IJCTA, 10(5) 2017, pp. 267-274

Submarine to Submarine Angles-Only Passive Target Tracking

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Abstract: This research effort is to track the target even though the range measurements are not available. Unscented angles-only Kalman filter (UAKF) is used for bearing and elevation target tracking. The mathematical modeling and simulation have been carried out. It is shown that UAKF algorithm effectively tracks the target in underwater environment.

Keywords: Stochastic theory, statistical signal processing, applied statistics, estimation theory.

1. INTRODUCTION

In underwater, passive target tracking is generally followed to track a submarine target [1]. The observer submarine is assumed to be moving at low speed to reduce self noise for tracking of the targets. These days, submarines with sonars are coming up having the facility to get target elevation measurements also. In this paper, research is towards submarine (observer) tracking another submarine using elevation and bearing measurements .As angle measurements are only available, the process is highly nonlinear and hence unscented angles-only Kalman filter (UAKF), a non linear filter is explored for this application, as shown in the Figure 1. [1-6].The estimated target range, course, bearing and speed (RCBS) are utilized in weapon guidance algorithm (which is not discussed here).

Section 2 deals with modeling of state vector, measurements and UAKF. In section 3, generation of measurements and creation of scenarios are discussed. Section 4 deals with results obtained in simulation.

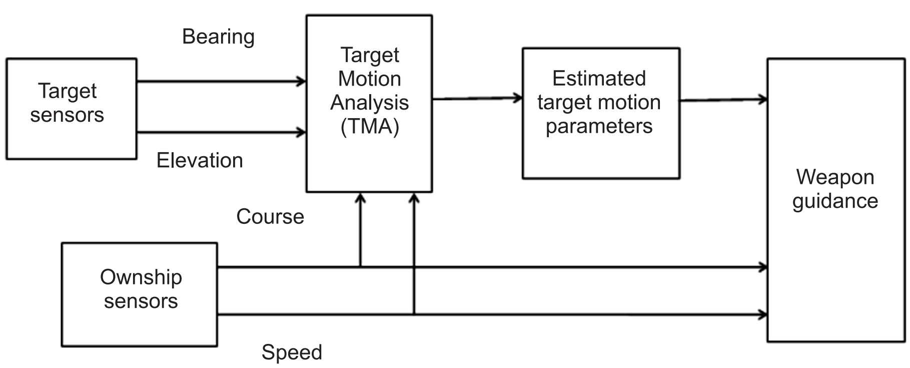

Figure 1: Block diagram of passive target tracking using bearing and elevation measurements

2. MATHEMATICAL MODELLING [3-4]

A. Measurements

Let $X_s(k)$ be state vector and it is defined as

$$X_S(k) = [\dot{x}(k)\ \dot{y}(k)\ \dot{z}(k)\ R_x(k)\ R_y(k)\ R_z(k)]^T \tag{1}$$

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where, $\dot{x}(k), \dot{y}(k), \dot{z}(k), \mathrm{R}_x(k), \mathrm{R}_y(k)$ and $\mathrm{R}_z(k)$ are velocity and range components in x, y and z directions. Azimuth and elevation angles are considered w.r.to True North. B_m is modeled as

$$\mathrm{B}_m(k+1) = \tan^{-1}\left(\frac{\mathrm{R}_x(k+1)}{\mathrm{R}_y(k+1)}\right) + \zeta(k) \tag{2}$$

Variance of $\xi(k)$ is σ_b^2. The measurement matrix is

$$\mathrm{H}(k+1) = \Big[0 \quad 0 \quad 0 \quad \hat{R}_z(k+1|k)/\mathrm{R}^2(k+1|k) \quad \hat{R}_y(k+1|k)/\mathrm{R}^2(k+1|k) \quad \hat{R}_x(k+1|k)/\mathrm{R}^2(k+1|k)\Big] \tag{3}$$

The target state dynamic equation is

$$\mathrm{X}_s(k+1) = \phi \mathrm{X}_s(k) + b(k+1) + \Gamma w(k) \tag{4}$$

where ϕ is given by

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ t & 0 & 0 & 1 & 0 & 0 \\ 0 & t & 0 & 0 & 1 & 0 \\ 0 & 0 & t & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

where t is sample time and

$$\Gamma = \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ t^2/2 & 0 & 0 \\ 0 & t^2/2 & 0 \\ 0 & 0 & t^2/2 \end{bmatrix} \tag{6}$$

b is $b(k+1) = [0 \quad 0 \quad 0 \quad -(x_0(k+1) - x_0(k)) \quad -(y_0(k+1) - y_0(k)) \quad -(z_0(k+1) - z_0(k))]$ (7)

$x_0(k)$ and $y_0(k)$ are observer position components.

$\omega(k)$, is Gaussian with variance equal to

$$\mathrm{E}[\Gamma(k)\omega(k)\omega^{\mathrm{T}}(k)\Gamma^{\mathrm{T}}(k)] = \mathrm{Q}\delta_{ij} \tag{8}$$

where $\delta_{ij} = \sigma_w^2$ if $i = j$

$= 0$ otherwise

$$\mathrm{Q} = \begin{bmatrix} ts^2 & 0 & ts^3/2 & 0 & 0 & 0 \\ 0 & ts^2 & 0 & 0 & ts^3/2 & 0 \\ 0 & 0 & ts^2 & 0 & 0 & ts^3/2 \\ ts^3/2 & 0 & 0 & ts^3/4 & 0 & 0 \\ 0 & ts^2/2 & 0 & 0 & ts^3/4 & 0 \\ 0 & 0 & ts^2/2 & 0 & 0 & ts^3/4 \end{bmatrix} \tag{9}$$

B. Unscented Kalman Filter Algorithm

A matrix of sigma vectors is formed [2] to calculate the mean and covariance of y.

$$\chi_0 = \bar{x}$$

$$\chi_i = \bar{x} + \left(\sqrt{(L+\lambda)P_x}\right)_i \qquad i = 1, \ldots, L$$

$$\chi_i = \bar{x} - \left(\sqrt{(L+\lambda)P_x}\right)_{i-L} \quad i = L+1, \ldots, 2L$$

$$W_0^{(m)} = \lambda/(L+\lambda)$$

$$W_0^{(c)} = \lambda/(L+\lambda) + (1-\alpha^2+\beta)$$

$$W_i^{(m)} = W_i^{(m)} = 1/\{2(L+\lambda)\} \; i = 1, \ldots, 2L \tag{10}$$

where $\lambda = \alpha^2(L+\kappa) - L$

α is a scaling parameter. Here α, κ, β are chosen as 0.001,0 and 2 respectively. The vectors χ_i are propagated as follows,

$$y_i = g(\chi_i) \; i = 1, \ldots, 2L \tag{11}$$

$$\bar{y} \approx \sum_{i=0}^{2L} W_i^{(m)} y_i \tag{12}$$

$$p_y \approx \sum_{i=0}^{2L} W_i^{(c)} \{y_i - \bar{y}_i\}\{y_i - \bar{y}_i\}^T \tag{13}$$

UKF implementation is as follows [3-9].

C. Unscented Kalman Filter Algorithm

1. Sigma point state vectors are presented as

$$X(k) = \left[X_S(k) \quad X_S(k) + \sqrt{(n+\lambda)\,p(k)} \quad X_S(k) - \sqrt{(n+\lambda)\,p(k)}\right] \tag{14}$$

2. The same are modified using eqn. (2),
3. The state vector is predicted as

$$X_S(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} X_S(i, k+1|k) \tag{15}$$

4. The covariance matrix is predicted as

$$P(k+1|k) = \sum_{i=0}^{2n} W_i^{(c)} \left[X_S(i,k+1|k) - X_S(k+1|k)\right]\left[X_S(i,k+1|k) - X_S(k+1|k)\right]^T + Q(k) \tag{16}$$

5. The state vectors are updated as

$$X(k+1|k) = \left[X_S(k+1|k) \quad X_S(k+1|k) + \sqrt{(n+\lambda)p(k+1|k)} \quad X_S(k+1|k) - \sqrt{(n+\lambda)p(k+1|k)}\right] \tag{17}$$

6. Then measurement predicted as

$$y(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} Y(k+1|k) \tag{18}$$

7. Covariance of innovation is

$$P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} \left[Y(i,k+1|k) - y(k+1|k)\right]\left[Y(i,k+1|k) - y(k+1|k)\right]^T + R(k) \tag{19}$$

8. The cross covariance is

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} \left[X(i, k+1|k) - X(k+1|k)\right]\left[Y(i, k+1|k) - y(k+1|k)\right]^T \tag{20}$$

9. Kalman gain is

$$K(k+1) = P_{xy} P_{yy}^{-1} \tag{21}$$

10. The state is estimated as

$$X(k+1|k+1) = X(k+1|k) + K(k+1)(y(k+1|k+1) - y(k+1|k) \tag{22}$$

11. And its covariance is

$$P(k+1|k+1) = P(k+1|k) - K(k+1)P_{yy}K(k+1)^T \tag{23}$$

Algorithm flow is shown in Figure 3. [3-9].

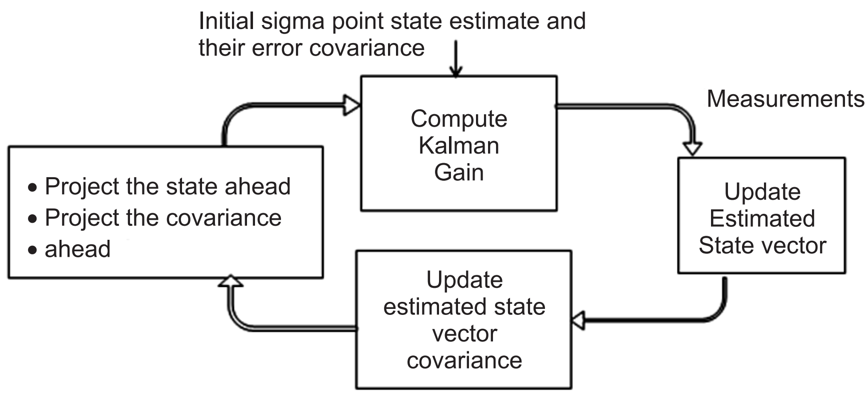

Figure 3: UAKF process

3. GENERALISED SIMULATOR

Let initial position of the target be (x_t, y_t, z_t) and the target moves with velocity v_t. After time t seconds, observer position changes and the change in the observer position is given by

$$dx_0 = v_0 \times \sin(ocr) \times \sin(oph) \times t \tag{24}$$

$$dy_0 = v_0 \times \cos(ocr) \times \sin(oph) \times t \tag{25}$$

$$dz_0 = v_0 \times \cos(oph) \times t \tag{26}$$

where *ocr* and *oph* are observer course and pitch respectively. Now the new observer position becomes

$$x_0 = x_0 + dx_0 \tag{27}$$

$$y_0 = y_0 + dy_0 \tag{28}$$

$$z_0 = z_0 + dz_0 \tag{29}$$

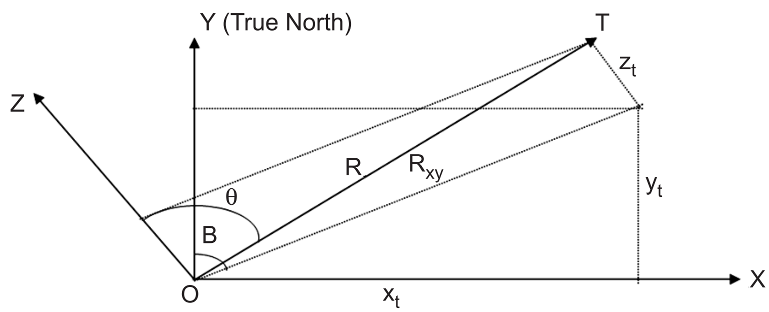

Figure 4: Target and observer positions

From Figure 4

$$x_t = R_{xy} \times \sin(B) \tag{30}$$

$$y_t = R_{xy} \times \cos(B) \tag{31}$$

$$\sin(\theta) = R_{xy}/R \tag{32}$$

Substituting equations (28) in (26) and (27)

$$x_t = R \times \sin(\theta) \times \sin(B) \tag{33}$$

$$y_t = R \times \sin(\theta) \times \cos(B) \tag{34}$$

$$z_t = R \times \cos(\theta) \tag{35}$$

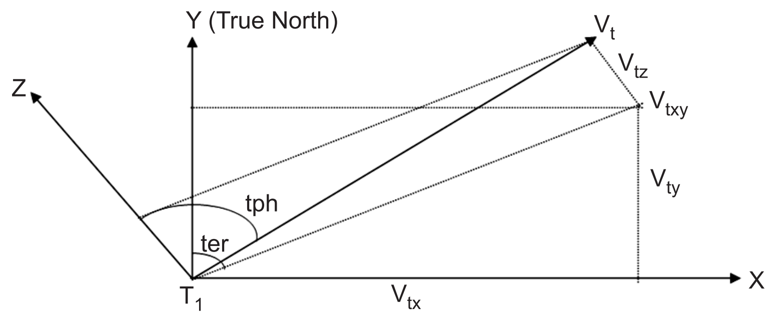

Figure 5: Target and observer velocities

When the target is in motion with velocity v_t, change in target position after t seconds, from Figure 5.

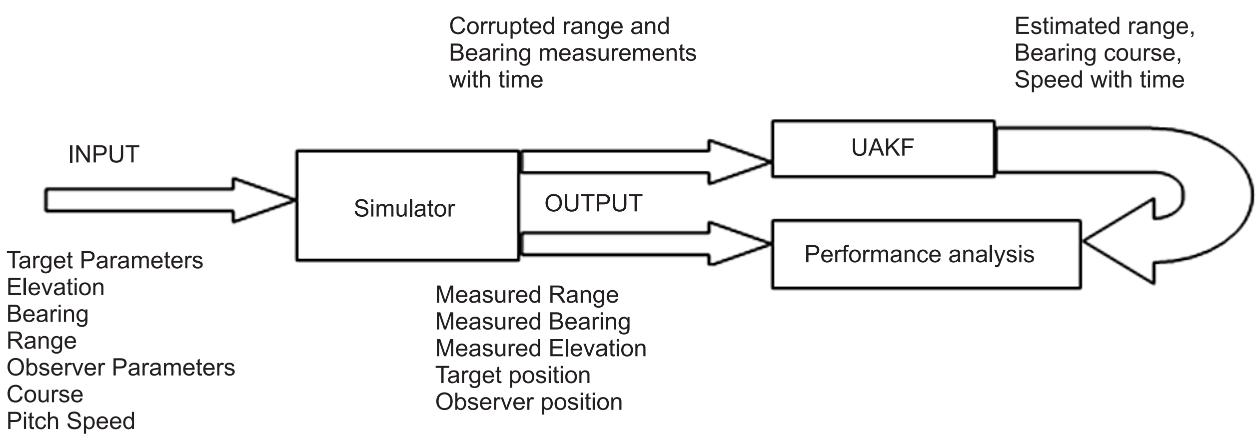

Figure 6: Block diagram of TMA in simulation mode

$$dx_t = v_t \times \sin(tcr) \times \sin(tph) \times t \tag{36}$$

$$dy_t = v_t \times \cos(tcr) \times \sin(tph) \times t \tag{37}$$

$$dz_t = v_t \times \cos(tph) \times t \tag{38}$$

where *tcr* and *tph* are target course and pitch respectively.

Now the new target position is

$$x_t = x_t + dx_t \tag{39}$$

$$y_t = y_t + dy_t \tag{40}$$

$$z_t = z_t + dz_t \tag{41}$$

Target true bearing, range and elevation are

$$\text{true bearing} = \tan^{-1}\left(\frac{x_t - x_0}{y_t - y_0}\right) \tag{42}$$

$$\text{true range} = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2 + (z_t - z_0)^2} \tag{43}$$

$$\text{true elevation} = \tan^{-1}\left(\frac{\mathrm{R}_{xy}}{z_t - z_0}\right) \tag{44}$$

Since the measurements are affected by noise in real situations, noise is added to these measurements.

Measured bearing = true bearing + sigma b

Measured range = true range + sigma r

Measured elevation = true elevation + sigma e

where sigma b, sigma r and sigma e are 1σ values of white Gaussian process. The details are shown in Figure 6.

4. SIMULATION AND RESULTS

It is assumed that experiment is conducted at favorable environmental conditions and hence the angle measurements are available continuously. Simulation is realised on a personal computer using Matlab. The scenarios chosen for evaluation of algorithm are shown in Table 1. For example, scenario1 describes a target moving with bearing of 45° with course and speeds of 255° and 10 m/s respectively. The elevation angle is 135°. The bearing and elevation measurements are corrupted with 0.33°(1σ) and 0.33°(1σ) respectively.

In simulation, estimated and actual values are available and hence the validity of the solution based on certain acceptance criterion is possible. The following acceptance criterion is chosen. The solution is converged when error in estimated course, speed and range are <= 3°, <= 5 m/s and <= 8% respectively.

The errors in estimated range, speed and course for scenario 1 are presented in Figure 7, Figure 8 and Figure 9 respectively.

The solution is converged when the course, speed and range are converged. The convergence time (seconds) for the scenarios is given in Table 2.

In simulation, it is observed that the estimated course of the target, speed and range of the target are converged at 405th sample, 65th sample and 152nd sample respectively for scenario1.So, for scenario 1, the total solution is obtained at 405th sample. Similarly for the other scenario the convergence time is shown in Table 2.

Table 1
Input parameters chosen for the algorithm

Scenario	*Initial range (m)*	*Bearing (deg)*	*Elevation (deg)*	*Pitch (deg)*	*Course (deg)*
1	3000	45	135	135	255
2	4500	45	135	135	275

Table 2
Convergence time in samples for the chosen scenarios

Scenario	*Course*	*Elevation*	*Range*	*Speed*	*Total solution*
1	405	126	152	65	405
2	409	3	64	60	409

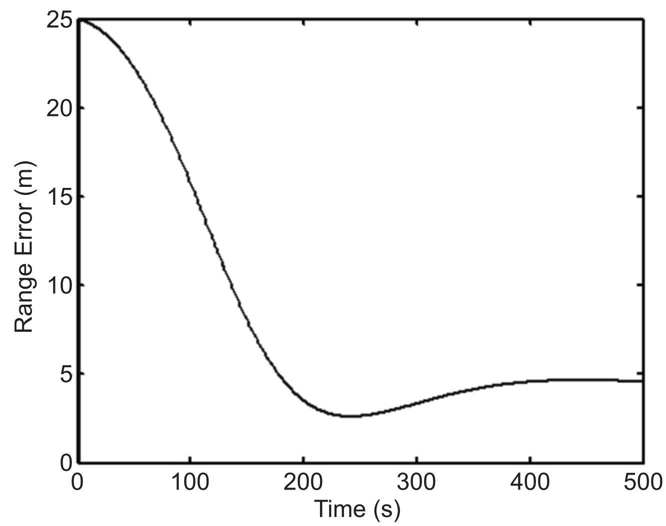

Figure 7: Error in range estimate

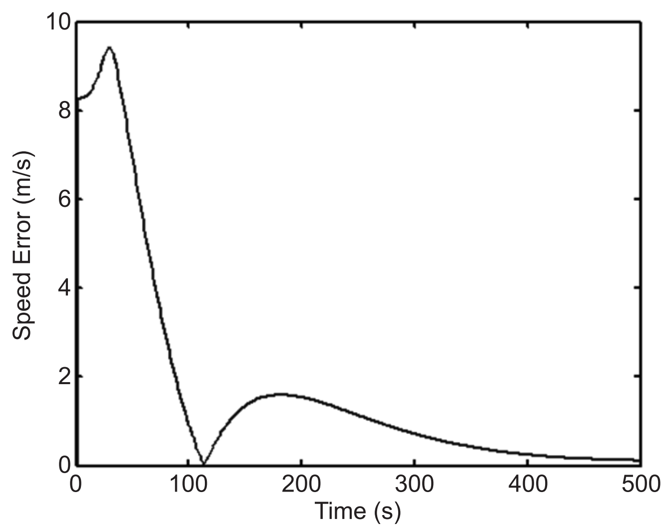

Figure 8: Error in speed estimate

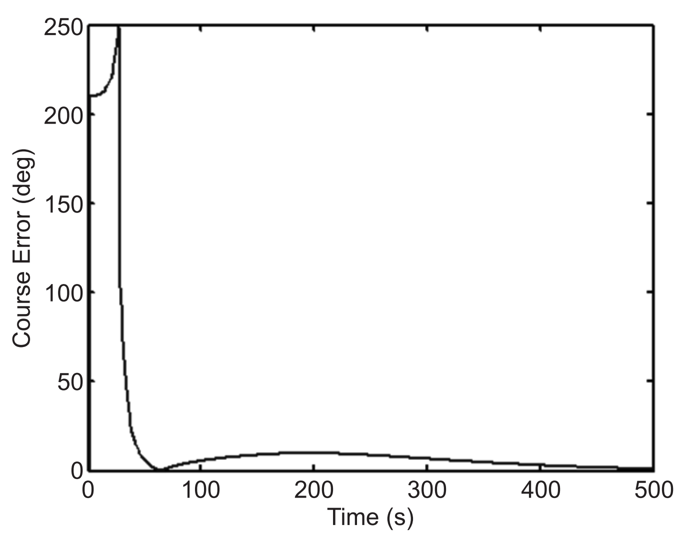

Figure 9: Error in course estimate

Based on these results, UAKF is recommended for passive target tracking and in particular, submarine to submarine scenario, when elevation measurements are also available along with bearing measurements.

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