma_i(W)$ respectively. If condition $\boldsymbol{U} = \boldsymbol{X}$ and $\boldsymbol{V} = \boldsymbol{Y}$, then $tr\left[WU^TAV\right]$ get as far as its upper limit. By the consequence of Lemma 1, the following postulate, which implies an equivalent convex transform of the original non-convex modified WNNM [8,9] problem in Eq. (6).

Statement 1: For any $Y \in R^{m\times n}$ unaccompanied by reduction of quality. Suppose that 5ØZÜe"5Ø[Ü, and let SVD of

$$\text{Y is } Y = U\sum V^T \text{, where} \sum = \begin{pmatrix} diag(\sigma_1,\sigma_2,...,\sigma_n) \\ 0 \end{pmatrix}. \tag{8}$$

Modified WNNM problem solution in Eq. (6) is denoted with $X^* = UDV^T$ where

$$D = \begin{pmatrix} diag(d_1,d_2,...,d_n) \\ 0 \end{pmatrix} \tag{9}$$

is a positive diagonal matrix and $diag(d_1,d_2,...,d_n)$ is solution of convex optimization [10] problem and denoted by

$$\min_{d_1,d_2,...d_n} \sum_{i=1}^{n} (d_i - \sigma_i)^2 + w_i d_i \tag{10}$$

to the extent that $$d_1 \geq d_2 \geq ..... \geq d_n \geq 0 \tag{11}$$

Proof: For $X \in R^{m\times n}$ and SVD of X is $X = \bar{U}D\bar{V}^T$ where $\bar{U}$ and $\bar{V}$ are unitary matrices, and $D = \begin{pmatrix} diag(d_1,d_2,...,d_n) \\ 0 \end{pmatrix}$ with $d_1 \geq d_2 \geq ..... \geq d_n \geq 0$. Then equation is

$$\min_X \|Y - X\|_F^2 + \|X\|_{w,*} \Leftrightarrow \min_{\bar{U},\bar{V},D} \|Y - \bar{U}D\bar{V}^T\|_F^2 + \|\bar{U}D\bar{V}^T\|_{w,*} \tag{12}$$

to the extent that $d_1 \geq d_2 \geq ..... \geq d_n \geq 0 \Leftrightarrow$

$$\min_{\bar{U},\bar{V},D} \|Y\|_F^2 - 2tr\left(Y\bar{U}^T D\bar{V}\right) + \|D\|_F^2 \|D\|_{w,*} \tag{13}$$

to the extent that $d_1 \geq d_2 \geq ..... \geq d_n \geq 0 \Leftrightarrow$

$$\min_D \left(\|D\|_F^2 \|D\|_{w,*} - 2 \min_{\bar{U}^T\bar{U}=I,\bar{V}^T\bar{V}} tr\left(Y\bar{U}^T D\bar{V}\right) \right) \tag{14}$$

to the extent that $d_1 \geq d_2 \geq ..... \geq d_n \geq 0$. According to Lemma 1, we have

$$\min_{\bar{U}^T\bar{U}=I,\bar{V}^T\bar{V}} tr\left(Y\bar{U}^T D\bar{V}\right) = \sum_i \sigma_i d_i \tag{15}$$

and the optimal solution is obtained at $U = \bar{U}$ and $\bar{V} = V$. After that we have

$$\min_X \|Y - X\|_F^2 + \|X\|_{w,*} \Leftrightarrow \min_D \left(\|D\|_F^2 + \|D\|_{w,*} - 2\sum_{i=1}^{n} d_i \sigma_i \right) \tag{16}$$

to the extent that $d_1 \geq d_2 \geq ..... \geq d_n \geq 0 \Leftrightarrow$

$$\min_D \left(\sum_{i=1}^{n} d_i^2 + w_i d_i - 2d_i\sigma_i \right) \tag{17}$$

to the extent that $d_1 \geq d_2 \geq ..... \geq d_n \geq 0 \Leftrightarrow$

$$\min_{D}\left(\sum_{i=1}^{n}\left(d_i-\sigma_i\right)^2+w_i d_i\right) \tag{18}$$

to the extent that $d_1 \ge d_2 \ge ..... \ge d_n \ge 0$

A mentioned above equation and the desirable solution of the modified WNNM problem in Eq. (6) is

$$X^* = UDV^T \tag{19}$$

But in Eq. (10) and Eq. (19), D is the constrained optimization issue.

Statement 1 allows the modified WNNM issue be equivalently transformed in Eq. (10). It is interesting to see that Eq. (10) is a convex problem. It indicates that convex problem and non-convex problem are equal, which is much easier to solve. Furthermore, in a specific yet very useful case, i.e., the weights 5ØdÜ 5ØVÜ=1, 2…5Ø[Ü are in a non-descending order, it can be shown that the global optimum of Eq. (10) has a closed form [11]. The corollary1 is nominated below.

Corollary1: If weights satisfy $0 \le w_1 \le w_2 \le ..... \le w_n$ the globally optimal solution of Eq. (10) is $D^* = \begin{pmatrix} diag\left(\bar{d}_1, \bar{d}_2, ..., \bar{d}_n\right) \\ 0 \end{pmatrix}$, where $\bar{d}_i = \max\left(\sigma_i - \frac{w_i}{2}, 0\right)$.

Proof: If we ignore the constraints of the problem in Eq. (10), we can obtain the following unconstrained problem

$$\min_{d_i \ge 0}\left(d_i-\sigma_i\right)^2 + w_i d_i \tag{20}$$

Since $d_i \ge 0$, i = 1, 2… n the above problem is equivalent to the following problem

$$\min_{d_i \ge 0}\left(d_i-\left(\sigma_i-\frac{w_i}{2}\right)\right)^2 \tag{21}$$

Then it is easy to see that the globally optimal solution of the problem in Eq. (21) is

$$\bar{d}_i = \max\left(\sigma_i - \frac{w_i}{2}, 0\right) \quad i = 1, 2\ldots n \tag{22}$$

Since $\sigma_1(X) \ge \sigma_2(X) \ge \sigma_3(X) \ge ......\sigma_n(X)$ and $w_1 \le w_2 \le w_3 \le ...... \le w_n$ we have $\bar{d}_i \ge \bar{d}_2 \ge \bar{d}_3 \ge ...... \ge \bar{d}_n \ge 0$ (23)

Thus, $d_{i=1,2\ldots}$ meet the constraint and they are nothing but original constrained solution in Eq. (10).

In real scenarios, the conclusion of corollary1 is able to be used for a practical purpose. For instance, by allot larger singular values to smaller weights modified WNNM leads the most recent stage in the development of image enhancement results. Actually, combining Statement 1 and Corollary 1 readily obtain globally and the following analytical form (when $0 \le w1 \le w2 \le \ldots \le wn$) have X^* that is

$$X^* = UD^* V^T \tag{24}$$

Where

$$D^* = \begin{pmatrix} diag\left(\bar{d}_1, \bar{d}_2, ..., \bar{d}_n\right) \\ 0 \end{pmatrix}, \bar{d}_i = \max\left(\sigma_i - \frac{w_i}{2}, 0\right) \tag{25}$$

and $$Y = U\Sigma V^T \text{ is SVD of 5Ø€Ü.} \quad (26)$$

3. MODIFIED WEIGHTED NORM MINIMIZATION WITH IMAGE DENOISING

Additive White Gaussian Noise (AWGN) is a diminution model, i, e, $\mathbf{Y}_i = \mathbf{X}_i + \mathbf{N}_i$ where $\mathbf{X}_i$ and $\mathbf{N}_i$ are matrices of clean image and noise image respectively. Under LRMA methods [12], the matrix of clean image ($\mathbf{X}_i$) computed from $\mathbf{Y}_i$. So, WNNM method is used to compute $\mathbf{X}_i$ with the design and operation of a system and is given by

$$\hat{X}_i = \arg\min_{X_i} \frac{1}{\sigma_n^2}\|Y_i - X_i\|_F^2 + \|X_i\|_{w,S_p}^p \quad (27)$$

Where σ_n^2 to mark the noise variance, in equation (9) F-norm data is first term and low rank regularization is the second term.

The following steps to indicate the denoising algorithm

1. Noisy image **y** is the input and denoised image $\hat{x}^K$ is the output
2. Assignment
3. Assign $\hat{x}^0 = y$, $\hat{y}^0 = \hat{y}$
4. For k=1: K
5. Frequentativeregularization $y^{(k)} = \hat{x}^{(k-1)} + \alpha\left(y - x^{(k-1)}\right)$
6. Find same sections to show matrix Y_j
7. SVD$\left[U, \sum, V\right] = SVD\left(Y_j\right)$
8. Compute weight vector $w_j = \dfrac{c\sqrt{n}}{\left(\delta_j^{\frac{1}{p}}\left(\hat{X}_i\right) + \varepsilon\right)}$
9. Compute ” by using $\Delta = diag\left(\delta_1, ..., \delta_r\right)$
10. To obtain $\hat{X}_j = U\Delta V^T$
11. end
12. Denoised image (Enhanced image) for $\mathbf{x}^k$ by cluster $\mathbf{X}_j$
13. End
14. Enhancement of image is obtained by $\hat{x}^K$

4. RESULTS AND DISCUSSIONS

The performance of proposed method MWNNM in image denoising is tested by using some images like house and monarch from data base. In the given Figure 1 indicates that clean image, noisy image, enhanced image and subtraction image. Clean image is added with AWGN based up on noise variance i.e., σ_n and its values are 0.5, 1, 2, 3, 5, 7, 40, 60, 70, 90, 100 and 150. In this test, 12 noise levels are used under that images have different PSNR values. For example, the image has $\sigma_n = 40$ and PSNR values 31.35.

Detailed analysis is given in Figure 2 when $\sigma_n = 40$ the PSNR is equal to 31.35, $\sigma_n = 60$ For PSNR value is 29.44, $\sigma_n = 70$ For PSNR value is 28.59, $\sigma_n = 100$ For PSNR value is 26.66, $\sigma_n = 150$ For PSNR value is 24.23.

In Figure 2, blue color bar indicates maximum averaged PSNR based up on the number of iterations. In figure 3 is the conclusion that when number of iteration increased the averaged PSNR increases.

In Figure 4, when σ_n values are increases the PSNR values decreases in case of house image more PSNR as compared to monarch image. So, image- to- image PSNR values are changed by using proposed method.

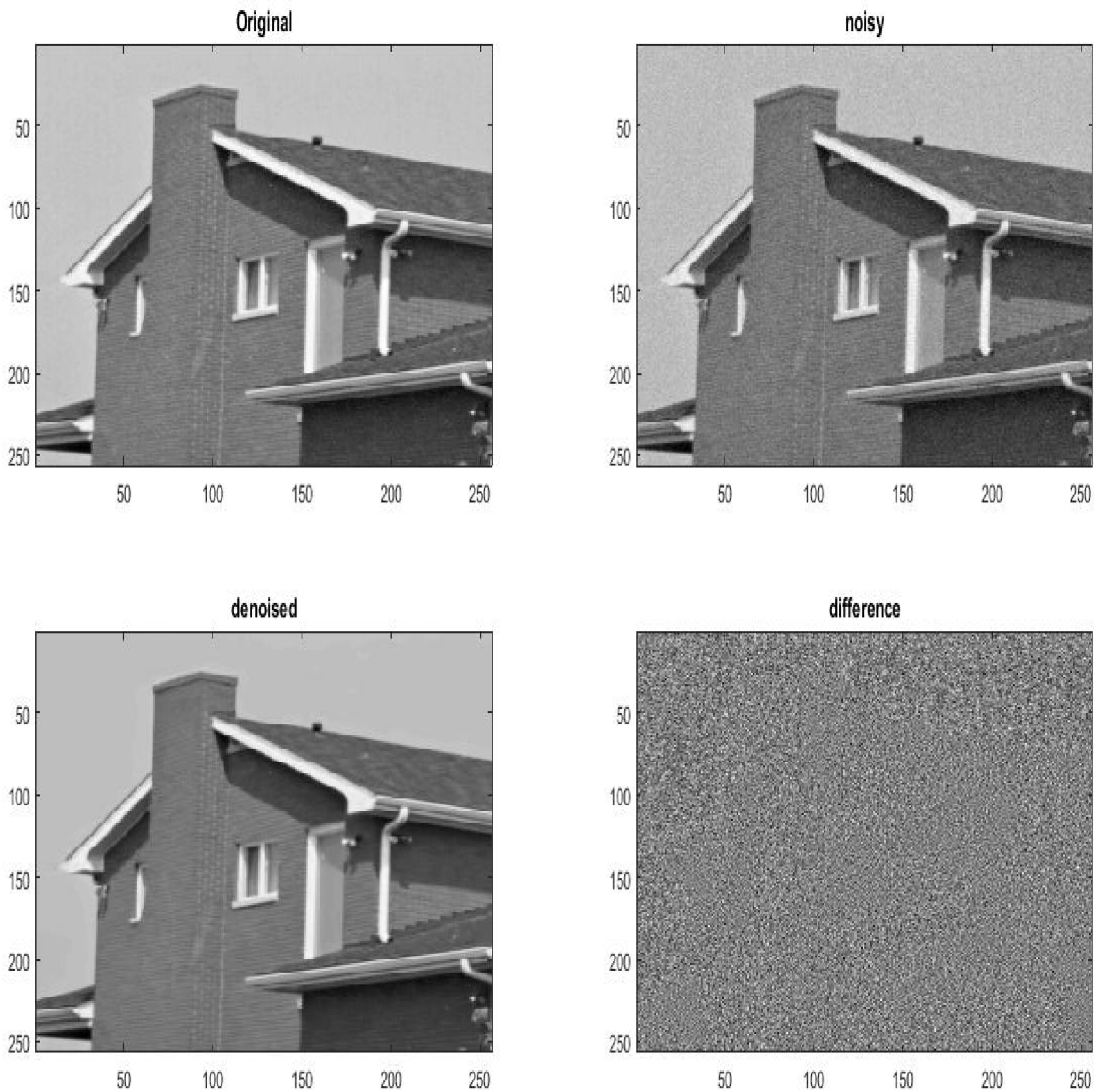
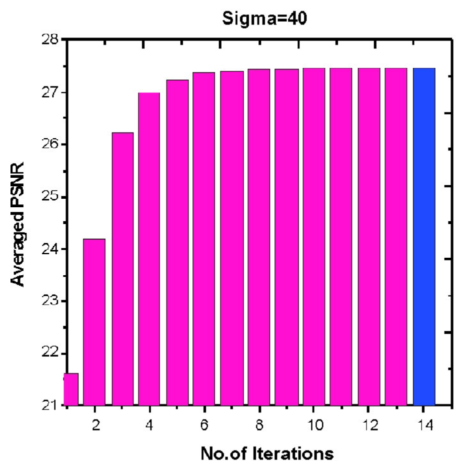
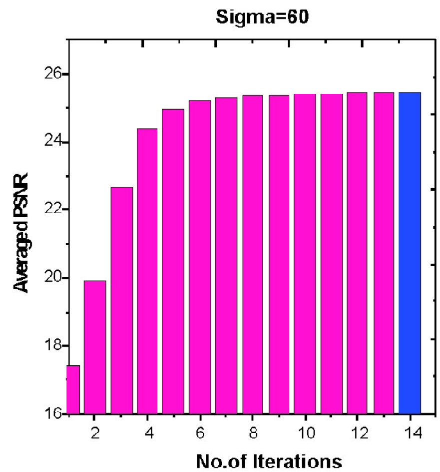


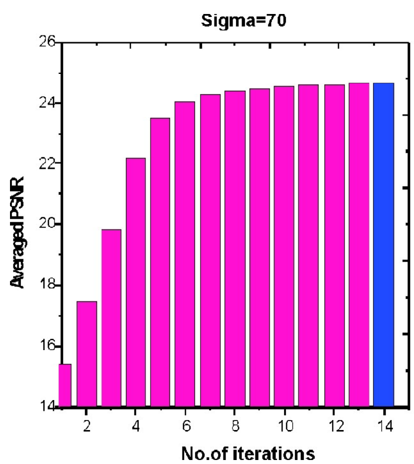
Figure 1: Modified WNNM Image Denoising Method



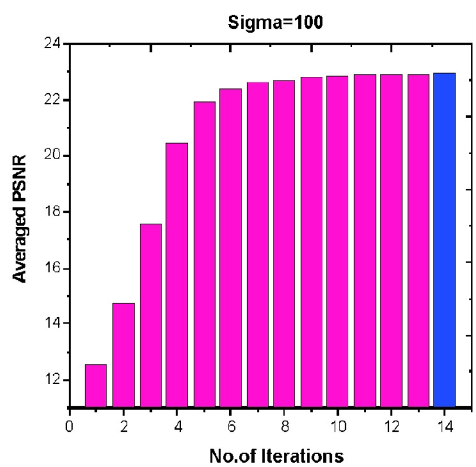
(a) $\sigma_n = 40$



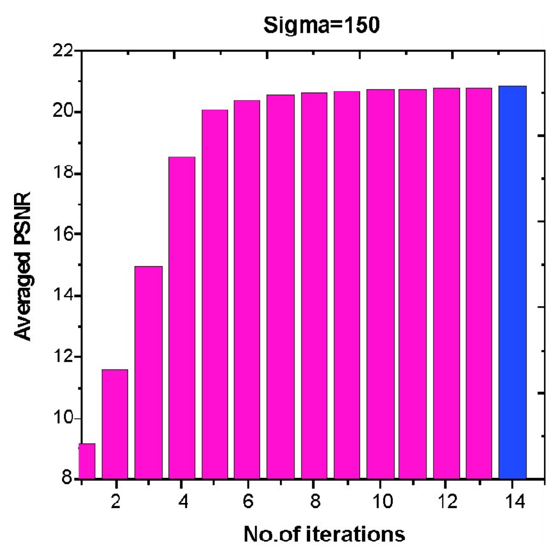
(b) $\sigma_n = 60$



(c) $\sigma_n = 70$



(d) $\sigma_n = 100$



(e) $\sigma_n = 150$

Figure 2: Enhanced image results for different noise levels

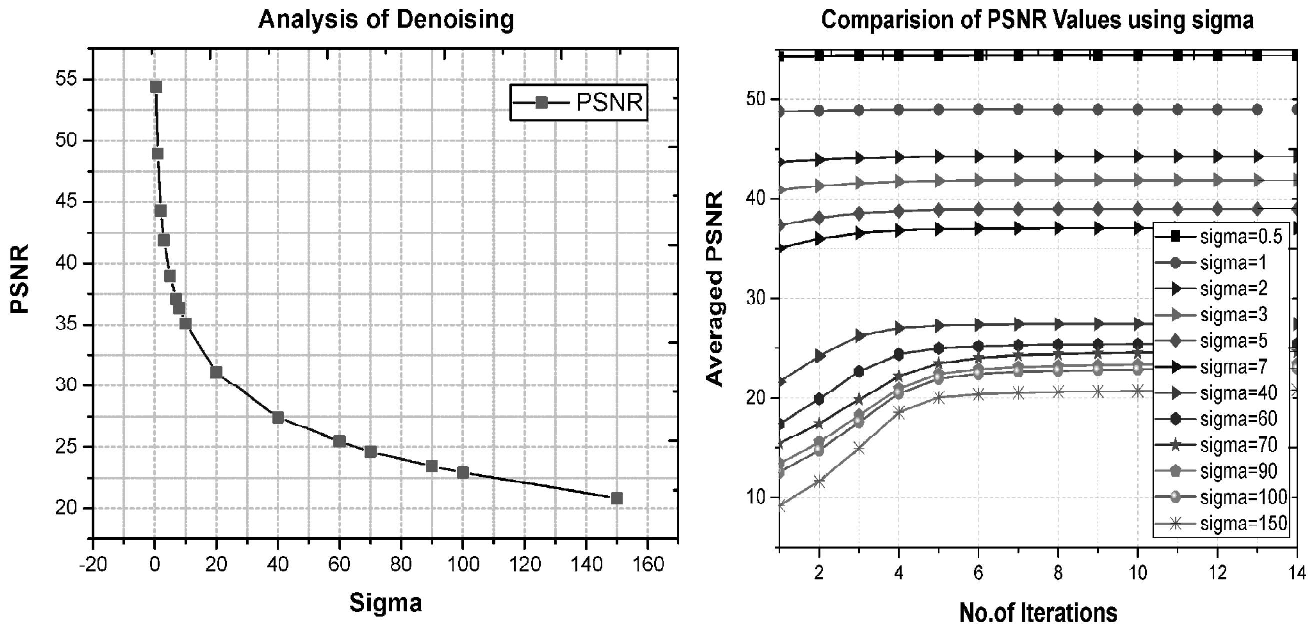


Figure 3: (a) Analysis of denoising and (b) comparisons of PSNR values using various σ_n

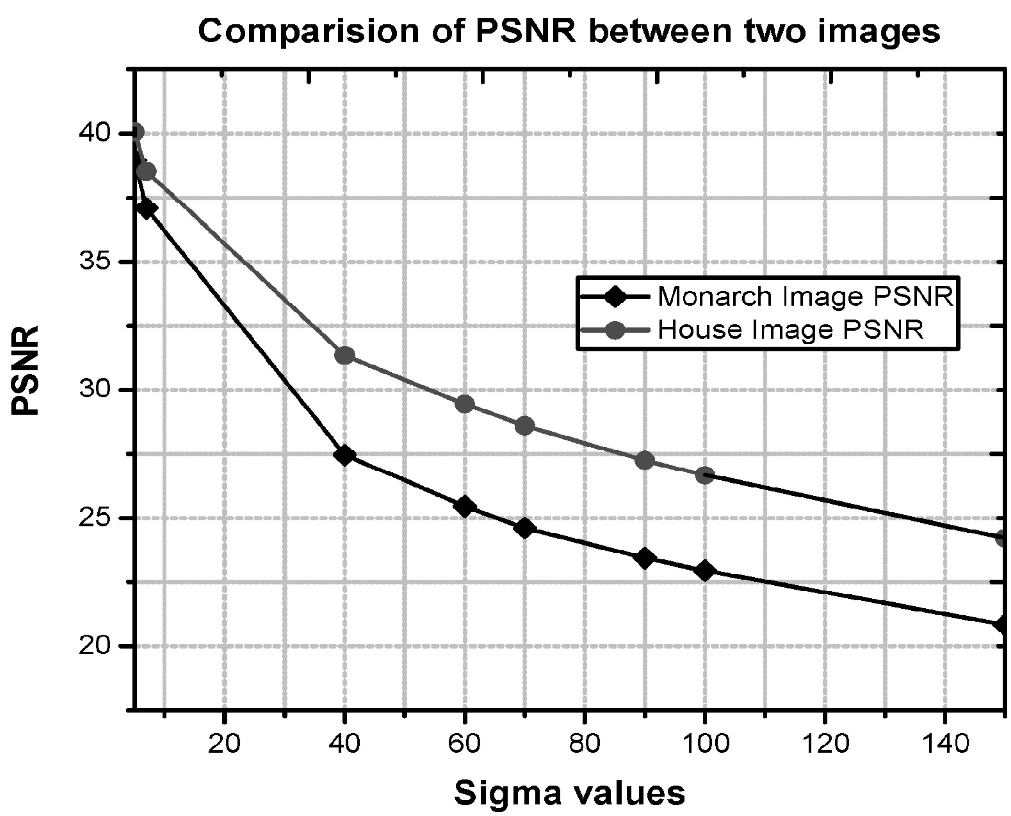


Figure 4: Comparison of PSNR between two different images (Monarch, House)

Table 1
Image Denoised PSNR results for Monarch and House images

Sigma Values	*PSNR Values (Monarch)*	*PSNR Values (House)*
5	38.98	40.07
7	37.08	38.53
40	27.46	31.35
60	25.45	29.44
70	24.62	28.59
90	23.45	27.25
100	22.95	26.66
150	20.83	24.23

Using Modified WNNM method, Test images like monarch and house has computed in terms of noise variance and PSNR in the table 1.

In table I taken images monarch for this PSNR values are low as compared to the PSNR values of house image. So PSNR values have changed from the image to image.

5. CONCLUSION

In this paper, the proposed method modified WNNM gives better results in terms of PSNR and average PSNR using house and monarch images compared to other conventional method. So, the proposed method modified WNNM be applicable to image enhancement along with image back ground difference to support its successfulness.

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