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DIVIDED DIFFERENCE METHOD USING TRIANGULAR FUZZY NUMBER

Abstract: *In this paper the polynomial interpolation of triangular fuzzy number is discussed. First general form of the polynomial with fuzzy coefficients is proposed. The divided difference interpolation method is studied with triangular fuzzy number and an example is provided to illustrate the algorithm.*

Keywords: *Interpolation, polynomial, triangular fuzzy numbers*

I. INTRODUCTION

In real life problem data's are imprecise and uncertain, fuzzy set theory is the best approximation theory to deal with uncertain values. Many researchers focus on fuzzy numbers to study mathematical programming, numerical problems, stochastic and statistical problems.

Interpolation method is the most popular method useful in fields of engineering, economy, business oriented problems. Chenyi Hu *et al.* (7) have discussed interval polynomial interpolation problems in Lagrange form. Hussein *et al.* (10) have discussed interpolation of fuzzy data using cubic splines.

In this paper interpolation polynomial of triangular fuzzy number is formulated on comparing with divided difference interpolation problem. The existence theorem and some of its properties are dealt with. An algorithm is proposed for divided difference interpolating method using triangular fuzzy number illustrated by an example.

II. PRELIMINARIES

Definitions 2.1

A triangular fuzzy number is represented by three points as $A = (a^l, a^c, a^r)$. Its membership function is interpreted as

$$\mu_A(x)=\begin{cases}0, & x<a^l\\ \dfrac{x-a^l}{a^c-a^l}, & a^l\le x\le a^c\\ \dfrac{a^r-x}{a^r-a^c}, & a^c\le x\le a^l\\ 0, & x>a^r\end{cases}$$

Definitions 2.2

Let $\tilde{a}=(a^l,a^c,a^r)$, $\tilde{b}=(b^l,b^c,b^r)$ be two triangular fuzzy numbers. Then $\tilde{a}=\tilde{b}$ if and only if $a^l=b^l$, $a^c=b^c$, $a^r=b^r$

Definitions 2.3

The arithmetic operations of triangular fuzzy numbers are defined as

(i) $\tilde{a}+\tilde{b}=(a^l+b^l,a^c+b^c,a^r+b^r)$

(ii) $k\tilde{a}=\begin{cases}(ka^l,ka^c ka^r)\,k\ge 0\\ (ka^r,ka^r,ka^r),else\end{cases}$

Definitions 2.4

Let $\tilde{a}_0,\tilde{a}_1\cdots\cdots\cdots\cdots\tilde{a}_m$ be triangular fuzzy numbers. A function $P_n(x,\tilde{a}_0,\tilde{a}_1\cdots\cdots\cdots\cdots\tilde{a}_m)$ denoted by $\tilde{P}_n(x)$, is called the n-order polynomial with triangular fuzzy numbers coefficients, if it satisfies the following conditions.

(i) $\tilde{P}_n(x)$ is an n-order polynomial about x,

(ii) $\tilde{P}_n(x)$ is a 1-order polynomial about $\tilde{a}_0,\tilde{a}_1\cdots\cdots\cdots\cdots\tilde{a}_m$

Definitions 2.5

Let $Q_0(x)$, $Q_1(x)$ ... ., $Q_m(x)$ be $m+1$ polynomials whose degree is no more than n and at least one of them is a polynomial of degree n. The fuzzy triangular polynomials $\tilde{P}_n(x)$ has the following form

$$\tilde{P}_n(x) = \tilde{a}_0\, Q_0(x) + \tilde{a}_1 Q_1(x) + \cdots \cdots \cdots \cdots + \tilde{a}_m\, Q_m(x)$$

The set of all triangular fuzzy polynomials of degree m ($m \le n$) is denoted by $\tilde{P}_n$

III. INTERPOLATION POLYNOMIALS OF FUZZY NUMBERS

In this section the interpolation problem of triangular fuzzy numbers is formulated in detail by comparing with the divided difference interpolation problem The existence theorem of solutions investigated and some related properties are developed.

Definition 3.1

Let $x_0, x_1, \ldots\ldots\ldots\ldots x_n$ be $n + 1$ distinct nodes.

Given $y_0 = f(x_0)$, $y_1 = f(x_1)$, ... , $y_n = f(x_n)$

The problem is to find a polynomial $P_n(x) \in P_n$, called an interpolating polynomials, such that

$$P_n(x_i) = y_i \ (i = 0, 1, 2\ldots\ldots\ldots n)$$

Theorem 3.1

There exists a unique polynomial $P_n(x) \in P_n$ such that $P_n(x_i) = y_i$ for $i = 0, 1, \ldots \ldots \ldots\, n$

The divided differences of $P_n(x)$ is defined by

$$P_n(x) = f(x_0) + \sum_{l=1}^{n} r_l(x)\left(\prod_{k=0}^{n}(x - x_k)\right)$$

where

$$r_l(x) = \sum_{i=0}^{n}\left(\frac{f(x_i)}{\prod_{\substack{i=j\\ i\neq j}}(x_i - x_j)}\right)$$

for the $n + 1$ pairs (x_i, y_i) in which (y_i) are triangular fuzzy numbers.

Theorem 3.2

Let $x_0, x_1, \ldots \ldots \ldots . x_n$ be $n + 1$ distinct nodes and let $\widetilde{y_l} - (y_i^l, y_i^c, y_i^r)$ $i = 0, 1, 2 \ldots \ldots n$ be $n + 1$ triangular fuzzy numbers. There exists at least one fuzzy polynomial such that $\tilde{P}_n(x_i) = \widetilde{y_l}$ for $i = 0, 1, 2 \ldots \ldots n$.

Proof

To prove existence let us use a constructive approach providing an expression for $\tilde{P}_n(x)$ for arbitrary y_i, $i - 0, 1, 2 \ldots \ldots . . n$

Suppose that $P_n(x, y_0, y_1, \ldots \ldots \ldots . . n)$ is an interpolating polynomial such that

$$P_n(x_i, y_0, y_1, \ldots \ldots \ldots .. n) = y_i, \quad (i = 0, 1, 2 \ldots \ldots \ldots n)$$

Define
$$\tilde{P}_n(x) = (P_n^l(x) = (P_n^l(x), P_n^c(x), P_n^r(x))$$

where
$$P_n^l(x) = \underset{i=0,1,\ldots\ldots n}{\overset{\inf}{\forall y_i \in \widetilde{y_l}}} P_n(x, y_0, y_1, \ldots \ldots \ldots . y_n)$$

$$P_n^r(x) = \underset{i-0,1,\ldots\ldots .n}{\overset{\sup}{\forall y_i \in \widetilde{y_l}}} P_n(x, y_0, y_1, \ldots \ldots \ldots . y_n)$$

$$P_n^C(x) = f(x_0) + \sum_{l=1}^{n} r_l(x) \prod_{k=0}^{n} (x - x_k)$$

Then $\tilde{P}_n(x)$ satisfies the interpolation condition

Since
$$P_n^l(x_i) = \underset{i=0,1,\ldots\ldots n}{\overset{\inf}{\forall y_i \in \widetilde{y_l}}} P_n(x_i, y_0, y_1, \ldots \ldots \ldots . y_n)$$

$$= \underset{i=0,1,\ldots\ldots n}{\overset{\inf}{\forall y_i \in \widetilde{y_l}}} y_i = y_i^l$$

$$P_n^r(x_i) = \underset{i=0,1,\ldots\ldots n}{\overset{\sup}{\forall y_i \in \widetilde{y_l}}} P_n(x_i, y_0, y_1, \ldots \ldots \ldots . y_n)$$

$$= \underset{i=0,1,\ldots\ldots n}{\overset{\text{sup}}{\forall y_i \in \widetilde{y_l}}} \quad y_i = y_i^r$$

$$\tilde{P}_n(x_i) = (P_n^l(x), P_n^c(x), P_n^r(x))$$

$$= (y_i^l, y_i^c, y_i^r)$$

$$= \tilde{y}_i$$

As a consequence the interpolating polynomial of triangular fuzzy number exists.

Theorem 3.3

The divided differences form of the interpolating polynomial of fuzzy numbers is given by

$$\tilde{P}_n(x) = f(x_0) + \sum_{l=1}^{n} r_l(x) \prod_{k=0}^{n} (x - x_k)$$

where

$$r_l(x) = \sum_{i=0}^{n} \frac{f(x_i)}{\prod_{\substack{i=j \\ i \neq j}} (x_i - x_j)}$$

3.4. Numerical Example

Construct divided difference table for the data

x	*f(x)*
0.5	(1.610, 1.625, 1.636)
1.5	(5.701, 5.875, 5.901)
3.0	(30.10, 31.0, 32.0)
5.0	(5.701, 5.875, 5.901)
6.5	(281.0, 282.125, 283.01)
8.0	(520, 521.0, 522.0)

And Find the interpolating polynomial and an approximation to the value of $f(7)$.

Solution : We have the following divided difference table.

x	$f(x)$	l	$r_l(x)$
0.5	(1.610, 1.625, 1.636)	1	(4.065, 4.250, 4.291)
1.5	(5.701, 5.875, 5.901)	2	(5.3773, 5.0, 4.747)
3.0	(30.10, 31.0, 32.0)	3	(0.9786, 1.0, 1.1199)
5.0	(5.701, 5.875, 5.901)	4	(0.0162, 0.0001, 0.0085)
6.5	(281.0, 282.125, 283.01)	5	(0.0003, 0, 0.0004)
8.0	(520, 521.0, 522.0)		

$$r_l(x) = \sum_{i=0}^{n} \frac{f(x_i)}{\prod_{\substack{i=j \\ i\neq j}} (x_i - x_j)'}$$

$$\begin{aligned} P_n(x) &= (1.610, 1.625, 1.636) + (4.065, 4.25, 4.291)\,(x - x_o) \\ &\quad + (5.3773,5,4.747)\,(x - x_o)\,(x - x_1) \\ &\quad + (.9024,1,1.1199)\,(x - x_o)\,(x - x_1)\,(x - x_2) \\ &\quad + (0.0162,0.0001,0.0085)\,(x - x_o)\,(x - x_1)\,(x - x_2)\,(x - x_3) \\ &\quad + (0.0003,0,0.0004)\,(x - x_o)\,(x - x_1)\,(x - x_2)\,(x - x_3)\,(x - x_4) \\ &= (1.610,1.625,1.636) + (4.065,4.25,4.291)\,(x - 0.5) \\ &\quad + (5.3773,5,4.747)\,(x - 0.5)\,(x - 1.5) \\ &\quad + (0.9024,1,1.1199)\,(x - 0.5)\,(x - 1.5)\,(x - 3) \\ &\quad + (0.0162,0.0001,0.0085)\,(x - 0.5)\,(x - 1.5)\,(x - 3) \end{aligned}$$

$(x - 5) + (0.0003, 0, 0.0004)\,(x - 0.5)\,(x - 1.5)\,(x - 3)\,(x - 5)\,(x - 6.5)$

Put $x = 7$

$$\begin{aligned} &- (1.610,1.625,1.636) + (26.42,27.62,27.89) + (192.23,178.75,169.70) \\ &\quad + (129.04,143,160.14) + (4.6332,0.0286,2.431) \\ &\quad + (0.0429,0,00.72) \end{aligned}$$

$$\tilde{P}_n(x) = (353.97, 351.02, 361.85)$$

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